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Wealth accumulation and aggregate demand stagnation in a two class economy with applications to the United States

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Abstract

I develop a structuralist model of long run growth and distribution with capitalists and workers. Wealth distribution offers a resolution to profit-led vs wage-led regimes. At a stable steady state, wealth must be shared by each class and the consumption of workers becomes a key determinant of capacity utilization. The paradox of thrift becomes a paradox of wealth. Capitalists’ tendency to over-accumulate has negative consequences for their own steady state wealth, through the mechanisms of demand driven economic growth. As an application, it is predicted that observed levels of wealth inequality can cost the US economy approximately $500 billion of annual output in current terms. The model offers support for public policies looking to equalize the distribution of wealth and income whilst also improving macroeconomic stability and performance.

JEL Classification: D3, E21, O4
Keywords: Wealth distribution, Economic Growth, Paradox of thrift

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1 Introduction

Does the concentration of wealth constrain aggregate demand in the long run? In this paper I study such an issue in a two class (capitalists and workers) framework. My theoretical findings are based on a demand driven approach to economic growth rather than using a traditional neoclassical production function technology. This method is able to track both patterns of wealth accumulation and their interaction with the functional and personal income distribution and the determination of the state of aggregate demand. As a model of economic growth, this puts an upper bound on wealth owned by a capitalists class as opposed to perpetual and patrimonial wealth accumulation predicted in Piketty (2014) and modeled in neoclassical theory by Stiglitz (1969).

New stylized facts have emerged over the last few decades, which contradict the Kaldor facts which formed the basis of neoclassical model of accumulation.\footnote{Solow (1956)}

- The US wealth income ratio has followed a century long U curve, returning to levels reaching as a high as 600\% in the beginning and end of the 20th century. Under the simple wealth income ratio formula that Piketty and Zucman (2014) use, a high savings rate relative to the rate of growth is identified as an important driver of this empirical pattern.

- Karabarbounis and Neiman (2014) report a global decline in the share of labor income, a finding which has been heavily used to build the story of patrimonial wealth accumulation by Piketty and Zucman. Those relying on labor are thus likely to concede their share of income to not just capital owners, but also well paid management classes whose income does not necessarily correspond to their marginal contribution to production.

- Income has been unequally distributed amongst persons over the same time frame but the distribution of wealth is even more concentrated amongst a small fraction of the population.\footnote{This finding is reflected in the data series of Saez and Zucman (2014). My independent calculations verify similar trends even when individuals are ranked by income.} By 2010, if individuals (tax units) in the US economy were ranked by wealth, then the share of
wealth attributable to the Top 1 percent was close to 40 percent. The corresponding share of income for this group was closer to 20 percent.

- Two main causes, themselves important, are identified for these trends. Firstly, there is a huge difference in savings necessary for a person in the Top 1 percent to drive their share of wealth to observed levels. Saez and Zucman find that the bottom 90th percentile of the wealth distribution have had declining (and on occasion negative savings) throughout the late 20th and early 21st century. Secondly, upward asset price trajectories since the early 1980s have allowed households with substantial assets to realize capital gains on pre-existing assets.

These facts require explanation with new models, accounting for not just distribution but also the central role played by investment and consumption demand in determining long run growth. Alternative perspectives from structuralist growth models have been around for some time, but lacked a treatment of wealth distribution between capitalists and workers. Although there were models with positive savings for each class, either wealth remained a missing variable or was latent in the underlying dynamic system.

This paper presents a new take on growth and distribution, consistently tracking wealth accumulation by both capitalists and workers. I find that the conditions for stability of the aggregate economic system are deeply linked to the steady state distribution of wealth. The degree of capitalist wealth ownership in the long run bifurcates profit-led and wage-led regimes, as they appear in the medium run. Once taken into account, aggregate demand is a strong force that offers a parsimonious explanation of the limits to wealth concentration. The paradox of thrift becomes a paradox of wealth as the medium run spirals into a long run steady state. Excess accumulation exerts downward pressure on profitability because any mechanism that crowds out the labor share ultimately crowds out the demand for goods since workers engage in the bulk of consumption. Thus capitalist thrift becomes its own poison and the tendency to outsaves the rest of the population ends up constraining aggregate demand. Capitalists only earn profits while workers (who

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3 The authors use the terms anticapital policies to describe policies which disrupted wealth accumulation in the period until the 1970s and pro capital for the reverse policies in place since then which facilitated asset price recoveries.

4 See for example Palley (2012)
engage in saving as well) have access to both components of the functional
income distribution - indirectly capitalists depend on the wages they pay out
to generate their own income. The first important result is thus that the
distribution of wealth depends on the saving behavior of each class in this
system. The second important outcome is that at the steady state, the rate
of utilization is determined independent of capitalist saving behavior.

The model has important counterparts when applied to empirical trends
in the US economy. Proxied using top fractiles of the personal wealth distri-
bution, capitalist wealth concentration constrains aggregate demand and the
annual loss in output due to wealth inequality is on the order of $500 billion.
The upshot is that policy targets which simultaneously improve aggregate
demand and distribution are clear. There is support for Keynesian interven-
tion by stimulating long run investment as well as stabilizing any downward
pressure on labor income. This paper serves as a small contribution to growth
theory and its discussion of wealth. The first part resolves some aspects of
structuralist models as well as long run limits to growth and accumulation.
While the second part is more speculative, it offers an alternate take on the
consequences of wealth inequality in the United States.\textsuperscript{5}

\section*{1.1 Related literature}
The literature on growth and distribution recognizes forces that equalize or
disequalize the distribution of wealth and income. A comprehensive and sem-
inial exercise of this flavor, is found in Stiglitz (1969). Stiglitz shows that
these forces are either income differentials which are \textit{efficiency} related or the
result of negative saving at zero income for some classes. Stiglitz' paper itself
was an entry into a sequence of US-UK Cambridge contributions on wealth
distribution in economies with capitalist and worker classes, notably start-
ing from Kaldor (1955) and followed up in seminal papers by Pasinetti
(1962) and Samuelson and Modigliani (1966). The seemingly received
convention was that the equalizing force of markets and the low return on
abundant factors would drive the wealth distribution to equality in most
cases. The literature on such models is plenty but is generally associated

\textsuperscript{5}By direct it is implied that inequality has consequences through the aggregate demand
channel, as opposed to indirect channels that relegate the effects to (quite plausible) claims
regarding the undermining of liberal democratic values
with a production function technology that allocates returns to labor and capital. In other words, problems of accumulation are restricted to the contribution due to Solow (1956). Another explanation of wealth inequality is the presence of differential rates of return to wealth - particularly a positive correlation of returns and size of wealth. The empirical finding of Saez and Zucman (2014) on the rate of return to different assets neutralizes any discriminatory behavior with regards to wealth levels. That is, rates of return on various assets are not affected by the proportion of assets held. This should not rule out theories that take into account such differential rates of return but the scope of this paper is restricted on such matters.

The treatment of savings behavior is often done through the use of models where agents undertake intertemporal optimization of lifetime resources. This paper on the other hand takes savings propensities as both given and differentiated by class (capitalists save more than workers). The notion of class is itself a substantiation of my simple differential savings claim. In the naive sense, if wages pay for subsistence and capitalist incomes are sourced from surplus value then they are naturally over and above the average level of subsistence and should be intuitively saved at a higher rate. In the not-so-naive sense, Michl and Foley (2004) propose a different take using a model where dynastic capitalists save for bequest and workers engage in lifecycle savings. Such a framework is useful in understanding the proportion of self-made vs dynastic wealth but if the purpose is to study aggregate demand and capital accumulation, then taken given rates of saving is a satisfactory first approximation. Constant saving rates are compatible microfoundations with logarithmic utility and differential savings can arise from a wide range of endogenous saving models such as wealth-in-the-utility function, pure and impure bequest, differences in the degree of patience and time preference correlated with initial wealth.

Two (not necessarily only) reasons justify the use of given rates of saving as an approximation for wealth accumulating behavior. First, as Carroll (1998) found some time ago, the simplest models that explain statistics such as wealth distribution and high savings of wealthy households are those where

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6Foley and Michl abstain from marginal conditions of the production function technology, but take the wage rate as given besides full employment capacity utilization.
accumulation of wealth is an end in itself.\textsuperscript{7} This resonates with the notion of a capitalist, the owner of means of production whose purpose, according to the political economists of the 19th century, was to accumulate wealth without limits. Secondly, differences in the propensity to save must surely be over and above simple differentials in people’s patience otherwise increases in impatience and decreases in the rate of saving may well be two sides of the same coin. That these optimizing microfoundations have limited useful behavioral insight is beyond the scope of this paper. For such reasons, it may be the case that \textit{Piketty and Zucman (2014)} relegate wealth accumulating behavior to reasons beyond the lifecycle.\textsuperscript{8}

2 A growth model with capitalists and workers

This section presents a note on a two class model of growth with inter class wealth distributions as well as its implications for the functional and class distribution of income. Capitalists own a share of wealth $Z = \frac{K_c}{K}$ and earn income in the form of profits, generated from the use of their capital in production. The stylized model considers a one good economy only\textsuperscript{9} so that wealth can only be accumulated as capital. Growth is demand driven, rather than the result of a production function technology and savings propensities are given by $s_c$ and $s_w$ for capitalists and workers respectively. To study the properties of medium run capital productivity, labor productivity is taken as predetermined.

2.1 The Pasinetti conjecture

Well before the spectre of patrimonial wealth accumulation was popularized by \textit{Piketty (2014)}, there was a long tradition of considering growth with capitalist, rentiers and workers in Classical and Keynesian modes of analy-

\footnotesize\textsuperscript{7}This argument starts with the statement that lifecycle savings and bequest don’t do an adequate job of explaining the saving behavior of the richest households.

\footnotesize\textsuperscript{8}See page 1273 in Piketty and Zucman (2014). The authors find on multiple occasions that using the lifecycle model or permanent income framework does an inadequate job of explaining the persistently high wealth income ratios in advanced economies

\footnotesize\textsuperscript{9}Implications of multi good economies are considered in the last section
Many contemporary models consider savings out of wages and profits, under the moniker of Kaldorian savings propensities. Picking up on a logical slip by Kaldor himself, Pasinetti (1962) refined this analysis by proposing that if workers and capitalists have positive propensities to save then there should be a wealth distribution rather than pure capitalist control of wealth. This is a valuable statement, because if workers are to save, then those savings are accumulating in a stock somewhere. The contribution of this analysis was enhanced by the fact that even with a distribution of wealth between two classes, it would be capitalists who would control the rate of profit.

In constructing his model, Pasinetti did not move too far away from neoclassical models - capacity was utilized at its limit and there was full employment. At equilibrium, the growth rate of capitalist wealth \( s_c r \) equaled a predetermined rate \( g \). The equilibrium rate of profit came to:

\[
r_{eq} = \frac{g}{s_c}
\]

But now consider the case where the rate of accumulation \( g \) in the economy is independent of the volume of savings and in fact responds to profitability. In a simple linear representation with an animal spirits term \( g_0 \) and a response parameter \( \alpha > 0 \) to the rate of profit, this can be written as:

\[
g = g_0 + \alpha r
\]

With \( r = \pi u \) implying the rate of profit is the product of the share of profits \( \pi \) and the output-capital ratio \( u \). The output-capital ratio or the utilization rate is the barometer of demand in the economy and labor and capital do not enter as factors into a production function technology. In the one good case, aggregate demand determines output \( uK \), by utilizing the capital stock. At equilibrium if capitalist wealth grows with the rest of the economy then:

\[
s_c r = g_0 + \alpha r
\]

\(^{10}\) See alternative distribution theories in Kaldor (1955)

\(^{11}\) Since \( \dot{K}_c = s_c \frac{rK}{K_c} = g \)

\(^{12}\) Following from: \( r = \frac{\pi}{X} \frac{u}{X} \)
Thus, $r^{eq} = \frac{g_0}{s_c - \alpha} \quad (2)$

The above expression implies that the rate of profit is non-negative as long as there is a positive animal spirits term and the response of accumulation to the rate of profit is less than the equivalent response of capitalists i.e $s_c > \alpha$.\textsuperscript{13} The larger the difference between capitalist thrift and the accumulation response, the lower the rate of profit given a fixed $g_0$. The Pasinetti case can be seen as a special case when there is no response of accumulation to profit so that $\alpha = 0$ and $r^{eq} = \frac{g_0}{s_c}$.

### 2.2 Demand and distribution with capitalist wealth

The equilibrium rate of profit in (2) can be decomposed into demand and distribution since $(r = \pi u)$ so that:

$$u^{eq} = \frac{g_0}{\pi^*(s_c - \alpha)} \quad (3)$$

Since excess demand is set to zero ($g = su$, where $s$ is the rate of saving) via the investment saving identity, the functional income distribution ($\pi^{eq}$) can be extracted.\textsuperscript{14} Assuming a uniform rate of profit across classes, savings can be decomposed into worker savings ($S_w$) and capitalist savings ($S_c$).

$$S = S_w + S_c = s_w((1 - \pi)X + rK_w) + s_c rK_c$$

Total wealth is decomposed into the wealth of workers ($K_w$) and capitalist wealth ($K_c$). With $Z$ being the ratio of capitalist wealth to total wealth, by dividing the above expression by income $X$, we get the aggregate saving rate ($s$) for the economy weighted by the income and wealth share of the two classes.

$$s = \frac{s_c Z \pi}{\text{capitalist saving rate}} + \frac{s_w (1 - Z) \pi}{\text{worker saving rate}} + s_w (1 - \pi) \quad (4)$$

Since $u = \frac{g}{s}$ therefore substituting (1) and (4), we get the expression:

$$u = \frac{g_0}{s_w + ((s_c - s_w)Z - \alpha)\pi} \quad (5)$$

\textsuperscript{13}This broadly resonates with the Keynesian stability condition viz the response of savings should exceed the response of investment

\textsuperscript{14}In canonical neoclassical models this share is taken as given by the coefficient of the capital labor ratio. In such cases, contrary to the class (sic) of models used in this paper, the supply of capital determines aggregate demand
If the economy is profit led, then utilization responds positively to the share of profits \( \left( \frac{\partial u}{\partial \pi} > 0 \right) \) which for the above expression only holds in the case where:

\[
Z < \frac{\alpha}{s_c - s_w} \tag{6}
\]

In simple terms this links both the Keynesian stability conditions and the profit led economy with the distribution of wealth. If \( \alpha > s_c - s_w \) then capitalists can in principle hold more than 100% of the economy’s wealth (with workers going into debt to them if borrowing is allowed). On the other hand, high saving rate differentials (ceteris paribus) between capitalists and workers are only compatible with low values of \( Z \).

### 2.3 Medium run

In the medium run, capitalists’ share of wealth \( (Z_{MR}) \) grows at the accumulation rate as in the demand driven Pasinetti expression (2). From (3), (4) and (5), this gives expressions for the medium run:

\[
\pi_{MR} = \frac{s_w}{s_c - Z_{MR} (s_c - s_w)} = \pi(Z_{MR}) \text{ with } \pi' > 0 \tag{7}
\]

\[
u_{MR} = \frac{g_0}{s_c - a} \frac{1}{\pi_{MR}} = u(Z_{MR}) \text{ with } u' < 0 \tag{8}
\]

To add further clarity to these expressions, note that \( \pi_{MR} \) and \( u_{MR} \) must be in the range of 0 and 1. As long as \( s_c > s_w \), both classes will exist at equilibrium and the share of wealth held by capitalists constrains aggregate demand through the utilization rate \( (u_{MR}) \). The larger the interclass savings differential, the stronger the effect of wealth concentration on the share of profits and consequently on the rate of utilization. Although the response of both distribution and demand is negative to the capitalist saving rate, the decline in \( u_{MR} \) tends to be much steeper, thereby exerting downward pressure on the rate of profit at equilibrium.

### 2.4 Long run

In the long run the aggregate economy is on a steady state if capital stock grows at the same rate as the population, i.e a constant capital stock per
person ($\kappa^*$). In the neoclassical scheme, this entails the flow-stock ratio $(su)$ adjusting itself to a natural rate of growth - bringing us back into the Pasinetti scheme. But with demand driven growth, there also exists a steady state $\kappa^*$ such that $\dot{\kappa} = 0$ as long as a model closure can be imposed using $\pi$ and its relation to $\kappa$.

The structuralist closure which relates the share of profit inversely to the employment rate ($\lambda = \frac{L}{N}$) closes the system in the long run. Thus:

$$\pi = \phi(\lambda), \phi' < 0$$
$$\lim_{\lambda \to 0} \phi(\lambda) = +\infty$$

The underlying intuition of this closure is simple - a prospering labor market increases the wage bill and compresses the share of profits. Algebraic manipulation\(^{15}\) of the above relationship yields a simpler generic form:

$$\pi = \phi(\kappa, Z)$$ (9)

The assumptions on the form of $\phi$ are not incompatible with neoclassical distribution theory,\(^{16}\) reflecting its simplicity. Since $g = g(\pi, Z)$ and as shown above, $\pi = \phi(\kappa, Z)$ therefore there exists a $\pi^*$ which brings the system into a steady state. The canonical growth and distribution system is:

$$u = u(\pi, Z)$$
$$\pi = \pi(\kappa, Z)$$
$$\dot{\kappa} = \kappa(\kappa, Z, \pi)$$
$$\dot{Z} = Z(\kappa, Z, \pi)$$

\(^{15}\)The employment rate is the ratio of the labor force ($L$) and the population ($N$),

$$\lambda = \frac{L}{X \cdot K \cdot N} = \frac{u \cdot \kappa}{\xi}$$

\(^{16}\)As an example, consider the case where $\pi = \frac{\beta}{\lambda} = \frac{\beta \xi}{u \cdot \kappa}$ where $\beta$ is a parameter. Since $r = \pi u$, this expression gives us $r = \beta \frac{\xi}{\kappa}$. The inverse relationship between $r$ and $\kappa$ under given levels of productivity $\xi$ indicates similarity to the neoclassical assumption of diminishing marginal returns or $r = f(\kappa)$, with $f' < 0$ if the elasticity of substitution is less than unity
2.4.1 Steady state

There are now six variables \((u, \pi, \kappa, Z, \dot{k}, \dot{Z})\) and the four equations given above. This system can be solved by setting the two state variables to zero i.e at \(\dot{k} = 0\) and \(\dot{Z} = 0\). It is now feasible to compute values \((\kappa^*, Z^*, \pi^*, u^*)\) in terms of the parameters \(s_w, s_c, \alpha, g_0\) at a constant rate of accumulation \(g = g^* = n\). Steady state expressions for the distribution of wealth \((Z^*)\) and the aggregate economy \((\kappa^*)\) are:

\[
Z^* = \frac{(n - g_0) s_w - n \alpha \pi^*}{\pi^* (g_0 - n) (s_c - s_w)} \tag{10}
\]

\[
\kappa^* = \xi \frac{\alpha s_w}{Z^* (g_0 - n) (s_c - s_w) + \alpha n} \varphi^{-1}(\pi^*) \tag{11}
\]

There are two results here that can be observed. First, with differential savings rates across classes the capital labor ratio \((c^* = \frac{\kappa^*}{\phi^{-1}(\pi^*)})\) is not independent of the distribution of wealth - a result that contrasts with the neoclassical-linear-savings-rate model of growth and distribution due to Stiglitz (1969).

The second important result is that the rate of utilization at steady state is constrained by high shares of capitalist wealth as long as the rate of accumulation exceeds the animal spirits term \((n > g_0)\). By substituting at the steady state, expression (5) becomes:

\[
u^* = \frac{Z^* (g_0 - n) (s_c - s_w) + \alpha n}{\alpha s_w} \tag{12}
\]

Therefore the higher the steady state share of capitalist wealth, the lower is the output capital ratio and this constraint is magnified by high savings differentials between capitalists and workers. The existence of positive steady state values relies on these two simple parametric restrictions (besides real and positive values of the parameters themselves) which as justified below:

- \(n > g_0\) i.e. steady state accumulation exceeds capital accumulation independent of the profit rate. Since at equilibrium, \(g^* = g_0 + \alpha \pi^* u^* = n\) therefore the equilibrium rate of profit \(r^* = \pi^* u^*\) is positive only when the rate of growth is greater than the exogenous accumulation rate.
• $s_c > s_w$, which is the core of differential saving in this model. In the converse case, capitalists would not exist. Given the sources of income, capitalists must *outsave* workers since they do not have access to wages.

The equilibrium for the aggregate economy ($\kappa^*$) is stable\(^{17}\) when $\phi'_\kappa < 0$ which follows from (9).

### 2.5 Self elimination from oversaving:

A paradoxical drama emerges when the economy’s wealth shares and utilization rates are evaluated at the steady state path. If the economy is on a balanced growth path so that the instantaneous and long run output capital ratio are aligned, then from (5), (7) and (12):

$$u^* = \frac{g_0}{s_w + ((s_c - s_w)Z^* - \alpha)\pi^*}$$

$$= \frac{Z^*(g_0 - n)(s_c - s_w) + \alpha n}{\alpha s_w}$$

This yields a staggeringly simple but intuitive result. The distribution of wealth at the steady state is:

$$Z^* = \frac{\alpha}{s_c - s_w}$$  \hspace{1cm} (13)

Thus the inequality condition in (6) provides the eventual clues to the distribution of wealth at the economy’s state of rest. If the economy is at a steady state then any tendency on the part of capitalists to save excessively becomes a curse on their own wealth share. The higher the relative savings rate differential (holding $\alpha$ constant) the lower is capitalist wealth relative to workers’ wealth at the steady state. *In the medium run, there is a paradox of thrift and its long run snowballing effect is the paradox of wealth - excess saving makes the equilibrium output capital ratio costly to sustain and the resulting drag limits profitability. When $s_c$ is large, capitalists limit their eventual wealth because their behavior exerts a drag on their only only source of income. When workers oversave, they exert a drag on consumption and the pressure on aggregate demand ends up restricting their eventual income and therefore their saving and wealth.*

\(^{17}\)The stabilizing condition for the aggregate economy is: $\frac{\partial \kappa}{\partial \kappa}|_{\kappa = \kappa^*} < 0$
The corollary result from this is that as long as $\alpha < (s_c - s_w)$ the two class economy is sustained and wealth is shared by both classes. Therefore this steady state expression depicts the bifurcation between a profit-led versus wage-led economy. A profit led economy can only be maintained with the existence of positive wealth for each class as long as the response of investment is less than the differential response of savings. A wage led economy, well known to be unstable\textsuperscript{18} allows capitalist wealth to exceed aggregate wealth implying either workers go into debt to finance their consumption, or if borrowing is not permitted then $Z$ forever tends to unity without ever getting there.\textsuperscript{19} Note that $Z^*$ is not defined at $s_c = s_w$ quite simply because capitalists would not exist on such a steady state path.

There are important consequences regarding workers of this stylized economy for two reasons. First note that their savings behavior controls their own steady state wealth. Secondly they completely control the steady state utilization rate since if $Z = Z^*$ then from (12):

$$u^* = \frac{g_0}{s_w}$$

Since their saving rate is low, workers contribute the most to consumption in this stylized capitalist economy. Excess worker saving in response to say, a wealth shock, therefore lowers aggregate demand through the rate of utilization. Additionally, animal spirits as metaphorically parameterized by $g_0$ must be less than the worker savings propensity to prevent over-utilization of the economy’s productive capacity. In other words, the parametric restriction for $0 < u^* < 1$ is $g_0 < s_w$.

This idea shows convergence between multiple theories of growth and distribution. On the one hand, with a low natural rate of growth there is a view popularized most recently in Piketty and Zucman (2014) and Piketty (2014) that in a dystopian future, patrimonial wealth may grow without bound (in the limit, the output capital ratio approaches zero). My analysis puts a boundary on this notion and relates it to a demand side response parameter

\textsuperscript{18}See ch 3, 4 of Taylor (2009)

\textsuperscript{19}This resonates with the anti dual results of Darity (1981) where local stability for the aggregate economy could still launch the wealth distribution purely towards capitalists. In Stiglitz (1969), a stable economy can distribute wealth purely to one class if they earn an income premium (given exogenously)
(α) which can control this tendency. In the medium run even in the unlikely case that robots are producing all output, the utilization rate is arrested at the rate of profit. In the Marxian scheme, since labor is the source of value there would be no profits without labor. As (14) shows, even with alternative schemes of accumulation, workers that comprise the labor force play a critical role in determining the long run state of aggregate demand.

2.5.1 Income inequality between capitalists and workers

To conclude this model, note that on the balanced growth path the class share of wealth equals the income share times the relative savings rate, or:

\[ Z^* = \frac{s_c}{s} y_c^* \]  

(15)

Where \( y_c \) represents the income share of capitalists. Solving at the steady state yields:

\[ y_c^* = \frac{\alpha s_w}{(s_c - \alpha)(s_c - s_w)} \]  

(16)

Figure 1: Numerical example: Capitalist income and wealth shares under stability conditions \( \alpha < s_c < 1 \) at \( \alpha = 0.2, s_w = 0.05 \)

This expression drives home the point regarding curse of capitalist accumulation. Firstly, inter-class wealth inequality will always exceed income inequality as long as one class only derives income from profits. Yet, the

\footnote{If there are no workers then the equilibrium profit share is \( \pi^* = 1 \) because no wages are paid out (the capital labor ratio \( c^* \) approaches \( \infty \) in the limit). From expression (2) this implies \( r^* = \frac{\pi^* u^*}{s_c - \alpha} = \pi^* u^* = u^* \).}
force of saving drives down the steady state share of income and wealth by lowering profitability and hence profits. The figure above gives a numerical example where capitalist wealth and income declines relative to workers as $s_c$ rises (ceteris paribus).

- **Corollary:** If at the steady state, both classes (workers and capitalists) exist then an egalitarian equilibrium simultaneously in wealth and income is impossible

Suppose capitalists are a fraction $a$ of the population. If the distribution of wealth between both classes is egalitarian then $Z^* = a$. In such a situation if income is distributed equally amongst all participants of society, then the share of capitalist income is $y_c^* = \pi^* Z^*$. This implies $a = \pi^* Z^*$. Since $Z^* = a$ therefore this means $a = a\pi^*$ which is only possible if $\pi = 1$ that is all income is profits and hence every member of society is then a capitalist.

3 Wealth distribution and stagnation in the US economy: Calibration

The results of the model discussed so far resolve a few puzzles in structuralist growth theory whilst simultaneously offering an explanation of the recently popularized secular stagnation debate.\textsuperscript{21} Consider a simple example where the wealth distribution starts out unevenly so that $Z > u$:

If $\frac{K_c}{K} > \frac{X}{K}$
then, $K_c > X$

If the stock of capitalist wealth exceeds national income by a significant margin, then the stagnationist effect is significant even in the short run. Suppose $K_c = 2X$ and the rate of profit $r$ is 5 percent. In the next period thus, capitalists earn as income $rK_c = 0.1X$. Now assume capitalists are 1% of the population and the rest are workers. 10% of total income then gets divided amongst one percent of the population while the remaining workers get 90%.

\textsuperscript{21}See Summers (2014) and Gordon (2012) for demand and supply side arguments respectively. The explanation offered by Summers is not Keynesian but rather Wicksellian, stemming from the loanable funds arguments about the Zero Lower Bound
If the working class consumes everything and the capitalist class consumes 40% then only 96% of the income forms consumption demand. The leakage from the spending multiplier contributes to capitalist wealth formation and wealth becomes even more concentrated as the effect *snowballs* into the long run. But as demand stagnates through the elimination of workers’ income share and the dynamics of distribution come into play, capitalists end up squeezing their own source of income via over-saving. In turn, this leads to the chronic underutilization of capacity and mitigates further wealth concentration.

One potential application of this model is to estimate the parameter $g_0$, since statistics on the distribution of wealth, income and savings can be observed. Note that this parameter is the exogenous or *minimum* growth in the demand driven system. At this point, this paper becomes speculative and provocative. Within the boundaries of the defined system, what rates of growth are compatible with the distribution of wealth in the United States?

In his popular paper and *TED* talk, Gordon (2012) does a subtraction exercise to assign a 0.2% long run growth rate to the US economy due to six *headwinds*. To assess a comparative case using only one headwind (wealth inequality), I calibrate the model from the previous section using observed parameters. While there are no capitalists and workers in the sense of these models, there is a break point in the personal income and wealth distribution signaling the existence of a two class economy. This is best shown in the statistical equilibrium estimates of Yakovenko and Silva (2005) - 97% of the economy is differentiated from the upper tail in the underlying process. Moving up even further, Saez and Zucman (2014) find that the top 1% of households on the wealth distribution own almost 40% of wealth and steadily increased their share of income through the kind of snowballing process described in the numerical example. Over the period 1980-2010, the authors (ibid) find a relatively stable 37% average rate of saving within the top fractile. The remaining households engaged in dis-saving to finance consumption

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22Inequality is in fact one of the headwinds, but more for its impact on democratic principles than demand effects
23There is strong evidence for a pareto power-law at the top of the distribution and exponential distribution for the rest of the population with the top being strongly correlated with booms and busts in the stock market index
24Prominently known as the neo-liberal or capital friendly era
and maintain aggregate demand. Since the shares of income and wealth are known, expression (16) can be used to calibrate $\alpha$ and make estimates of $g_0$.

A simple linear fit is applied to the relationship between Top 1% income and wealth shares, using the data in Saez and Zucman (2014). Notably, the fit reveals negative income shares for the this group at zero wealth shares, which is fully consistent with the idea of wealth related income for the capitalist class.

$$y_c = \gamma_0 + \gamma_1 Z \text{ with } \gamma_0 = -0.0498, \gamma_1 = 0.58545$$ (17)

Using (2), (16) and (17) the steady state is calibrated at $s_c = 37\%$ and $s_w =$
5%. The latter values are parametric estimates using Saez and Zucman’s data. The right panel in the figure below reveals calibrated values of $g_0$ as $Z^\ast$ goes from 0 to 1. The left panel shows simulated paths of GDP growth based on the 1980-2010 growth rate, the more recent 2010-2015 growth rate and the share of growth due to $g_0$. The starting value is $14$ trillion, in order to mimic 2010 real GDP from the NIPA estimate.

![Figure 3: Growth rates and implications for current and past wealth inequality](image)

Two points are noteworthy. Firstly, the 10 year cost of long run vs contemporary growth rates is on the order of $1.7$ trillion and significantly approximately 33% (or $0.5$ trillion) of this is due to wealth inequality shooting up from 20% to 40%. Secondly and on a related note the drag of $Z^\ast$ on the calibrated estimate $g_0$ brings about a sharp decline for plausible levels of wealth inequality. Since US wealth concentration amongst the top fractiles has historically\(^{25}\) been in the range of 20-50%, this suggests an autonomous accumulation level between 0.6 to 1.3%.

### 3.1 Policy targets

Since the model of growth and distribution in this paper tracks closely the stylized facts of the recent US economy, it is only natural to envisage which policy variables can both increase aggregate demand and improve the distribution of wealth and income. The clearest and simplest policy variable target is the worker saving rate $s_w$ which not only reduce influences wealth concentration but also controls the long run utilization rate. Thus stabilizing this behavioral parameter is crucial to macroeconomic stability - if

\(^{25}\)See estimates in Piketty (2014) and Saez and Zucman (2014)
agents’ thrift behavior is stable with a stable income. Wealth distribution can also be controlled and used to augment capacity utilization through a tax-redistribution scheme which transfers any excessive saving differential \( s_c - s_w \) towards \( g \) within the stable boundaries. However the model in this paper offers clear support for independent and not-profit-related investment as a driver of faster longer run growth. The combination of policy targets upon \( g_0 \) and \( s_w \) would be simultaneously beneficial for long run output and its distribution as income amongst the members of the population.

3.2 Other theories

There are other theories that discuss different reasons for wealth accumulation. This model is limited by the fact that it is a one good framework. Relaxing this assumption invites the possibility of price effects and capital gains. A discussion of some neoclassical variants in this framework is done in Stiglitz (2015b) where wealth can increase with much more modest increases in capital stock.

Secondly, The spectre of overaccumulation and oversaving generally emerges as dynamic inefficiency in lifecycle models with no transversality condition. Under the assumption that all households transfer their savings for investment by firms, overaccumulation analogously generates a burden on consumption due to the high capital intensity at equilibrium. High capital accumulation requires even more investment to sustain it and eats into agents’ golden rule consumption thereby causing an inefficiency in the terminology of welfare economics. The result under full certainty manifests itself in the rate of profit falling below the rate of growth. Empirically this causes an issue, since there is no single rate of return. Originally, Abel et al. (1989) found that under uncertainty, a sufficient condition for dynamic efficiency is that capital income exceed investment in every period - easily fulfilled for most advanced economies.

Recently Geerolf (2013) has littered doubts over these claims, using adjustments to land rents, mixed income and other factors showing that it need not be the case that the US economy has always satisfied Abel et al’s sufficiency condition. Geerolf expresses support for a savings glut but remains confined to these efficiency considerations because there exists (in these models)
a given rate of growth. These are important and complementary findings to my note. However the emphasis here is that long run growth can be demand determined and even possibly find a steady state path. Depending on whether the economy actually ever gets on this path, the spectre of wealth inequality can exert a deep effect.

References


