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Informational Performance, Competitive Capital-Market Scaling, and the Frequency Distribution of Tobin’s Q

Paulo L. dos Santos and Ellis Scharfenaker*

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Abstract

We develop a systemic interpretation of the functioning of capital markets that formally accounts for the observed frequency distribution of Tobin’s q, reported in Scharfernaker and dos Santos, 2015. Considering Tobin’s q as a ratio of expected total rates of return, we draw on an epistemological understanding of the tools of statistical mechanics to interpret capital markets as a competitive informational system. The strong modality in the distribution of q is taken to be conditioned by the arbitrage operations of corporate insiders. We take the persistent spread in the distribution of q to reflect the presence of obstacles to that agency, which impose an informational constraint on the operation of capital markets. This spread is also shaped by the fact that the measure of Tobin’s q effectively scales the expected returns for an individual corporation relative to those expected of all corporations. This scaling reflects aggregate measures of bullishness in investors’ valuations that insiders do not seek to exploit. In addition to accounting for the frequency distribution of q observed for the past 50 years, this interpretation points to a systemic diagnostic for the presence of speculative equity-price bubbles, and offers a new informational characterization efficiency in capital markets. According to the latter, U.S. capital markets have experienced a steady secular loss in their informational efficiency since the early 1980s.

Keywords: Tobin's q, Information Theory, Statistical Mechanics, Observational Economics

JEL codes: C46, E10, G1, L1

1 Historical and Analytical Situation

The relationship between capital-market valuations of corporate securities and the value of corporate assets has been considered by a large number of contributions spanning a broad range of analytical traditions. Despite considerable theoretical and methodological differences, those contributions can be roughly grouped around two broad lines of argumentation. The first line contends that competitive, capital-market developments operating independently of ‘fundamental’ measures...
of profitability may ensure that valuations of corporate securities deviate systematically from measures of corporate asset values. This line originates in Classical Political Economy, which sought to identify and characterize the specific distributional relations established by capital-market competition. Marx (1894) offered an early treatment of the distinction between the capital value of joint-stock companies and their market capitalization, which he thought effectively reflected the present, discounted price of future profit flows. Marx termed market capitalization ‘fictitious capital,’ and understood it to be established through competitive capital-market processes prone to destabilizing, and temporarily self-fulfilling, speculation.\(^1\) On the same broad analytical bases, Hilferding (1910) argued that market capitalization typically exceeds the value of corporate assets because the rate of return demanded by passive investors is typically below the rate of profit on productive assets. To Hilferding, this configuration opened the possibility for distinctive capital-market wealth redistributions upon the issuance of new equity, as corporate insiders could appropriate “founders’ profits” arising from the difference between the value of the corporation’s assets and the higher price paid by outsiders for equity claims on profits generated by those assets.

Contributions drawing on other analytical traditions also looked at market capitalization as a distinctive arena of competition between existing shareholders, corporate managers, and the broad investing public. Idiosyncratic U.S. institutionalist Thorstein Veblen (1978 [1904]), for instance, provided an early argument concerning the possibility that managers may profit from informational advantages they posses over other capital-market participants. Managers may be able to inflate public expectations concerning future earnings and equity prices, opening the possibility for capital-market appropriations through issuance, sale, or short sale of equity by managers.\(^2\) A broader consideration of the competitive interaction between corporate managers and less-informed outsiders was offered more recently by the influential contribution of Myers and Majluf (1984). Their “Pecking-Order Hypothesis” (POH) is grounded on the idea that less-informed outsiders may take the observed actions of managers as signals of the true condition of the corporation. Thus, management decisions to issue new shares may suggest to the investing public that shares are expensive relative to insiders' knowledge, resulting in immediate price falls. These strategic actions by investors are taken to increase the cost of equity capital, and to condition a preference by corporate managers for retention of earnings and borrowing as a source of capital. As such, they may be understood to limit the scopes for the types of capital-market appropriations by corporate insiders identified by Hilferding and Veblen.

A second line of argumentation is centered on the idea that deviations between security valuations and corporate assets should be short-lived. They define arbitrage opportunities that well-informed and well-funded capital-market participants – notably, corporate managers – can prof-

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1. While not emphasized by Marx, the central and destabilizing ‘fiction’ created by financial-asset prices is the imputation of the existence of a stock (market capitalization) on the basis of present prices that reflect only the most recent transaction, i.e. a flow.

2. See Bolbol and Lovewell (2001), particularly for a discussion of Veblen’s contributions on these issues.
itably exploit and, by so doing, eliminate. This line originated with the influential contribution of Brainard and Tobin (1968). Seeking to give form to Keynes (1936) arguments on the influence of stock-market developments on investment, the paper introduced the measure now known as “Tobin’s $q$,” the ratio between market valuations of a corporation’s liabilities and measures of the replacement cost of its assets. $q$ values in excess of unity were argued to stimulate investment, since they suggest investors are demanding rates of return that are lower than the “marginal efficiency of capital.” Investment funded by new equity issuance would result in immediate financial gains to incumbent shareholders, but also put downward pressure on profitability. Conversely, low values for $q$ were taken to discourage investment. Corporate investment was thus posited to regulate the evolution of $q$, limiting the scopes for departures from a value of one. Approaching the issue from a perspective informed by financial theory, Fischer and Merton (1984) put forward an influential version of this argument, pointing to the full array of arbitrage actions corporate managers may take whenever $q$ diverges from unity – including investment and issuance, as well as divestment and buybacks. These operations are taken to guide real and financial investment decisions and to create strong forces tending to keep Tobin’s $q$ near unity.

Following the contribution of Brainard and Tobin (1968), debates concerning the relationship between security valuations and the value of corporate assets primarily took the form of arguments in favor or against the “$q$-theory of investment.” Those debates identified at least four reasons why obstacles may prevent corporate insiders from undertaking the arbitrage actions implied by the theory. First, as argued by Lindenberg and Ross (1981), high $q$ values may reflect non-reproducible Ricardian rents conferring distinctive cost or revenue advantages to some corporations, or reflect broader forms of monopoly power. In such cases high capital-market valuations would be necessary to equate effective, risk-adjusted rates of return across corporations, but would not result in increases in investment. The presence of monopoly power is a particular instance of the broader possibility, identified in Blanchard, Rhee, and Summers (1993), that high (low) $q$ values may be persistent in any setting where average profitability is thought to be falling (rising) on capital stock. Second, corporate managers are likely to be better informed than outsiders, ensuring that investment and financing decisions are driven primarily not by the arbitrage opportunities of Fischer and Merton (1984), but by the strategic interactions and signaling considerations of Myers and Majluf (1984). Third, corporate governance concerns may keep managers from seeking to exploit all potential short-term capital-market appropriations. Those appropriations amount to transfers from new or future shareholders, to whom management will eventually be accountable. Further, if managers act in line with the interest of long-term investors, they may be unwilling to undertake

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3 An explicitly neoclassical approach to Tobin’s $q$ and investment was provided by Hayashi (1982).

4 The authors were so confident in the influence of capital-market prices on corporate operational investment that they mooted the possibility that counter-cyclical interventions by a central monetary authority include position-taking in corporate equities.

5 As argued by Blanchard, Rhee, and Summers (1993).
actions that would only directly benefit short-term shareholders ready to sell their positions.

Fourth and finally, security prices may reflect expectations of capital gains or losses on corporate securities, particularly equities. As noted by Blanchard, Rhee, and Summers (1993), there is no general theoretical expectation concerning how managers will react in such cases. Like those authors, we find it implausible that managers would act on those prices when such action was not already warranted by the prospects for profitability in a corporation’s undertakings. Persistent high or low valuations of a corporation’s securities may thus arise when they are sustained by expectations of capital gains or losses that are unrelated to the underlying manager expectations concerning the profitability of the corporation’s existing and potential projects.

An extensive empirical literature has also considered the purchase of the “q-theory of investment”. Positive associations between q and investment have been established by a number of contributions. But the hypothesis that these associations follow from a compelling, independent influence of capital-market valuations on investment, and not from a confounding influence of fundamentals like profitability or the growth rate of sales and revenues on both of these measures, has not been well supported empirically. Using average, aggregate measures of investment, market valuation, and profitability, Blanchard, Rhee, and Summers (1993) find their measure of the direct, independent role of market valuation in shaping corporate investment is positive but limited. Using firm-level, multi-year data, Mork, Shleifer, and Vishny (1990) similarly argue that once account is given to fundamentals like sales and revenue growth, the evolution of equity prices is by-and-large a “sideshow,” capable of accounting for only a very small percentage of the total variability in corporate investment. One of the serious difficulties this literature has faced follows from the sheer complexity of the manifold interdependencies involved in the determination of individual values of q, which throw up formidable problems of endogeneity that are likely impossible to address at the frequencies for which data is available.

This paper is part of a new line of work that seeks to open a broader line of inquiry into the determinations and real implications of measures of Tobin’s q. This work draws on detailed firm-level data available for U.S.-listed non-financial corporations, and on the epistemological interpretation of statistical mechanics advanced by mathematical physicist E.T. Jaynes. Specifically, the paper develops a systemic competitive account of the observed frequency distribution of Tobin’s q reported in Scharfernaker and dos Santos (2015). In that paper we established that since 1962 the end-of-year frequency distribution of \( \ln [q] \) for U.S. non-financial corporations has informationally conformed to the Asymmetric Laplace or double-exponential function, which is fully characterized by three time-varying parameters. The remarkable, long-term stability in this functional form makes it possible to use the Principle of Maximum Entropy (PME) to infer the aggregate mathematical form taken by the complex, interdependent processes shaping individual values of Tobin’s q.

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6 Notably including Bosworth (1975) and Fama (1981) and Barro (1990).


8 Scharfenaker and Semieniuk (2016) find similar results for profit rates as the result of firm competition.
This observation sets a new scientific benchmark for assessing the purchase of different accounts or theorizations of the determinations of Tobin’s \( q \), which will be successful only to the extent that they are formally – that is, mathematically – equivalent to processes generating the observed Asymmetric Laplace distributions.

The systemic theorization we develop here accounts for the observed distribution of \( \ln[q] \) as the result of competitive interactions between investors, corporations, and non-corporate liability issuers. It is based on an understanding of Tobin’s \( q \) as a ratio of two rates of return: the rate of what we term total returns on a corporation’s assets investors expect, and the total rate of return they demand on the corporation’s liabilities. We consider that the latter rate is endogenous to the process of capital-market competition, and is conditioned by the average, expected total rate of return on the assets of all corporations. Under this light, Tobin’s \( q \) appears as a measure that is subject not only to individual determinations, but to systemic, competitive ones as well.

On these bases we propose a characterization of capital markets as a competitive informational system. We consider that the strong observed modality in the distribution of \( q \) follows from the presence of potential arbitrage opportunities for (would-be) corporate insiders whenever the expected total rate of return on a corporation’s assets diverges from the average, expected total rate of return across all corporations. As a result, it is possible to think of capital markets as a system that functions as if it were trying to allocate capital across corporations so as to equate expected, effective rates of return across all assets, taking all individual measures of \( q \) to a modal value. At the same time, the persistent and formally consistent spread in the distribution of \( \ln[q] \) reflects the presence of obstacles to arbitrage. At any given point in time, this implies that there are deviations from the mode which insiders are either unwilling or unable to undo.

The persistent shape in the spread of the observed distribution allows us to draw upon Information Theory to make two inferences about the functioning of the capital-market system. First, while the obstacles to arbitrage are varied in nature, they may be understood to impose an aggregate informational or entropy constraint on the operation of capital markets. Second, under the theorization we offer in this paper Tobin’s \( q \) embodies a scaling of the expected rate of total returns on a corporation’s assets relative to the average expected total rate of return across all corporations. In line with the “Principle of Social Scaling” postulated in dos Santos, 2016, we consider that this scaling imposes a first moment constraint on the distribution of \( \ln[q] \), in a manner that reflects a measure of excess bullishness in investors’ expectations of returns on the assets of individual corporations.

Our characterization can account for the observed frequency distribution of \( \ln[q] \) from which we can make inferences about the functioning of capital markets over time by studying the time varying parameters of the distribution’s functional form. It also allows us to develop two new, systemic measures of capital-market performance. First, we put forward an indicator for the presence of equity-price speculative bubbles, based on the observation of high aggregate measures of bullishness.
on investors’ valuations of individual corporations that are accompanied by a greater unwillingness of insiders to make investment decisions based on those valuations. Second, our informational characterization of capital markets suggests a new conceptualization of the informational efficiency of capital markets as an allocative system, based on the systemic shadow cost of information, measured as aggregate foregone returns to arbitrage opportunities. These two measures yield interesting insights into the recent cyclical and secular behavior of U.S. capital markets. This includes the observation that those markets have become steadily less informationally efficient since the early 1980s, which could be the result of a steady increase in insiders’ reluctance to use investor valuations to inform their investment decisions, or of an increase in the significance of monopolistic investment behavior by U.S. corporations.

The paper is organized as follows. Section two presents the observed facts, the explanatory burdens they pose, and the systemic nature of the approach we take toward meeting them. Section three discusses the central analytical elements of our theorization. Section four develops the formal framework and lays out the salient inferences it supports. Section five concludes.

2 The Facts, Explanatory Burdens, and Systemic Approaches

This section sets the stage for the framework developed in this paper by reporting the basic facts concerning the frequency distribution of Tobin’s $q$, as recently established in Scharfernaker and dos Santos, 2015, identifying the explanatory burdens posed by the form taken by the observed distributions, and by explaining and motivating what we mean by a systemic theorization of capital-market behavior that can account for those distributions.

2.1 The Data and Distribution

Scharfernaker and dos Santos, 2015 examined the frequency distribution of what may be called the observable measure of $q$, defined by the ratio of the market value of equity and the book value of debt to total assets, calculated with data from the Compustat fundamentals annual database for North America spanning the years 1962-2014.9 The summary statistics for the pooled dataset are presented in Table 1.

The end-of-year frequency distributions of $\ln [q]$ exhibit strong modalities and a remarkably consistent degree of organization. Figure 1 shows on a semi-log probability scale the individual, end-of-year distributions for a measure of this variable centered about its modal value $\ln [q_d]$.

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9 Using updated data from the Wharton Research Data Services (WRDS) made available from the University of Missouri, Kansas City, we discard government (SIC classification 9100-9999) and financial services (SIC classification 6000-6800) as well as the upper and lower quantiles due to extreme outliers. We use the complete-cases dataset for our variables of interest in order to deal with missing values. Our final dataset contains 230,594 observations, or on average 4351 observations a year. Market value is calculated as the product of common share outstanding (CSHO) and price close annual fiscal (PRCC_F) plus long-term debt, (DLTT) and short-term debt (DLC).
denoted by \( y = \ln[q] - \ln[q_d] \),

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Max</th>
<th>Obs.</th>
<th>Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln[q] )</td>
<td>-1.375</td>
<td>-0.300</td>
<td>0.075</td>
<td>0.232</td>
<td>0.621</td>
<td>3.642</td>
<td>230,594</td>
<td>20,166</td>
</tr>
</tbody>
</table>

Figure 1: Stacked empirical densities of centered \( y \) on a log probability scale, 1962-2014, with maximum likelihood fitted Asymmetric Laplace distributions (dashed lines). Each point corresponds to the center of a histogram bin and each shape represents a particular year.

We contend that these distributions of \( \ln[q] \) embody a type of “statistical equilibrium,” defined by their consistent conformance to an Asymmetric Laplace (AL) distribution, with time varying parameters.\(^{10}\)

\(^{10}\)The information distinguishability statistic developed by Soofi and Retzer (2002) confirms that the estimated AL distributions can account for about 98 percent of the informational content of the observed frequency distributions.
The functional form of the AL distribution can be parametrized in several different ways, including as a piecewise distribution about its mode \( c = ln [q_d] \). As shown in Scharfernaker and dos Santos (2015), the AL distribution is the solution to a maximum entropy program where the infra-modal mean deviation from the mode is constrained to \( M_L \), and its supra-modal converse to \( M_H \). In this case the distribution of \( x = ln [q] \) could be expressed as,

\[
\mathcal{AL}[x; c, M_L, M_H] = \frac{1}{(\sqrt{M_H} + \sqrt{M_L})^2} \begin{cases} 
  e^{\frac{(x-c)}{\sqrt{M_H} + M_L}} & \text{if } x \leq c \\
  e^{\frac{(x-c)}{\sqrt{M_H}M_L + M_H}} & \text{if } x > c 
\end{cases}
\]

(1)

The Asymmetric Laplace distribution is presented in Figure 2 alongside a Normal distribution on both linear and logarithmic probability scales. The semi-log scale facilitates the identification of (double) exponential distributions, as their fat tails make them appear linear. The AL distribution is distinguished by its sharp peakedness around the mode, fat tails, and skewness, which implies the mean, median, and mode do not coincide with one another. The AL distribution reduces to the symmetric Laplace distribution when \( M_L = M_H \), has a right skew when \( M_H > H_L \), and a left skew when \( M_L > M_H \).

\[
\begin{array}{c}
\mathcal{N}(0,0.2) \\
\mathcal{AL}(0.0.1,0.3)
\end{array}
\]

Figure 2: A comparison of the Normal distribution, with \( \mu = 0 \) and \( \sigma = 0.2 \), to the Asymmetric Laplace distribution, parameterized as Eq. 1, with location parameter \( c = 0 \), and shape parameters \( M_L = 0.1 \), and \( M_H = 0.3 \).

The maximum likelihood estimates for the three parameters in definition 1 are plotted in Fig. 3. It is notable that the modal value \( c \) varies over time, typically over values just below zero, and that the infra-modal mean deviation, \( M_L \), is considerably more stable than its supra-modal converse, \( M_H \).

\[ \text{over the entire period.} \]
Figure 3: Maximum likelihood fits and error bars of $M_L$, $M_H$ from (Eq. 1) corresponding to the piecewise mean constraint (left) and modal $c$ (right).

In this paper we use a more conventional parametrization, in line with Kotz, Kozubowski, and Podgórski (2002), who characterize the Asymmetric Laplace distribution as the entropy-maximizing distribution for a continuous random variable $x$ on a non-vanishing support $(-\infty, \infty)$ with a constrained first moment, $E[x] = c_1 \in \mathbb{R}$, and absolute first moment, $E[|x|] = c_2 > 0$,

$$AL[x; c, \sigma, \kappa] = \frac{\kappa}{\sigma(1 + \kappa^2)} \begin{cases} e^{\frac{(x-c)}{\sigma \kappa}} & \text{if } x < c \\ e^{-\frac{\kappa(x-c)}{\sigma}} & \text{if } x \geq c \end{cases}$$

(2)

where $\kappa = \left(\frac{c_2 - c_1}{c_2 + c_1}\right)^{1/4}$, $\sigma = \frac{1}{2} \left(c_2^2 - c_1^2\right)^{1/4} \left(\sqrt{c_2 + c_1} + \sqrt{c_2 - c_1}\right)$, and $c$ is a location parameter.

Following this parameterization we plot the time evolution of $c_1$ and $c_2$ in figure 4 as well as the maximum likelihood estimates of $\kappa$ and $\sigma$.

Figure 4: Left: Time evolution of $c_1$ and $c_2$. Right: Maximum likelihood estimates of $\kappa$ and $\sigma$. 
A casual examination of these findings already affords a broader perspective on hitherto debates concerning Tobin’s $q$. The strong modality in its distribution hints at the presence of strong regulations toward a particular value in the vicinity of $q = 1$. But its persistent and formally consistent spread also hints at important and formally regular counter-regulations. In what follows we offer a first attempt to identify the nature of these tendencies and what they may tell us about the content of capital-market competition.

2.2 Explanatory Burdens and Methodological Approach

By starting with observation of the actual frequency distribution of observable Tobin’s $q$, we are following the conventional, inductive approach of observational science. Instead of starting by positing the existence and robustness of first principles of individual behavior and using them to deduce micro-level “laws of motion” that may or may not be refuted by empirical evidence, we take our observation to define the explanatory burden for successful theorization. The observation of an informationally persistent pattern of organization in the distribution of observable $q$ ensures this approach is distinctively fruitful relative to previous work on this issue in at least two ways.

First, it affords a new perspective on extant debates concerning the “correct” measure of $q$.\(^{11}\) Two possibilities arise. Observable $q$ may effectively be a proxy for a more behaviorally relevant “correct” measure of $q$, in which case the formal consistency in the regulation of observable $q$ merely reflects the fact that differences between its individual measures and “correct” measures are informationally insignificant. Alternatively, it may be that it is observable $q$ that is directly regulated by capital-market competition in the formally consistent ways revealed by its distribution. In what follows we develop an account that can be applied to either possibility.

Second, the persistent pattern of organization in observable $q$ also allows us to rely upon the Principle of Maximum Entropy (PME) to infer the aggregate, mathematical form taken by the determinations of its individual values. As explained in Scharfernaker and dos Santos, 2015, the Asymmetric Laplace distribution is the maximum-entropy distribution for a quantity subject to two systemic constraints—one on its expected value and to another on the expected value of its absolute value. Put differently, if the only coherent regulations of individual values of a quantity effectively boil down to these two aggregate constraints, Asymmetric Laplace frequency distributions will be generated by the greatest multiplicity of micro-level states compatible with the constraints. Conversely, consistent observation of an Asymmetric Laplace frequency distribution is convincing evidence that the determinations of that quantity are formally equivalent to these two aggregate

\(^{11}\)There is a significant literature discussing relative merits of using different observable, imputed, or otherwise estimated measures in both the numerator and denominator of Tobin’s $q$. In the numerator, debates have dealt with the use of readily available firm-level data on the book value of debt versus attempts to estimate market value of debt; as well as with attempts to estimate the total market value of preferred shares. In the denominator, debates have focused on the use of book or estimated replacement value of assets. See Lindenberg and Ross, 1981, Lewellen and Badrinath, 1997, and Whited, 2001, for instance.
The theoretical task at hand thus consists of developing an explanation of capital-market functioning that can account for the three parameters defining the observed Asymmetric Laplace distributional forms: The mode, the first-moment constraint, and the expected absolute value constraint. Such an account must also be able to accommodate and yield insights into three observed features in the distributions: The mode of $y_i$ almost always takes on near-zero negative values; the distribution has a consistent positive skew; and the infra-modal mean is remarkably stable over time.

In what follows we develop a systemic account of capital-market functioning that meets this explanatory burden. Instead of offering a detailed characterization of the individual, micro-kinetic behavior of investors, managers, and arbitrageurs, we consider capital markets as a single system whose behavior may be inferred from the shape of the frequency distribution of observable $q$. We contend that fundamental competitive interdependences in capital markets ensure that the behavioral significance of any characteristic or course of action of a corporation is inherently systemic: In competitive capital markets, improvements in the prospects for returns in one corporation are of necessity comparative dis-improvements in the prospects of all other corporations; investors may not react to any given set of actions by management of a corporation in isolation from the actions being undertaken by other managers; and their decision effectively to become more bullish on a particular corporation may well reduce their ability to be similarly bullish on others; etc. The account we develop below relies on the existence of competitive behavior, arbitrage, and these fundamental interdependences between individual corporations. We contend that the systemic processes it describes are robust across a variety of fine-grain or micro-kinetic details of individual capital-market behavior.

It may be objected that the resulting approach is not in line with extant approaches to analysis of capital-market behavior. Such an objection would be entirely correct. But the relatively simple systemic, competitive account developed below is hitherto the only explanation of the functioning of capital markets that can account for an observational set with $\sim 10^5$ points effectively covering the population of U.S.-listed private, non-financial corporations over a period of 50 years. And it does so without recourse to strong and empirically doubtful assumptions about the functional form taken by individual beliefs concerning returns, or about the robustness of the informational association between returns in different securities. Of course it may be possible to relate the account below to more conventional, individualist theorizations that grapple with the fine-grained, micro-kinetic structure of the capital-market system. While posing significant conceptual and empirical difficulties, such efforts are not necessary to account for the processes driving the persistent form of organization in the frequency distribution of $y$. Neither are they necessary for us to construct and motivate the systemic measure of equity-market price bubbles, and the informational measure of capital-market allocative efficiency developed below.
3 A Systemic, Competitive Interpretation

This section develops the analytical foundations of the systemic account of the functioning of capital-markets consistent with the empirical distribution of $q$. First and most generally, the account is founded on a characterization of Tobin’s $q$ not as a ratio of stocks, but as an implicit ratio of expected rates of total returns in capital markets. As such, $q$ is a behaviorally relevant moment in the dynamic processes of capital-market competition. Systemic regularities and interdependences in those processes sustain the high measure of functional stability observed in the distribution of $q$.

We take the strong modality in the distribution of individual measures of $y_i = \ln[q_i] - \ln[q_d]$ to reflect the exhaustion by corporate insiders of arbitrage gains that may be present whenever the rate of total return on assets investors expect from a corporation diverges from the total rate of return they expect from all corporate assets. The exact location of the mode reflects, in turn, the fact that all corporate liabilities compete with the liabilities of other issuers for investment. As a result of that competition, investors apply discounts in their valuations of expected total returns on corporate liabilities relative to returns on other, potentially safer investments.

We account for the consistent spread in the distribution on the basis of two processes, each corresponding to one of the aggregate moment constraints defining the Asymmetric Laplace distribution. First, the absolute value of $y_i$ appears as a measure both of allocative capital-market inefficiencies, defined relative to investors’ expectations, and of returns to arbitrage operations foregone by insiders. This allows us to contend that competitive capital markets function like a system that is attempting to minimize this quantity across all assets. In this task, we consider that the system faces informational or entropy constraints defined by the “obstacles” to arbitrage discussed above. Such a constrained minimization is formally equivalent to the presence of an aggregate constraint bearing on $E|y_i|$. Second, in our interpretation, $y_i$ is in fact a scaled measure of expected total returns for an individual corporation relative to the average expected rate of total returns for all corporations. Drawing on the “Scaling Principle” first articulated by dos Santos, 2016, we show conditions under which this scaling is formally equivalent to a first-moment constraint on the distribution of $y_i$. Significantly, in our account the form taken by this scaling hinges on measures of investor bullishness on individual corporate returns, which shape the observed right-skew in the distribution of $y_i$.

3.1 Expected Returns, Competition, and the Modality of $q$

Perhaps the most useful and behaviorally relevant way to understand Tobin’s $q$ is as an implicit ratio of two unobservable, forward-looking rates of return expected by investors: the effective, total expected returns to liability holders generated by a corporation measured relative to its assets, and the effective opportunity rate of return demanded by all investors. By total expected returns we mean present valuations of all expected gains accruing to liability holders. This includes expected
future cashflow yields on present assets, expected capital gains, as well as valuations of future operational and capital-market gains on projects the corporation may or may not undertake.\textsuperscript{12} We also take total returns to include all risk adjustments effectively made by investors in their valuations, including risk premia and any possible hedging or broader portfolio-management gains associated with the expected returns of any given corporation.

Formally, let the rate of total returns on a corporation’s assets expected by investors be given by $\rho_i$. Given that all conceivable gains to liability holders are included in this definition, and that all investors can always hold assets with non-negative total real yields, we can be assured that $\rho_i$ is always non-negative for active corporations. This risk-adjusted expected rate of return must give liability holders their opportunity rate of return, $\bar{r}$. Thus, Tobin’s $q$ is given by,

$$q_i = \frac{\rho_i}{\bar{r}}$$

We also suppose that the rate of total returns expected by outside investors is not necessarily equal to the rate expected by managers. Since it is extremely unlikely that investors posses better knowledge about a corporation than its managers, we take management’s expected rate to total return to be the best estimate of the prospects of a corporation. Let this rate be given by $\pi_i$. It is reasonable to presume that its value is dynamically conditioned by developments in output-market competition, and by the investment decisions of managers.

The rate of total return expected by investors contains a possible adjustment by a factor $\beta_i$, defined as a multiplicative measure of investor “bullishness” over and above the rate expected by managers. While we have little \textit{a priori} knowledge about its mercurial determinants, we suppose that they include possible reactions by investors to the actions of managers. Formally, we are supposing that,

$$\rho_i = \pi_i \beta_i$$

Consider now the “opportunity” rate of return demanded by investors. We argue that this rate is determined competitively. It is defined by the average rate of return investors expect to receive across all corporate assets. Like the expected total returns on the assets of individual corporations, we suppose that this average rate of return consists of the average expected rate of return expected by managers, denoted by $\bar{\pi}$, and by a multiplicative adjustment for investors’ expectations $\beta$. We also suppose that in their valuation of all corporate liabilities, investors apply a discount factor $\alpha$.

\textsuperscript{12}In the latter point we follow Roll and Weston, 2008, who argue that this inclusion of valuations of possible future projects (say as call options) ensures that observable measures of average $q$ have a closer relationship to unobservable measures of marginal $q$. 

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reflecting the competitive, risk-return prospects of alternative investments in non-corporate assets. It is important to note that in this specification, the opportunity rate of return is endogenous to capital-market competition, as it will be conditioned by measures of the expected rate of total return of each corporation. This dependence conditions the emergence of the systemic regulations on individual values of \( y_i \) responsible for its frequency distribution.

Under these suppositions \( q_i \) becomes,

\[
q_i = \frac{\pi_i \beta_i}{\pi \beta \alpha}
\]  

(5)

Here Tobin’s \( q \) appears as a measure of expected excess returns on the assets of a corporation relative to the expected returns on all corporate assets, as perceived by investors. We take it to be a behaviorally relevant quantity in the processes of capital-market competition among corporations, whose collective prospects define \( \bar{\pi} \), and between all of them and outside investors, whose beliefs define \( \beta \), the \( \beta \)'s, and \( \alpha \). This leads to a characterization of the modality in the distribution of \( y_i \) as the result of competitive, arbitrage interventions by corporate insiders.

Present or would-be managers of a corporation, “insiders” henceforth, are capable of taking and disposing of positions in corporate assets. As such, they can generate returns by acquiring or disposing of assets for a corporation whenever differences between the expected total rate of return on those assets diverges from the cost of capital. We consider the determinations of that cost systemically. Corporations are in competition with each other for investment, ensuring that cost is in line with the expected total return investors expect across all corporate assets. There will thus be arbitrage opportunities for insiders whenever \( \pi_i \beta_i \neq \bar{\pi} \beta \). Whether or not insiders take advantage of this opportunity depends on whether they are willing and capable of doing so. We suppose that if they are, their actions will tend to reallocate capital and bring about changes in \( \pi_i \) and \( \beta_i \) that ensure \( \pi_i \beta_i = \bar{\pi} \beta \). In the present interpretation, we consider that such actions condition the modality in the distribution of \( y_i \).\(^{13}\)

Formally, arbitrage operations by insiders along these lines ensure that the modal value \( q_d \) is given by,

\[
q_d = \frac{1}{\alpha}
\]  

(6)

The value taken by the mode reflects the bullishness present in investors’ valuation of aver-

\(^{13}\)It is entirely possible to postulate a slightly more complex variant of the situation considered here, where modality is also explicitly conditioned by arbitrage operations by investors whose knowledge is in line with that of insiders. Such an account would not, however, add anything fundamentally new to the broad findings we can motivate here on the basis of the simpler account we are developing. It also requires a more involved treatment of two variables, \( \pi_i \) and \( \beta_i \) and moments of their joint distribution.
average expected total rates of return across all corporations, \( \alpha \). Empirical observation suggests this measure is typically greater than one, suggesting investors are in effect discounting expected total rates of return when when making valuations of the prospects of individual corporations. This is plausible in any setting where large stocks of low-risk assets are available to investors and there are significant measures of risk aversion.

It bears noting that our account considers that capital-market competition is effectively a two-fold process, pitting corporate issuers of liabilities against each other, and all of them collectively against non-corporate issuers of liabilities. The location of the mode in the distribution of \( y_i \) reflects the latter competitive moment. As we will now see, the shape of that distribution reflects the former, inter-corporate competition.

Under the characterizations developed above there is a fairly natural economic interpretation of \( y_i \), which emerges as,

\[
y_i = \ln \left( \frac{q_i}{q_d} \right) = \ln \left( \frac{\pi_i \beta_i}{\pi \beta} \right)
\]

We may understand \( y_i \) as a forward-looking measure of the total returns on a corporation’s assets expected by investors, scaled to the more general rate of total return they expect on all corporate assets. This measure may be understood to contain important information about the allocative performance of capital markets, defined not in general, but relative to investors’ expectations; and about the presence of possible arbitrage opportunities. It should thus not be surprising that competition strongly regulates its systemic distribution.

Modal values \( y_d = 0 \) can be understood as “efficient,” relative to investors’ expectations, in that they correspond to a state in which no reallocation of capital between the individual corporation and all other corporations can improve on the average rate of total return expected by investors.\(^{14}\) Conversely, off-modal values may be understood as “inefficient,” in that reallocations of capital could result in an increase in that average rate of return. Further, at off-modal values, insiders are also foregoing potential arbitrage returns. Notably, \( E |y_i| \) offers a measure of both the social and insider losses posed by a particular valuation. It is a measure of the rate of return insiders could realize if they were to long the undervalued and short the overvalued assets defining Tobin’s \( q \). It also offers a measure of the possible aggregate gains in total returns arising from such arbitrage operations. Self-interest may thus be understood to be potentially capable of generating allocations that are, in the specific sense motivated here, efficient. The strong modality in the distribution of \( y_i \) suggests capital markets succeed in significant measures in doing this, relative to the beliefs of investors.

\(^{14}\)This capital-market “efficiency” translates into allocative efficiency relative to the knowledge state of better informed managers if capital-market pricing ensures that \( \beta_i = \beta \). Arbitrage by insiders will yield efficient allocations only when investors’ expectations, even when incorrect relative to insiders’ knowledge, do not distort relative prices.
At the same time, the persistent spread in the distribution of $y_i$ points to the presence of many obstacles to such outcomes. Insiders may be unable to take advantage of possible arbitrage opportunities. More interestingly, they may be unwilling to act in order to collect short-term gains that may be offset in the long run, including because they do not share the investor beliefs sustaining security valuations. The next section offers accounts of how these obstacles can be understood to generate the aggregate moment constraints the formally characterize the persistently observed shape of the spread in the frequency distribution of $y_i$.

### 3.2 Systemic Behavior and Moment Constraints

In order to account for the observed spread in the distribution of $y_i$, we consider capital markets as an informationally constrained system that effectively functions as to minimize private and social losses due to allocative and pricing inefficiencies, measured relative to investors’ beliefs. On this bases we can account for the aggregate constraint on $E|y|$. We also note that like all measures of Tobin’s $q$, $y_i$ is a quantity that reflects the competitive scaling by the capital-market system of total rates of return on the assets of individual corporations expected by investors relative to the average total rate of return on all corporate assets. In line with the argument advanced by dos Santos, 2016, we show how this type of scaling may be formally equivalent to an aggregate first-moment constraint on $y_i$. We outline these arguments in turn.

We consider capital markets as a system that functions as to minimize the aggregate or average measure of $E|y|$. In competitive markets we should expect this quantity to be kept as small as possible in the face of the various obstacles to arbitrage facing those agents. It is also true that $E|y|$ is a measure of average aggregate social losses due to misallocations of capital, relative to investors’ expectations. It is thus conceivable that the self-interest of arbitrageurs results in capital allocations that are in some way socially desirable. If arbitrage by insiders was always instantaneous, costless, possible, and desirable, its exercise in competitive capital markets would generate frequency distributions for Tobin’s $q$ that would approximate a degenerate, Dirac Delta distribution at $q_d$. This would correspond to a “perfect,” or more appropriately, Platonic state in capital markets.

But such an outcome is an abstraction, as attested by the observed residual indeterminacy in the distribution of $q$. It is reasonable to conclude that capital markets will always exhibit some measure of inefficiency. Valuations at which $y_i = 0$ will be observed for a variety of reasons. Even if equity prices quickly come to reflect information that conditions the investment behavior of insiders, actual investment or divestment projects take time.

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15 An Austrian political economist may rightly point out that the power of competitive market processes is not defined by the absence of such inefficiencies, but by the ability of those processes constantly to reduce them in light of new facts.

16 It is sensible to conclude, thus, that the observation of price movements pre-dating investment decisions with which they have an informational association cannot be taken as _prima facie_ evidence that insiders are “responding”
It is also possible that valuations at which \( y_i \neq 0 \) persist because insiders are not planning to act to undermine them—either because they cannot or because they do not find it in their long-term interest to do so. There may be fixed costs and threshold effects in the exercise of arbitrage operations. More interestingly, valuations may reflect the presence of the obstacles to arbitrage discussed above. Non-reproducible Ricardian rents, the presence of monopolistic returns in output markets, strategic and governance constraints will all inhibit insiders from undertaken arbitrage operations that will tend to move valuations to levels at which \( y_i = 0 \). It is also possible that insiders do not think that taking advantage of investor expectations they do not share is in their long-term interest. This may include situations where valuations are conditioned by investor fads or fashions, or by rational or irrational investor expectations of capital gains or losses. We will return below to this latter possibility and its likely purchase over speculative cycles.

While these obstacles to arbitrage are diverse in form and content, we argue that the observed distribution of Tobin’s \( q \) suggests that capital markets function as if they collectively imposed an informational constraint on the ability of the capital-market system to informationally organize investors’ expected rates of return, prices of corporate liabilities, and, thus, the distribution of \( q \). Formally, we posit that all of these obstacles effectively impose, at any given point in time, a lower-bound on the entropy of the distribution of \( y_i \). Under this supposition, any decreases in the capacity and willingness of insiders to undertake arbitrage operations impose a higher lower-bound on the entropy of the distribution of \( y_i \).

Given a minimum entropy, competition will effectively allocate the system’s scarce informational or organizational capacities so as to minimize the total measure of foregone returns on arbitrage operations. Formally, we are contending that the informational “problem” effectively solved by the competitive capital-market system may be formally represented by,

\[
\min \int_{\mathbb{R}} |y| f(y) \, dy \quad \text{subject to } \int_{\mathbb{R}} f(y) \, dy = 1
\]

\[
H[y] = - \int_{-\infty}^{\infty} f[y] \ln[f[y]] \, dy = S
\]

Where \( H[y] \) denotes the entropy of a distribution of \( y_i \), and \( S \) is the binding minimum-entropy constraint. As will be clear below, this interpretation of the functioning of capital markets is formally the dual to the imposition of an aggregate constraint on the absolute value of \( y \).

The first-moment constraint bearing on the distribution of \( y_i \) can be understood to follow from

\[17\] The idea of an entropy constraint can also be found in Sims (2003), who applies it to the stimulus-response behavior of individuals. Here we apply this idea to the effective functioning of an economic system.
the competitive process of capital-market scaling that is embodied in that variable. It should be evident from 7 that the variable \( y_i \) represents a scaling of the rate of total return on a corporation’s assets expected by investors relative to the average rate of total return they expect across all corporate assets. As shown in dos Santos, 2016, variables embodying such scaling will under a variety of conditions be subjected to a first-moment constraint, in a manner that is formally analogous to a “conservation” principle. In the present case, an increase in an individual measure of \( \pi_i \) brings about a decrease in the valuations \( y_j \) of all other \( \pi_j \) by increasing the average expected total rate of return \( \bar{\pi} \). Similar relationships would exist if \( \beta \) is an averaging function of the individual values \( \beta_i \). Systemic interdependences created by this type of valuation can be formally equivalent to a first-moment constraint on the distribution of a variable like \( y_i \).

The frequency distribution of \( y_i \) suggests the capital-market system behaves as if that measure is indeed subject to such a constraint. Formally, the system functions as if,

\[
E[y_i] = E\left[\ln \left( \frac{\pi_i \beta_i}{\bar{\pi} \beta} \right) \right] = c_1
\]

Under the interpretation developed here, the presence of such a constraint reveals information about the systemic determination of investors’ excess expectations measures given by \( \beta \) and by the values of \( \beta_i \). Using a first-order Taylor approximation in 9 we obtain,\(^{18}\)

\[
E[y_i] \approx E\left[\frac{\beta_i}{\beta} \right] - 1
\]

The presence of an aggregate, first-moment constraint on \( y_i \) is seen here as equivalent to the presence of a first-moment constraint on the measure of investors’ excess bullishness concerning the total rate of return they expect for individual corporations relative to their excess bullishness concerning the average total rate of return they expect across all corporations. In line with the Principle of Social Scaling, such a constraint may be understood to suggest that \( \beta \) is in fact an averaging function of the individual \( \beta_i \) values. Economically, this would represent a state where the market-wide measure of investors’ excess bullishness in their total-return expectations is an average measure of their excess bullishness on the returns of individual corporations. Formally, this may arise if, in effect, the formation of investors’ expectations ensure that,

\[
\beta = \frac{1}{a} E[\beta_i]
\]

\(^{18}\)This is a reasonably good approximation for the near-zero \( y_i \) values accounting for most of our observed distributions. See Appendix A for the derivation of more general, second- and higher-order approximations.
In which case \( c_1 = a - 1 \). Here the extent to which average excess bullishness represents an under- or over-weighted averaging of bullishness of the returns of individual corporations determines the value taken by \( c_1 \) and the skew in the distribution of \( y_i \). As we overwhelmingly observe distributions with a right skew, this line of interpretation would suggest that investors’ bullishness on individual returns conditions their bullishness across all returns, but in a way that ensures the later is on average smaller than the former. Put differently, investors are on average more bullish or exuberant concerning the returns of individual corporations than they are concerning the returns on the market as a whole.

4 The Formal Framework and Salient Inferences

It is now possible to develop the formal account of capital-market functioning offered by this paper. The resulting framework sustains a number of interesting inferences concerning the functioning of capital markets from the observed distribution of \( y_i \).

4.1 The Formalism

Capital markets are taken to constitute a system that is effectively attempting to minimize the systemic measure of foregone returns to arbitrage operations involving off-modal corporations. The system does this in the face of an entropy constraint, conditioned by the (in)ability and (un)willingness of insiders to undertake timely arbitrage operations, which can be understood as representing the perceptual or control limitations that individual investors face when dealing with their own behavioral determinations or market environment signals. It is also subject to a first-moment constraint conditioned by the average measure of excess bullishness present in the scaling by investors of individual expected total rates of return relative to the expected market rate of total returns.

Formally, then, the capital-market system effectively functions as if it were solving the programming problem,

\[
\begin{align*}
\min_{f[y]>0, y \in \mathbb{R}} \int_{-\infty}^{\infty} |y| f(y) \, dy \\
\text{s.t.} \int_{-\infty}^{\infty} f(y) \, dx = 1 \\
H[y] = -\int_{-\infty}^{\infty} f[y] \ln[f[y]] \, dy = S \\
\int_{-\infty}^{\infty} y f[y] \, dy = c_1
\end{align*}
\]

(12)

It should be obvious that this problem is equivalent to the problem of maximizing a Lagrangian
function defined on the frequency distribution, the given values for the distribution’s first moment and entropy, and the three Lagrange multipliers,

\[
\mathcal{L}[f[y], \mu, \gamma; S, c_1] = -\int_{-\infty}^{\infty} |y| f[y] dy + \mu \left( \int_{-\infty}^{\infty} f[y] dx - 1 \right) - \\
\lambda \left( \int_{-\infty}^{\infty} f[y] \ln[f[y]] dy - S \right) + \gamma \left( \int_{-\infty}^{\infty} y f[y] dy - c_1 \right)
\]

(13)

As shown in detail in Appendix B, this problem is solved by the Asymmetric Laplace function,\(^\text{19}\)

\[
f[y] = e^{-S} \begin{cases} 
1 - S & y < 0 \\
1 - S + \frac{2y}{c_1} & y \geq 0
\end{cases}
\]

(14)

The minimum value taken by \(E|y|\) will be given by,

\[
E|y| = \frac{1}{2} \left( e^{S-1} + c_1^2 e^{1-S} \right) = c_2
\]

(15)

Note that at this optimal distribution the Lagrange multipliers \(\lambda\) and \(\gamma\) take on their optimizing values,

\[
\gamma^* = c_1 \ e^{-S}
\]

(16)

\[
\lambda^* = \frac{1}{2} \left( e^{S-1} - c_1^2 e^{1-S} \right)
\]

(17)

These values for the multipliers can be interpreted in line with conventional practice in Economic analysis as shadow prices or costs, measured as an average foregone rate of return to arbitrage operations. Along these lines, the optimizing multiplier on the entropy constraint, \(\lambda^*\), offers an interesting, informational measure of the allocative performance of capital markets, defined relative to investors’ expectations. The multiplier measures the marginal reduction in foregone arbitrage returns that would result from a marginal decrease in the measure of entropy in the distribution of \(y_i\). Put differently, it measures the average marginal gains that would accrue to insiders if they were to become marginally more aggressive in pursuing arbitrage operations, effectively reducing the system’s entropy. We contend that \(\lambda^*\) is thus also providing us with a measure of the marginal losses that insiders would incur if they were indeed marginally to reduce the entropy of the system. It is a pecuniary measure of the significance of the obstacles to insider arbitrage at any given point of time. Under poorly performing capital markets, we should expect this measure to be large. Insiders

\(^{19}\)Confirming the earlier “duality” claim that the entropy constraint on a minimization of the expected absolute value of \(y\) is formally equivalent to an absolute-value constraint in an entropy maximization problem.
will leave large measures of short-term returns on arbitrage operations on the table because they believe the pursuit of those returns poses estimated losses that exceed estimated gains. Under well performing capital markets, we should expect this measure to be small, as obstacles to arbitrage represent smaller potential losses.

Note finally that it is possible to relate this parametrization of the Asymmetric Laplace distribution to its expression as a function of the infra- and supra- modal means $M_L$ and $M_H$ respectively,

$$M_H = \frac{1}{4} \left( e^{\frac{s-1}{2}} + c_1 e^{\frac{1-s}{2}} \right)^2$$

(18)

$$M_L = \frac{1}{4} \left( e^{\frac{s-1}{2}} - c_1 e^{\frac{1-s}{2}} \right)^2$$

(19)

4.2 Salient Inferences

This formal interpretation allows us to make a few interesting inferences concerning the performance of U.S. capital markets from the observed characteristics of the distribution of $y_i$. First, Eq. 19 shows that the relative stability in the observed measures for the infra-modal mean deviation $M_L$ can be understood as conditioned by a positive association between the aggregate measure of investor bullishness $c_1$ and the system’s entropy level $S$. This association is evidently strong, as seen in the estimated empirical values taken by each variable.

Figure 5: Scatterplot of $c_1$ and $S$ with fitted linear regression line. Entropy $S$ is calculated in nats.

Figure 5 shows the strong correlation between $c_1$ and $S$ with an adjusted $R^2$ of 0.564. The fitted linear regression implies that a one percent increase in the average modal deviation of $\ln[q]$
results in increase of 0.35 Nats of entropy in the system. Mathematically, the relative stability in $M_L$ reflects the fact that as the distribution’s mean increases, so does its measure of entropy. Put differently, as the distribution becomes more strongly skewed to the right, it also becomes less organized. The conjugation of these two effects tends to maintain this measure of infra-modal mean deviation from the mode at a reasonably constant level.

The economic content of this observation suggests a second interesting inference supported by our interpretation. Within the terms developed above, the close association between $c_1$ and $S$ suggests that as investors’ average excess bullishness on individual returns increases, so does the measure of insiders’ inability or unwillingness to take advantage of arbitrage opportunities posed by existing valuations. Put differently, the greater the aggregate measure of investor bullishness, the more insiders act as if security-market valuations are a “sideshow.” As investors’ expectations concerning individual corporations effectively become more exuberant relative to those of insiders, insiders tend to become less willing to take advantage of possible short-term arbitrage gains created by bullish individual valuations. On these bases we contend that measures of $c_1$ may be taken as a systematic diagnostic of the presence of speculative equity-price bubbles, as higher values reflect situations of high investor bullishness but a lower willingness or ability of insiders to invest on the resulting security prices.

![Figure 6: Time evolution of the average modal deviation constraint $c_1$ and the system’s entropy $S$ measured in nats.](image)

The empirical evolution of $c_1$ is interesting in this regard. It suggests U.S. capital markets have tended to exhibit higher measures of investor excess bullishness since the early 1980s. It also suggests that, at the annual, end-of-year frequency at which data is available, the late 1990s dot.com bull market stands out singularly as a clear episode of high investor bullishness and low insider appetite to invest on those valuations. Within the terms developed here, this would constitute a *prima facie* evidence of an equity-price bubble.
Perhaps most significantly, the evolution of $c_1$ and $S$ point to considerable variability in the relationship between capital-market prices and the information corporate insiders possess about the prospects for returns on the assets of individual corporations. Prices may always reflect investors expectations rather quickly, but our interpretation of the evidence presented above suggests those expectations are often wrong—at least in part, according to insiders with direct knowledge about the corporations they manage. We strongly believe further work to develop systemic diagnostic measures of speculative episodes along these lines may be useful both to our understanding of capital markets and to practical matters of financial-stability and monetary policy.

Third and finally, the empirical evolution of the informational measure of capital-market performance offered by $\lambda^*$ points to important, secular changes in the functioning of U.S. capital markets.

![Figure 7: Time evolution of $\lambda$ with Hodrick-Prescott filtered trend.](image)

Note that this measure does not exhibit any clear cyclical pattern at equity-boom frequencies. A relatively bearish market (like that at the end of 1997) may exhibit a high measure of entropy, ensuring that its informational performance, as measured by $\lambda^*$, is rather poor. Conversely, a relatively bullish market (like that at the end of 1983) may reflect good informational performance as a result of a low measure of entropy. What is most striking in the dynamic evolution of this measure is what appears to be a clear secular upward trend since the early 1980s. This would suggest that the past thirty-five years have witnessed a worsening informational performance in U.S. capital markets. According to the interpretation developed above, insiders have estimated rising long-term costs if they move to take advantage of short-term arbitrage gains possible under the capital-market valuations they face. This has ensured that securities markets have steadily become, informationally speaking, more of a “sideshow,” for any given configuration.
This could be the result of at least two processes. First, it may be that over this period insiders have become steadily less willing to take advantage of any given measure of investor excess bullishness. Under this scenario, the worsening informational performance of U.S. capital markets would suggest increasingly inefficient pricing of corporate securities, in the sense that prices increasingly reflect investor expectations that are not shared by insiders. A second possibility is that this period has witnessed a growth in monopoly power and rent-like appropriations in product markets. If that is the case, insiders may have steadily become less willing to increase investment for high measures of $\pi_i$ since they estimate such moves would reduce their future profitability. Under this scenario, the worsening performance of U.S. capital markets would originate in a growing lack of competitiveness in product markets and/or a growing reluctance of insiders to respond to high rates of return with increases in the pace of investment. Both these possibilities suggest very interesting and consequent lines of further inquiry.

5 Conclusions

Following the methodological approach of observational sciences, this paper has offered a systemic theorization of capital-market functioning that can account for the persistently observed functional form of the frequency distribution of Tobin’s $q$ across private, non-financial, U.S.-listed corporations. Taking $q$ as a ratio of the expected rate of total return on a corporation’s assets and the endogenous, required expected rate of return on its liabilities, and drawing on Information Theory, we have proposed an understanding of capital markets as an informationally constrained system. The system’s behavior is conditioned by the competitive interaction between corporate insiders and investors, and results in regulations of the distribution of Tobin’s $q$ that condition its mode and subject its mean and expected absolute value to constraints. This is to our knowledge the first theorization of capital-market behavior that can account for the strikingly well-organized and functionally stable frequency distribution of $\ln[q]$.

In addition to its explanatory ability, our theorization suggests three properties of that frequency distribution contain significant information about the functioning of capital markets. The distribution’s mode may reflect the relative attractiveness of corporate and non-corporate liabilities to investors. The expected value of its deviations from the mode or right skew may reflect measures of investor excess bullishness on the expected returns of individual corporations. And its implicit shadow cost of information, measured by $\lambda^*$, may provide an informational measure of the allocative performance of capital markets. These three new measures suggest new systemic characterizations of portfolio behavior, possible investor exuberance, and market performance. We hope that this paper helps encourage further work developing such measures as tools for deepening our understanding of capital market functioning and informing financial-stability and monetary policy.
References


6 Appendix A - The Constraint on $E[y_i]$

Starting from the original expression of $y_i$ in Eq. 7,

\[ y_i = \ln \left( \frac{\pi_i \beta_i}{\bar{\pi} \bar{\beta}} \right) = \ln [\pi_i] - \ln [\bar{\pi}] + \ln [\beta_i] - \ln [\bar{\beta}] \]  

(20)

A Taylor expansion of $\ln [\pi_i]$ centered on $\ln [\bar{\pi}]$, and of $\ln [\beta_i]$ centered on $\ln [\bar{\beta}]$ provides a good approximation for the values of $y_i$ in question, and yields the second-order approximation. Expressing this as a second-order approximation,

\[ y_i \approx \left( \frac{\pi_i}{\bar{\pi}} - 1 \right) - \frac{1}{2} \left( \frac{\pi_i}{\bar{\pi}} - 1 \right)^2 + \left( \frac{\beta_i}{\bar{\beta}} - 1 \right) - \frac{1}{2} \left( \frac{\beta_i}{\bar{\beta}} - 1 \right)^2 \]  

(21)

Using the fact that $\bar{\pi} = E[\pi_i]$ and that $a \beta = E[\beta_i]$, this becomes,

\[ y_i \approx \left( \frac{\pi_i}{E[\pi_i]} - 1 \right) - \frac{1}{2} \left( \frac{\pi_i}{E[\pi_i]} - 1 \right)^2 + \left( \frac{a \beta_i}{E[\beta_i]} - 1 \right) - \frac{1}{2} \left( \frac{a \beta_i}{E[\beta_i]} - 1 \right)^2 \]  

(22)

Taking expectations of both sides of this expression yields,

\[ E[y_i] \approx (a - 1) - \frac{1}{2} \left( 1 - 2a + a^2 \right) \]  

(23)

The first term constitutes the first-order approximation used above. Simplifying the second
term, we get the final, second order approximation,

\[ E[y_i] \approx 2a - \frac{a^2}{2} - \frac{3}{2} \]  

(24)

Two things bear noting. First, it should be evident that the first-order expression used above is a very good approximation for the values of \(a\) near one implicit in our observations for \(c_1\). Second, a constraint on this polynomial (or on those generated by higher-order approximations) is still effectively a constraint on \(a\), as motivated above.

7 Appendix B - Derivation of Distribution

The interpretation of the capital market as an informationally constrained system developed by this paper results in an account of the observed Asymmetric Laplace form for the frequency distribution of \(y\) via the following constrained minimization problem,

\[
\min_{f[y] > 0, y \in \mathbb{R}} \int_{-\infty}^{\infty} |y| f[y] dy
\]

\[
\int_{-\infty}^{\infty} f[y] dx = 1
\]

\[
H[y] = -\int_{-\infty}^{\infty} f[y] \ln[f[y]] dy = S
\]

\[
\int_{-\infty}^{\infty} y f[y] dy = c_1
\]

(25)

It should be obvious that this problem is equivalent to the problem of maximizing the following Lagrangian function,

\[
L[f[y], \mu, \lambda, \gamma; S, c_1] =
\]

\[
-\int_{-\infty}^{\infty} |y| f[y] dy + \mu \left( \int_{-\infty}^{\infty} f[y] dx - 1 \right) - \lambda \left( \int_{-\infty}^{\infty} f[y] \ln[f[y]] dy - S \right) + \gamma \left( \int_{-\infty}^{\infty} y f[y] dy - c_1 \right)
\]

(26)

In addition to throwing up the normalization, entropy, and expected value constraints in Eq. 25, this maximization yields the following necessary first-order condition,

\[
-|y| + \mu - \lambda \ln[f[y]] - \lambda + \gamma y = 0
\]

(27)
which results in a preliminary expression for the distribution of $y$,

$$f[y] = e^{\frac{\mu - \lambda}{\lambda}} e^{\gamma y - |y|} \quad (28)$$

Expressing this distribution piecewise henceforth,

$$f[y] = e^{\frac{\mu - \lambda}{\lambda}} \begin{cases} e^{\frac{\gamma y - 1}{\lambda}} & y < 0 \\ e^{\frac{\gamma y - 1}{\lambda}} & y > 0 \end{cases} \quad (29)$$

As long as $\lambda > 0$ and $\gamma \in (-1, 1)$, it is possible to obtain a definite result from substitution of this function into the normalization constraint and integration, which yields the condition,

$$e^{\frac{\mu - \lambda}{\lambda}} = \frac{1}{\lambda \left( \frac{1}{1-\gamma} + \frac{1}{1+\gamma} \right)} \quad (30)$$

which substituted into 29 yields,

$$f[y] = \frac{1}{\lambda \left( \frac{1}{1-\gamma} + \frac{1}{1+\gamma} \right)} \begin{cases} e^{\frac{\gamma y - 1}{\lambda}} & y < 0 \\ e^{\frac{\gamma y - 1}{\lambda}} & y > 0 \end{cases} \quad (31)$$

Following an analogous process, substitution of this distribution function into the expected value constraint in reflossmin and integration throws up the condition,

$$c_1 = -\frac{2\gamma \lambda}{\gamma^2 - 1} \quad (32)$$

which can be substituted into refsecond, yielding,

which substituted into 33 yields,

$$f[y] = \frac{\gamma}{c_1} \begin{cases} e^{\frac{-2\gamma y}{c_1(1-\gamma)}} & y < 0 \\ e^{\frac{-2\gamma y}{c_1(1+\gamma)}} & y > 0 \end{cases} \quad (33)$$

Finally, substitution of this expression into the entropy constraint and integration yields the condition,

$$S = 1 - \ln \left[ \frac{\gamma}{c_1} \right] \quad (34)$$
With which it is trivial to arrive at the final expression for the distribution function that solves
the programing problem in 25,

\[
f[y] = e^{1-S} \begin{cases} 
  e^{x^{2y-1_c}} & y < 0 \\
  e^{x^{2y+1+c_1}} & y > 0 
\end{cases}
\]  

(35)