Christian Schoder

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An estimated Dynamic Stochastic Disequilibrium model of Euro-Area unemployment.*

Christian Schoder†

August 24, 2017

Abstract

An empirical variant of the Dynamic Stochastic Disequilibrium (DSDE) model proposed by Schoder (2017a) is estimated for the Euro Area using Bayesian inference. Unemployment arises from job rationing due to insufficient aggregate spending. The nominal wage is taken as a policy variable subject to a collective Nash bargaining process between workers and firms with the state of the labor market affecting the relative bargaining power. A consumption function is implied by a precautionary saving motive arising from an uninsurable risk of permanent income loss. Comparing the estimated DSDE model to the corresponding estimated Dynamic Stochastic General Equilibrium (DSGE) model with frictional unemployment yields the following results: (i) The DSDE model stabilizes through fiscal policy, the DSGE model through monetary policy. (ii) To capture the persistence in the data, the DSDE model requires less persistence in the shocks than the DSGE model. (iii) As observed empirically, the DSDE model predicts the real wage to move pro-cyclically with a lag whereas the DSGE model predicts a counter-cyclical movement. (iv) In the DSDE model, a productivity shock is contractionary in the short run and expansionary in the medium run. (v) In the DSDE model, strengthening the workers’ bargaining power is expansionary in the short run and contractionary in the medium run. (vi) In the DSDE model, output is driven by demand shocks, wage inflation by bargaining power shocks, and unemployment by productivity and demand shocks. In the DSGE model, however, output is driven by productivity shocks, wage inflation by mark-up shocks, and unemployment by labor supply shocks.

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1 Introduction

Persistently high unemployment following the Great Recession in large parts of the Euro Area despite the implementation of labor market reforms have challenged unemployment theories based on search and matching frictions alone (cf. Shimer 2005, Yashiv 2007, Moreira et al. 2015, and Karamessini 2015). In particular, Michaillat (2012) finds that, in recessions, unemployment arises mainly due to job rationing rather than frictions. Farmer (2008) and Mian and Sufi (2014) identified recent US business fluctuations to be mainly a demand side phenomenon. This is in line with a renewed interest in disequilibrium theory that has emerged after the Great Recession (cf. Mankiw and Weinzierl 2011, Caballero and Farhi 2014, Korinek and Simsek 2014). Recently, Blanchard (2016), Romer (2016), and Farmer (2017) have forcefully argued to explore models beyond the general equilibrium paradigm.

Introducing rationing unemployment to a tractable macroeconomic framework, Schoder (2017a) proposed a Dynamic Stochastic Disequilibrium (DSDE) model for studying unemployment in highly centralized labor markets such as the ones in countries of the Euro Area. At the core of the DSDE model is the assumption of an administered rate of wage inflation, similar to the interest rate in Dynamic Stochastic General Equilibrium (DSGE) models which is assumed to follow a policy rule. The rate of wage inflation in the DSDE model has no tendency to clear the labor market. However, the state of the labor market affects wage formation through a collective bargaining process. At the steady state, involuntary unemployment prevails. The labor market is in permanent disequilibrium, and the labor supply is not fully utilized. Since steady-state output is determined by spending decisions, the underlying consumption theory needs to link consumption and income at the steady state. Hence, the DSDE model features an uninsurable risk of a permanent income loss which induces the rational households to accumulate a buffer stock of savings (Carroll 1997, Carroll and Jeanne 2009, Carroll and Toche 2009). This modeling device implies a consumption function similar to the one in the textbook IS model and allows the DSDE model to be closed, i.e. to have full rank. As argued by Schoder (2017b), the DSDE model can be interpreted as a traditional disequilibrium model in the spirit of Keynes (1936), Barro and Grossman (1971, 1976), Malinvaud (1977), Benassy (1993), Chiarella et al. (2000), Chiarella and Flaschel (2000), Chiarella et al. (2005) with the difference that the behavioral assumptions are derived from inter-temporal optimization and rational expectations.

While Schoder (2017a,b) studied the theoretical properties of the DSDE model, it was yet to be confronted with empirical data. Hence, the aim of the current contribution is to study historical unemployment in the Euro Area through the lens of the DSDE model and compare its empirical performance to the corresponding DSGE model with frictional unemployment similar to Christoffel et al. (2009) and Stähler and Thomas (2012). I employ Bayesian estimation techniques as is standard in the macroeconomic literature (cf. An and Schorfheide 2007, Smets and Wouters 2003, Del Negro and Schorfheide 2004, and Del Negro et al. 2007). Based on the estimated parameters, I discuss the predicted macroeconomic responses to structural shocks as well as the driving forces of macroeconomic fluctuations.

As a deviation from Schoder (2017a), I introduce to the present model rule-of-households that are assumed to be the only provider of labor services and to consume all of their income. This is mainly for the sake of keeping the empirical model tractable. Nevertheless, empirical evidence suggests a high propensity to consume out of wages. Berger-Thomson et al. (2009) find for Australia that the marginal propensity to consume is about 1 for income tax cuts. Souleles (1999) finds that consumption increase by 0.65 within a quarter following a 1 unit tax refund on wage income in the
Note that I maintain the same assumption in the corresponding DSGE model. Also, two of the standard features present in empirical DSGE models are missing: consumption habit formation and variable capital utilization. This is because of two reasons: First, I am mainly interested in studying European unemployment through the lens of a tractable DSDE model and it is not clear that habit formation or variable capital utilization add to the empirical performance of a DSDE model with a built in multiplier effect on consumption. This question is delegated to future research. Second, the benchmark DSGE model should be as close as possible to the corresponding DSDE model as I am interested in how different ways of modeling unemployment perform empirically. Adding features to the DSGE model that are missing in the DSDE model would blur the picture. Hence, I do not claim that I assess the empirical performance of DSGE models in general. With certainty, there are DSGE models available that perform far better than the one chosen.

I obtain the following results: (i) The data interpreted through the DSDE model assign an important role to fiscal policy for economic stabilization. In the DSGE model, stabilization is achieved by monetary policy. The fiscal multiplier is considerably higher in the former than in the latter. This is because a fiscal stimulus raises the real wage in the DSDE model. It originates from a labor market tightening and it stimulates consumption. (ii) In the DSDE model, the estimated shock persistence is lower than in the DSGE model. Hence, the former requires less persistence in the shocks than the latter in order to capture the persistence in the data. (iii) The DSDE model predicts the real wage to move pro-cyclically with a lag as observed empirically. In the DSGE model, the real wage moves counter-cyclically. (iv) The results for the DSDE model suggest that a productivity shock is contractionary in the short run and slightly expansionary in the medium run. Rising productivity primarily lowers the demand for labor in the short run. This increases unemployment and cuts into consumption. At the same time it triggers and investment boom that drives output above the steady state after 8 quarters. (v) Strengthening the worker’s bargaining power is expansionary in the short run and contractionary in the medium run. This holds only for the DSDE model and not for the DSGE model where output decreases persistently. The expansionary effects in the DSDE model are driven by the rise in the real wage which stimulates consumption. However, it also depresses investment which starts dominating after 8 quarters. (vi) The forecast error variance decomposition reveals that, in the DSDE model, it is mainly demand shocks that drive the variation in output. In particular more than 70% of the output variation in the Euro Area can be explained by demand shocks. In the DSGE model, it is primarily productivity shocks. While wage inflation is determined on the labor market and driven by bargaining power shocks in the DSDE model, it is determined on the goods market and driven by mark-up shocks in the DSGE model. Finally, the variation of unemployment, in the DSDE model, is primarily caused by demand and productivity shocks that sum up to roughly 85%. In the DSGE model, 60% of the unemployment variation is explained by labor supply shocks. Demand shocks are not important.

The remainder of the paper proceeds as follows: In Section 2, I briefly summarize the model which is presented in detail in Appendix A. The corresponding DSGE model is discussed in Appendix B. Section 3, discusses calibration choices and presents the results of the Bayesian parameter estimation. In Section 4, I discuss the macroeconomic responses to different shocks as predicted by the empirical DSDE and DSGE models. Sections 5 studies the forecast error variance decomposition. Section 6 concludes the paper.
2 The model

The economy comprises households, final good firms, intermediate good firms, a firm representative, a worker representative, a fiscal authority and a monetary authority. Appendix A presents the complete model in detail. Here, I limit myself to discuss the setup and the core assumptions. Schoder (2017a,b) provides a thorough discussion of the micro-foundations. The two core differences of the present model with respect to Schoder (2017a) are the assumption of an exogenously growing labor supply and the introduction of rule-of-thumb households which receive all wage income and do not save.

2.1 The household sector

The household’s problem builds on the precautionary savings theory proposed by Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009). Individuals are born randomly into generations of two types of households: optimizing and rule-of-thumb households. Generations grow in size by a constant growth rate. Each optimizing household is born active, i.e., receives income. The active household faces an uninsurable per-period risk of permanently losing all income. Once inactive, the household faces a per-period risk of death. In this setting, the household will accumulate precautionary savings in order to hedge against the risk of permanent income loss. The rule-of-thumb household faces the same per-period risk of death. Note that this setup implies the share of each household type to be constant.

I follow Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009) and assume that inactive households have access to a perfectly competitive Blanchard (1985) type of insurance market transforming wealth into annuities. Once the inactive household dies, bequests will be transferred to the insurance company which, in turn, distributes this wealth to the inactive households still alive. This modeling device facilitates aggregation as no accidental bequests remain on the aggregate level. The inactive household’s problem is standard, i.e., to choose an inter-temporal path of consumption and wealth that maximizes its expected discounted stream of log-utilities. As shown in Appendix A, the core and well-known result is that the inactive household’s per-period consumption is proportional to its wealth.

The active household faces a risk of permanent income loss. I assume that households cannot lose income and die in the same period. I further assume a transfer ensuring that all active households have the same wealth independent from when they were born. This allows for aggregation over all households. To keep the model tractable I assume that active households do not work but receive only profit income. In the optimum, the active household will choose a consumption path such that the marginal utility of consumption in \( t \) equals the discounted expected marginal utility of consumption in \( t + 1 \). This expected marginal utility takes into account the risk of becoming inactive. In this case, consumption will be chosen according to considerations of the inactive household discussed above. Hence, the active household’s consumption choice today is affected by the inactive household’s consumption choice which may become relevant tomorrow. Since the active household internalizes this solution of the inactive household’s problem, the former’s expected marginal utility of consumption depends on the level of previously accumulated wealth. Knowing that savings will be the only source of consumption when inactive, the active household accumulates a certain wealth in equilibrium for a given level of income. Hence, wealth is not merely a temporary consumption-smoothing device but a permanent buffer stock. This is the crucial property of the active household’s solution and gives rise to a Keynesian type of consumption function in the steady state.
state.

Rule-of-thumb households choose consumption based on the same utility function as active households. They face a per-period probability of death. Unlike, the previous household types, they are excluded from financial markets. Neither do they pay taxes. Through the budget constraint, consumption is directly linked to the labor supply for a given real wage. As in Groshenny (2009), I assume labor supply to follow an exogenous stochastic process.

2.2 The firm sector

A perfectly competitive firm aggregates intermediate goods into a final good, and a continuum of monopolistically competitive firms produce a differentiated good using capital and labor input. The only non-standard feature of the firm sector is the assumption of capital to be firm specific (Woodford 2005, Sveen and Weinke 2007, 2009). Labor services are rented from rule-of-thumb households.

The final good firm bundles differentiated intermediate goods into a homogenous final good using a Constant Elasticity of Substitution aggregator. Taking as given the input prices, it chooses the optimal amount of each intermediate under the constraint that a certain final output needs to be achieved. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index one can show that the First Order Condition (FOC) relates input demand inversely to its relative price. This is a standard result in the literature.

Any given intermediate good firm produces a certain quantity of its differentiated good according to a Cobb-Douglas production function in capital and labor. Labor embodied productivity grows at a deterministic rate. Intermediate goods are sold on a monopolistically competitive market. Facing a quadratic capital adjustment cost, the firm purchases investment goods to accumulate capital. These adjustment costs affect capital depreciation. I assume Rotemberg (1982) price setting which is subject to quadratic adjustment costs which are assumed to destroy output. As will become clear below, I assume the firm to maintain a certain debt-capital ratio. Taking as given total output, the overall price level, the capital stock, and the wage rate as well as the law of motion of capital, the production function, the demand function for intermediate goods, and the target debt-capital ratio, the firm chooses inter-temporal paths for its price, labor input, investment, external finance, and capital stock to maximize discounted inter-temporal distributed profits.

The firm chooses external finance such that a target debt-capital ratio is achieved. The Lagrangian multiplier of this constraint will be zero, i.e. the Modigliani-Miller theorem holds and the financial structure is irrelevant for the firm. Note, however, that the financial structure of the firm affects the households’ income through the distributed profits. The higher the share of external finance, the lower retained earnings and the higher distributed profits. In the conventional model without precautionary saving, a change in the distributed profits does not affect the consumption behavior of the household at the steady state. For instance, if the firms decide to increase external finance, thus leaving more profits for distribution to the households, the households’ consumption behavior will not be affected since the additional income will be saved completely. The financial structure is also irrelevant for the household. Things are very different in the present model due to the Keynesian-type of consumption function. Increasing the distributed profits by one unit at the steady state will cause savings not to increase exactly by one unit since a fraction is consumed. The solution of the intermediate goods firm’s problem has the following noteworthy implications: First, firms set the price with a mark-up on marginal costs minus a term arising from the cost of price adjustment. The adjustment cost of prices ensures that faster wage inflation leads to an
under-proportional increase of price inflation and, hence, to a larger real wage.\footnote{To obtain this implication, I assume price rigidities in the vein of Rotemberg (1982) instead of Calvo (1983). With Calvo pricing, an acceleration of wage inflation does not increase the real wage. Moreover, aggregation is difficult in a model featuring both Calvo price setting and firm-specific capital (cf. Sveen and Weinke 2007, 2009).} Second, the firm’s marginal product of labor depends on its previously chosen capital stock. Hence, the pricing decision is not independent from the capital accumulation decision since capital is not purchased on a spot market. Third, the marginal return to capital is not measured by the firm’s marginal revenue product of capital due to the absence of a rental market for capital services. Rather, it captures the reduction of nominal labor costs that can be afforded after a one-unit increase in the capital stock in order to produce a given level of output.

### 2.3 Policy and the labor market

Fiscal policy is characterized by a balanced structural budget and the deficit responding persistently to changes in output in the short run. The monetary policy rule links the interest rate to deviations of inflation from the target and to changes in output.

I assume that the rate of wage inflation is subject to a bargaining process between a workers’ and a firms’ representative. I take the steady-state real wage as the worker’s return and the steady-state profit rate as the firm’s return. The former can be shown to increase and the latter to decrease in the rate of wage inflation which is due to the assumption of quadratic price adjustment costs. Hence, I assume that the bargaining parties are concerned with the long-run implications of the bargaining. Nevertheless, the bargaining game is affected by the short run by assuming that the state of the labor market determines the relative bargaining power.

### 2.4 Model implications for unemployment

The precautionary saving motive in the household’s behavior and the assumption of non-accommodating collective wage bargaining instead of accommodating wage adjustment have vast implications for the model characteristics and, in particular, the nature of unemployment. Without the assumption of an accommodating nominal wage and with consumption depending on current income, the DSDE model has a strong Keynesian character. This is because I have left the realm of general equilibrium theory. The nominal wage does not adjust to eliminate any disequilibrium going beyond frictional unemployment. Removing any frictions and rigidities would eliminate unemployment in a DSGE model but not in the DSDE model. Since labor is not fully employed and involuntary non-frictional unemployment persists in the steady state, the resource constraint known from DSGE models now has the interpretation of a goods market equilibrium condition stating that aggregate output needs to equal aggregate spending.

In DSGE models business fluctuations are supply-driven. This means that output changes to shocks mediated by equilibrium adjustments on the factor markets. Nevertheless, aggregate demand shocks play a role. Consider a government shock. Both demand and output will increase and unemployment will drop. This is because, given product and labor market frictions as well as price and wage setting rigidities, the rise in demand will cause a possible staggered wage and price adjustment such that agents find it beneficial to provide more resources. Aggregate demand is limited by resources brought to the market, i.e. supply, at any period of time. In the DSDE model business fluctuations are demand-driven. A demand shock affects output without requiring households to provide more resources since at the given real wage labor is unemployed even in...
the absence of frictions and rigidities. Nevertheless, labor supply shocks do have effects. This is because of the labor market feedback on the wage formation. As labor market conditions change after a supply shock, so does the relative bargaining power and, hence, the nominal rate of wage inflation which, then, affects demand.

3 Estimation results

I estimate the proposed DSDE model for the Euro Area. For the sake of comparison, I also estimate an equivalent DSGE model with unemployment arising from search and matching frictions. The DSGE model is described in Appendix B. In short, the DSGE model differs from the DSDE model in two ways: First, the risk of income loss is zero and, hence, the household’s FOC collapses to the standard Euler equation. Second, instead of assuming wage inflation to be a policy variable, I assume privately efficient wage contracts in a search and matching setting of frictional unemployment.

For the sake of tractability, I chose to keep the empirical DSDE model simple. The core assumptions made to achieve this are the following: First, rule-of-thumb households receive wage income while optimizing households receive profit income. Second, I assume log utility implying a parameter of relative risk aversion of one and an inverse Frisch labor supply elasticity of one. Third, there is no habit formation or variable capital utilization. These are features that the DSGE literature has introduced to improve the empirical performance of DSGE models, in particular to generate the persistence observed in the data (Smets and Wouters 2003). To assess the empirical performance of the DSDE model compared to the DSGE model, I consider one of the latter which is as similar as possible to the former rather than one that is theoretically more elaborate.

Data and method. The identified parameters of the DSDE and DSGE models are estimated for the Euro Area using Bayesian inference (cf. An and Schorfheide 2007 and Smets and Wouters 2003). The data cover the period 1980Q1-2016Q4 and comprise real GDP, real consumption, real investment, the short-term interest rate, price inflation rate, the wage inflation rate and unemployment rate, all detrended by the Hodrick-Prescott (HP) filter as in Christoffel et al. (2009). Before, output and its components have been detrended by average population and labor productivity growth. Data have been taken from the ECB’s Area Wide Database and extended by the relevant Eurostat time series.

Various robustness tests have been performed and none of them fundamentally change the results obtained. In particular, I have considered a dataset with output and the demand components in growth rates as in Smets and Wouters (2003) rather than the cyclical part of the HP filter. The only noteworthy difference is that the persistence in the data increases which translates into higher estimates of the shock persistence. Moreover, I chose to include the period of malfunctioning monetary policy transmission starting around 2011 in the sample, but the main results are robust to excluding these observations. Finally, I have tried to include government spending and the labor force as observed variables. However, I ran into problems of (near) stochastic singularity despite introducing additional shocks to the system.

To compute the posterior mode of the estimated parameter distributions, I employed a Monte-Carlo based optimization routine. The first 4 observations serve as a presample for the Metropolis-Hastings sampler. I have run two parallel Markov chains to obtain the posterior distributions of the parameters. The number for replications in each chain has been chosen to be 100,000 which
### Table 1: Calibration and parameter restrictions

<table>
<thead>
<tr>
<th></th>
<th>DSDE</th>
<th>DSGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$ Households’ discount rate</td>
<td>0.995</td>
<td>0.995</td>
</tr>
<tr>
<td>$\delta$ Rate of capital depreciation</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>$\Gamma$ Gross growth rate of effective labor force</td>
<td>1.009</td>
<td>1.009</td>
</tr>
<tr>
<td>$\Gamma_n$ Gross growth rate of labor force</td>
<td>1.002</td>
<td></td>
</tr>
<tr>
<td>$\theta_c$ Share of rule-of-thumb households</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>$\lambda$ Debt-capital ratio</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$s$ Job separation rate</td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>$\kappa_c$ Matching elasticity</td>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td><strong>Steady state calibration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G$ Government expenditures</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Pi_p$ Gross price inflation rate</td>
<td>1.005</td>
<td>1.005</td>
</tr>
<tr>
<td>$V_p$ Price mark-up</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$R$ Gross interest rate</td>
<td>1.008</td>
<td></td>
</tr>
<tr>
<td>$\nu$ Workers’ bargaining power</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Steady-state restrictions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ Capital elasticity of output</td>
<td>$\alpha = 0.28$ such that $\bar{Y}/\bar{K} = 0.15$</td>
<td>$\alpha = 0.39$ such that $\bar{Y}/\bar{K} = 0.15$</td>
</tr>
<tr>
<td>$\kappa$ Inact. household’s wealth effect on cons.</td>
<td>$\kappa = 1 - \beta (1 - D)$ such that $\theta_c/(\theta_a + \theta_c) = 0.2$ (old age dependency ratio)</td>
<td></td>
</tr>
<tr>
<td>$U$ Risk of permanent income loss</td>
<td>such that $\theta_c/(\theta_a + \theta_c) = 0.2$ (old age dependency ratio)</td>
<td></td>
</tr>
<tr>
<td>$V_c$ Consumption utility scaling parameter</td>
<td>such that $\bar{Y} = 1$</td>
<td></td>
</tr>
<tr>
<td>$V_a$ Labor supply scaling parameter</td>
<td>such that $u = 0.091$</td>
<td>such that $N = 1$</td>
</tr>
<tr>
<td>$\nu$ Workers’ bargaining power</td>
<td>such that $\Pi_w = \Pi_p$</td>
<td></td>
</tr>
<tr>
<td>$R$ Gross interest rate</td>
<td>$R = \Pi_p \Gamma / \beta = 0.019$ such that a steady state exists</td>
<td></td>
</tr>
<tr>
<td>$V_a$ Steady-state total factor productivity</td>
<td></td>
<td>$V_a = 0.505$ such that $\bar{Y} = 1$</td>
</tr>
<tr>
<td>$\kappa_m$ Matching elasticity</td>
<td></td>
<td>such that $p_v = 0.2$</td>
</tr>
<tr>
<td>$c_v$ Vacancy posting costs</td>
<td></td>
<td>such that $u = 0.091$</td>
</tr>
</tbody>
</table>

is sufficient for the moments across the chains to converge. The scale of the jumping distribution has been initialized to 0.5 in the DSDE model and 0.3 in the DSGE model in order to have the acceptance rate in the neighborhood of 24%.

**Steady state and calibration.** A few weakly identified parameters have been calibrated according to microeconomic evidence, others in order to match the models’ steady states roughly with averages of the Euro Area. A few parameter restrictions have been imposed in the estimation.

Table 1 summarizes these calibration choices for the DSDE and DSGE models considered. In both models, the discount factor is assumed to be $\beta = 0.995$ which is in line with the literature (Smets and Wouters 2003). As is the convention, I calibrate the depreciation rate to $\delta = 0.025$. The average population growth rate and the effective population gross growth rate, which includes productivity growth, are $\Gamma_n = 1.002$ and $\Gamma = 1.009$, respectively. In the DSDE model, I set the share of rule-of-thumb households to 0.3. Note that since all wage income is consumed in the aggregate this share is only relevant for the restriction on the probability of income loss $U$ for a
given probability of death $D$ such that a certain old age dependency ratio is met (see below). I calibrated the debt-capital ratio to 0.2. In the DSGE model, I calibrate the job separation rate with 0.02 and the matching elasticity with 0.6. These values are consistent with Christoffel et al. (2009) and Stähler and Thomas (2012).

Regarding steady state calibration, I set government spending to 0.2 in the steady state which corresponds to 20% of the national product. I take the steady state gross price inflation rate to be $\Pi_p = 1.005$ which is close to the target of the ECB. The price mark-up at the steady state is 0.5. This is higher than in the related DSGE literature which calibrate/estimate the mark-up around 0.35 (Christoffel et al. 2008, Martins et al. 1996, Jean and Nicoletti 2002). With a lower mark-up, a stable steady state does not exist. Despite the large mark up the wage share turns out to be above 50% at the posterior mean. In the DSDE framework, the steady-state interest rate can be calibrated freely as long as, at the steady state, the real interest rate is sufficiently lower than the deterministic growth rate, i.e. $R/\Pi_p < \Gamma$ (Schoder 2017a). I set the steady-state interest rate to 1.008. In the DSGE model, the steady-state rate of interest is restricted by the Euler equation of the household’s problem. For a given inflation rate, it requires $R = \Pi_p \Gamma / \beta$ at the steady state which implies a gross interest rate of 1.019. Note that this is considerably higher than the deterministic growth rate of 1.009. In the DSGE model, the steady state of the workers’ relative bargaining power is 0.5 as in Christoffel et al. (2009). In the DSGE model, it is restricted to ensure that, at the steady state, the wage inflation outcome of the bargaining process is equal to the price inflation target of the monetary authority.

In both models, I restrict the capital elasticity of output such that an output-capital ratio of 0.15 is obtained. This corresponds roughly to the empirical ratio between output and non-residential capital in most countries of the Euro Area. The proportionality factor in the DSDE relation between inactive consumption and wealth depends on the discount factor and the death probability. I restrict it accordingly. The risk of permanent income loss is restricted such that the ratio of inactive to non-inactive households (active and rule-of-thumb) is 0.2. This corresponds roughly to the old age dependency ratio in the Euro Area. In the DSDE model, the restriction on the consumption utility scaling parameter ensures that output is one at the steady state. In the DSGE model, this parameter has no steady state implication. Hence, in this model, a restriction on the steady state total factor productivity normalizes output to one. A restriction on the labor supply scaling parameter ensures a steady state rate of unemployment of 0.091 in the DSDE model and a steady state labor supply of one in the DSGE model. This restriction makes the latter model comparable to related studies such as Christoffel et al. (2009) and Stähler and Thomas (2012) who consider unemployment on the extensive margin only. Finally, restrictions on the matching elasticity and vacancy posting costs in the DSGE model ensure an empirically observed vacancy filling probability and unemployment rate, respectively (Christoffel et al. 2009).

**Estimation results.** The distributions of the remaining model parameters are estimated using Bayesian inference. Table 2 reports the results for the DSDE and the DSGE model. Note that the prior distributions are equal across models in order to have the results as comparable as possible. However, a direct comparison requires some caution. This is because I have imposed different parameter restrictions in the two models as discussed above. Moreover, the steady states are not exactly the same.

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2For $R/\Pi_p = \Gamma$, Schoder (2017a) showed the consumption-income ratio to equal one. A steady state does not exist.
Table 2: Estimation results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DSDE dist</th>
<th>DSDE mean</th>
<th>DSDE stddev</th>
<th>DSGE dist</th>
<th>DSGE mean</th>
<th>DSGE interval</th>
<th>90% HPD interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>B</td>
<td>0.010</td>
<td>0.005</td>
<td>0.005</td>
<td>G</td>
<td>50.000</td>
<td>45.573</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>G</td>
<td>50.000</td>
<td>10.000</td>
<td>33.018</td>
<td>G</td>
<td>50.000</td>
<td>52.260</td>
</tr>
<tr>
<td>( \tau_i )</td>
<td>G</td>
<td>50.000</td>
<td>10.000</td>
<td>49.478</td>
<td>G</td>
<td>50.000</td>
<td>45.573</td>
</tr>
<tr>
<td>( \phi_{gy} )</td>
<td>G</td>
<td>1.100</td>
<td>0.200</td>
<td>1.154</td>
<td>G</td>
<td>1.100</td>
<td>2.442</td>
</tr>
<tr>
<td>( \phi_{ry} )</td>
<td>G</td>
<td>50.000</td>
<td>200</td>
<td>0.384</td>
<td>G</td>
<td>50.000</td>
<td>0.059</td>
</tr>
<tr>
<td>( \phi_{nu} )</td>
<td>G</td>
<td>2.000</td>
<td>300</td>
<td>1.374</td>
<td>G</td>
<td>2.000</td>
<td>1.094</td>
</tr>
<tr>
<td>( \xi )</td>
<td>B</td>
<td>0.700</td>
<td>0.100</td>
<td>0.819</td>
<td>B</td>
<td>0.700</td>
<td>0.913</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>B</td>
<td>0.500</td>
<td>0.100</td>
<td>0.784</td>
<td>B</td>
<td>0.500</td>
<td>0.948</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>B</td>
<td>0.500</td>
<td>0.100</td>
<td>0.813</td>
<td>B</td>
<td>0.500</td>
<td>0.888</td>
</tr>
<tr>
<td>( \rho_i )</td>
<td>B</td>
<td>0.500</td>
<td>0.100</td>
<td>0.547</td>
<td>B</td>
<td>0.500</td>
<td>0.270</td>
</tr>
<tr>
<td>( \rho_y )</td>
<td>B</td>
<td>0.500</td>
<td>0.100</td>
<td>0.347</td>
<td>B</td>
<td>0.500</td>
<td>0.894</td>
</tr>
<tr>
<td>( \rho_i )</td>
<td>B</td>
<td>0.500</td>
<td>0.100</td>
<td>0.225</td>
<td>B</td>
<td>0.500</td>
<td>0.113</td>
</tr>
</tbody>
</table>

Standard deviation of shocks

| \( \epsilon_g \) | IG        | 0.010     | 0.050       | 0.020     | IG        | 0.010     | 0.055       | 0.014     | 0.013 , 0.016 |
| \( \epsilon_a \) | IG        | 0.010     | 0.050       | 0.004     | IG        | 0.010     | 0.050       | 0.004     | 0.004 , 0.005 |
| \( \epsilon_r \) | IG        | 0.010     | 0.050       | 0.001     | IG        | 0.010     | 0.050       | 0.002     | 0.002 , 0.002 |
| \( \epsilon_u \) | IG        | 0.010     | 0.050       | 0.005     | IG        | 0.010     | 0.050       | 0.002     | 0.002 , 0.002 |
| \( \epsilon_p \) | IG        | 0.010     | 0.050       | 0.090     | IG        | 0.010     | 0.050       | 0.002     | 0.002 , 0.002 |
| \( \epsilon_o \) | IG        | 0.010     | 0.050       | 0.015     | IG        | 0.010     | 0.050       | 0.002     | 0.002 , 0.002 |
| \( \epsilon_i \) | IG        | 0.010     | 0.050       | 0.046     | IG        | 0.010     | 0.050       | 0.002     | 0.002 , 0.002 |

Notes: \( D \) is the per period risk of death, \( \tau_p \) is the price adjustment cost scaling parameter, \( \tau_i \) is the capital adjustment cost scaling parameter, \( \phi_{gy} \) is the output elasticity of government spending, \( \phi_{ry} \) is the inflation elasticity of the interest rate, \( \phi_{nu} \) is the unemployment elasticity of the bargaining power, \( \xi \) is the Calvo parameter for wage adjustment, \( \rho_x \) is the autoregressive parameter for any shock process \( V_x \), and the \( \epsilon_s \) are disturbances. B is the beta distribution, G is the gamma distribution, and IG is the inverse gamma distribution.

The prior means are roughly in line with the empirical literature on the Euro Area (cf. Smets and Wouters 2003, Ratto et al. 2009, Christoffel et al. 2008). The priors for the responses of monetary policy to inflation and output as well as for the price and investment adjustment costs are in line with Smets and Wouters (2003) and Ratto et al. (2009). Note that the prior mean for \( \phi_{r\pi} = 1.1 \) is in accordance with the Taylor principle even though it is not required for the DSDE model (Schoder 2017a). I set the prior mean for \( D \) as 0.01. For the population growth rate, the old-age dependency ratio and the share of rule-of-thumb households give as discussed above, this implies the optimizing household to be in the active state for about 67 years and in the inactive state for 25 years. For the prior standard deviations relatively large values are used since I am primarily interested in what posterior distributions the data assigns to the parameters for the two different models.

I obtain the following posterior results. A posterior per-period death probability of 0.005 implies a per-period probability of a permanent income loss of around 0.002 under the restriction that the old-age dependency ratio is 0.2. Hence the average duration of a household in the active or inactive state is around 50 years and in the active state around 125 years. These long durations may be explained by the fact that I do not consider social insurance. Hence, the model understates the
risks of income loss and death, respectively, in order to capture the limited extent of precautionary saving in the data. The implied annualized wealth to national income and wealth to disposable household income are around 1.9 and 3.1, respectively. These findings are in line with the data (cf. Piketty and Zucman 2014).

On the firm side, the parameters capturing price and capital adjustment costs are around 33.0 and 49.5, respectively, in the DSDE model and 45.6 and 52.3, respectively, in the DSGE model. Hence, the latter model assigns slightly more weight to price rigidities than the former. This finding is robust to different specifications as discussed below.

The response of government spending to a decline in output is found to be 1.84 in the former and only 0.24 in the latter. At the same time, the response of monetary policy to inflation is less aggressive in the DSDE model than in the DSGE model: 1.15 vs. 2.44. Note that a inflation elasticity of the interest rate below one is feasible in the DSDE model. In both the DSDE and DSGE model, the estimated monetary policy response to output is small: 0.38 vs. 0.06. Overall, the data seems to emphasize fiscal policy in the DSDE model and monetary policy in the DSGE model as the core stabilization policies. The reason for this difference can be found in the respective transmission mechanisms of the two models: The DSDE model favors quantity adjustment, the DSGE model price adjustment. In the former, therefore, excessive volatility of output is mitigated by aggressive counter-cyclical fiscal policy. In the latter, excessive volatility in price inflation is mitigated by aggressive monetary policy.

Regarding the labor market, the unemployment elasticity of the workers’ bargaining power is around 1.37. Given the expansionary short-run effects of rising wages, labor market tightening reinforces economic fluctuations. This is another reason, why the DSDE model exhibits a strong fiscal stabilization mechanism. In the DSGE model the Calvo wage parameter is 0.82 which is similar to the findings of Christoffel et al. (2009).

The estimated persistence of exogenous shocks is higher in the DSGE than in the DSDE model except for monetary policy, mark-up and investment shocks. Especially, fiscal policy, productivity, labor supply, and bargaining power shocks exhibit strong persistence in the DSGE model. Again, this result can be explained by the Keynesian multiplier mechanism built into the DSDE model which allows temporary shocks to have persistent quantity effects.

To assess which model describes the data better, I have simply compared the log-likelihoods of the two models. However, the findings are not robust. In the current specification the DSGE model slightly outperforms the DSDE model. Yet, adding further observed variables such as government spending or labor supply or taking output and the demand components as growth rates rather than trend deviations reverses this result.

4 Bayesian impulse-response analysis

This section discusses the estimated macroeconomic responses predicted by the DSDE model. I contrast them with those of the corresponding Economic Theory Vector Auto-Regression (ET-VAR) proposed by Del Negro and Schorfheide (2004) and Del Negro et al. (2007) and also known as DSGE-VAR as well as to those of the corresponding DSGE model with search and matching frictions on the labor market. The idea of the ET-VAR is to represent the economic model as a

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3Since this measure is somewhat sensitive to the number of estimated parameters, we follow Smets and Wouters (2003) and subtract from each log-likelihood of the full sample the log-likelihood of a training sample covering the first 40 quarters.
Figure 1: Deviations from the steady state after a one standard deviation impulse to the fiscal policy disturbance, $\epsilon_g$.

Bayesian VAR and control the confidence in the theory implied cross-coefficient restrictions on the parameters of the VAR by a hyperparameter. I chose a value of the hyperparameter associated with the smallest confidence in theory implied parameter restrictions that is possible and then estimated the BVAR. Hence, I give low weight to theory and let the data speak as freely as possible. This is helpful to detect weaknesses of the theory in terms of impulse-response analysis. I shall refer to the ET-VAR as DSDE-VAR as it rests upon the DSDE model.

As impulses, I consider a government spending shock, a monetary policy shock, a total factor productivity shock, and a shock to the worker’s bargaining power. Note that the size of the impulses in the DSDE, ET-VAR, and DSGE model are the same: one standard deviation of the corresponding shock as estimated for the DSDE model.
**Government spending shock.** I consider a disturbance to government expenditures, $\varepsilon_g$, which amounts to 2% of the steady state. The Bayesian impulse-response functions (IRFs) are plotted in Figure 1 for the DSDE, DSDE-VAR and the DSGE model. The following observations are worth to note:

First, fiscal policy is expansionary in the DSDE model. The impact multiplier is around 1.25.\footnote{The mean impulse is an increase of government spending at the steady state by 2% which is the posterior mean of the standard error of the government spending disturbance. This yields a level increase in government spending of 0.020*0.2=0.004. The change of output on impact is around 0.5% of the steady-state level. This yields a level increase of 0.005*1=0.005. The impact multiplier, then, is 0.005/0.004=1.25.} For a Keynesian model, this might be considered rather low. It is because of the counter-cyclical response of government spending already on impact. Note that government spending increases only by 1% despite an impulse of 2%. The multiplier measured as the change in output relative to the actual change in government spending is 2.5. High multipliers are feasible due to the presence of unemployed labor that can be exploited without frictions. The multiplier is mainly driven by an accelerator effect on consumption. The consumption multiplier on impact is around 0.7. Consumption is crowded in as an increasing real wage raises the purchasing power of the rule-of-thumb households. The real wage increases because the labor market tightens which strengthens the workers’ bargaining power. Hence, the real wage moves pro-cyclically with a lag as observed empirically (cf. Barbosa-Filho and Taylor 2006, Zipperer and Skott 2010, Flaschel 2009, and Galí et al. 2012). A rise in government spending slightly crowds out investment. An initial rise of government consumption of 2% reduces investment by 0.4% at the peak. The mechanism behind this result is not straightforward to disentangle. Note that it is not driven by capital adjustment costs. Due to the assumption of firm specific capital rather than a rental market, the adjustment of the capital stock in place is always gradual even without adjustment costs and with Tobin’s $q$ fixed at one. What is driving the negative response of investment is the interplay of the expected real wage, expected output, and the real interest rate, as discussed in detail by Schoder (2017a) and as can be seen by studying the firm’s FOC with respect to capital (equation A.12 in Appendix A). In the DSDE model the real interest rate decreases on impact. Given an expected expansion of output, it may seem beneficial for the firm to expand its capital stock. However, it anticipates a rise in the real interest rate and, hence, a falling expected Tobin’s $q$. The firm therefore reduces its capital stock on impact. As the reduction of the discount factor unfolds due to a decreasing real interest rate and output returns towards the steady state, future profits are valued less and the desired capital stock decreases further.

Second, despite a quick adjustment towards the steady state in the first 10 quarters, the DSDE model exhibits a very persistent long-run response to shocks. There are three stable eigenvalues close to one each generating excessive persistence. The first two arise from the active and inactive households’ budget constraints and the fact that consumption depends on the stock of precautionary savings. Consider a one-time rise in the interest rate. This causes the level of wealth to shift for both households which feeds back into persistently higher consumption. In Figure 1 this persistent positive effect on consumption is dampened by a third mechanism which leads to a persistent reduction in income and reduces consumption of the rule-of-thumb households. This third mechanism is a direct consequence of the assumption of firm specific capital. It is caused by the interaction of the firm’s choices of investment and prices. The long downward swing of the real wage is driven by the firm’s decision to keep price inflation above steady state. This is because its marginal cost is above average which, in turn, is due to a low capital stock. The choice of the desired capital stock

\[\text{Government spending shock.}\]
depends on the real interest rate and on expected output which are above and below steady state, respectively, which both induce to firm to cut back its capital stock.

Third, relaxing the cross-coefficient restrictions implied by the DSDE model, the IRFs of the DSDE-VAR reveal some of the weaknesses of the DSDE model estimated: (i) The fiscal multiplier is found to be around 2.5 after relaxing the cross-coefficient restrictions which is considerably larger than the DSDE multiplier of 1.25. This difference can be explained by a strong fiscal stabilization estimated for the DSDE model. (ii) The DSDE-VAR indicates a weakness of the underlying investment theory as it is not able to explain the crowding-in of investment as observed empirically. (iii) Macroeconomic responses seem to be hump-shaped and adjustment seems to be cyclical. Both features are not replicated by the DSDE model. (iv) The sensitivity of unemployment to changes in output seems to be slightly overstated by the DSDE model. (v) Persistent long-run effects do not seem to be a feature of the data. This comes as no surprise as trends have been removed. Nevertheless, the DSDE model captures well the direction, magnitude and duration of the macroeconomic responses to a fiscal shock.

Fourth, the predicted responses of the DSDE model differ considerably from those of the DSGE model. (i) The fiscal multiplier is only around 0.2 in the latter as Ricardian equivalence dominates despite the presence of rule-of-thumb households. While DSDE firms can simply hire additional labor from the pool of unemployed without any frictions, DSGE firms need to post open vacancies and search for candidates which is costly. To obtain large multipliers, DSGE models typically incorporate a binding zero lower bound to monetary policy. (ii) Consumption is crowded out in the DSGE model. (iii) The real wage moves counter-cyclical on impact, a finding shared by Christoffel et al. (2009). A depressed real wage cuts into the purchasing power of the rule-of-thumb households and lowers consumption. (iv) Persistent long-run effects can also be observed for the DSGE model as it shares the assumption of firm specific capital.

**Monetary policy shock.** Figure 2 plots the Bayesian IRFs for a contractionary monetary policy shock. The impulse is an increase in the interest rate by 10 basis points. In the DSDE model, output drops by 0.04% on impact; consumption and investment by 0.08% and 0.01%. Government spending increases due to automatic stabilizers. Unemployment increases by 0.05%-points. The real interest rate increases. This makes future profits of production less valuable and induces the firms to reduce the capital stock which implies investment to decrease. At the same time, lower employment at a given real wage reduces consumption of the rule-of-thumb households. The response of the consumption of optimizing households is ambiguous. On the one hand, the substitution effect induces the household to save more and consume less. On the other hand, the income effect increases the wealth holdings and, hence, consumption. In the long run the latter mechanism dominates the former. As unemployment returns towards the steady state after 10 quarters, consumption exceeds the steady state despite a lower real wage. This is because optimizing household have accumulated wealth in the periods before due to higher real interest rates and higher savings. Even though output returns to the steady state after 12 quarters, unemployment is persistently below the steady state as firms have substituted labor for capital. At the same time, larger marginal costs induce the firm to raise inflation above steady state depressing the real wage persistently. In the DSGE model, the impact adjustment of inflation is slightly stronger than in the DSDE model. Due to firm specific capital, the long run adjustment is slow.

The responses predicted by the DSDE-VAR are rather inconclusive as the confidence bands are broad. The DSDE and DSGE model predict similar responses to monetary policy. One noteworthy
Figure 2: Deviations from the steady state after a one standard deviation impulse to the monetary policy disturbance, $\epsilon_r$.

difference is the expansionary real wage on impact in the latter which is not predicted by the former. The reason is the more aggressive response to inflation in the DSGE model. It constrains price inflation and causes the real wage to increase.

**Technology shock.** Figure 3 plots the IRFs of a technology shock. We can make the following observations:

First, a technology shock is contractionary in the DSDE model. Higher productivity reduces the labor demand for a given output and a given stock of capital. Lower labor demand causes a strong initial rise in unemployment. Hence, consumption of the rule-of-thumb households drops causing output to decline. The drop in inflation increases the expected real profits of the intermediate
Figure 3: Deviations from the steady state after a one standard deviation impulse to the productivity disturbance, $\epsilon_a$.

good firms which raise investment. The initial rise in investment, however, is not strong enough to compensate the fall in consumption. Only after 8 quarters does the productivity induced investment boom push output slightly above the steady state.

The DSDE-VAR predicts these mechanisms to be more pronounced. Initially higher productivity lowers output, consumption and investment which move together in the data. After 10 quarters, investment pushes output above the steady state before it returns to the equilibrium.

The DSGE model differs considerably from the DSDE model. A productivity shock is expansionary. In contrast to the DSDE model, the DSGE model predicts the real wage to increase strongly. Hence, unemployment goes up only moderately and consumption increases causing output to expand. The IRFs exhibit excessive persistence.
Figure 4: Deviations from the steady state after a one standard deviation impulse to the workers' bargaining power disturbance, $\epsilon_{\nu}$.

Wage bargaining power shock. Finally, Figure 4 presents the Bayesian IRFs for a shock to the relative bargaining power of workers. In the DSDE model, strengthening the workers’ bargaining power is expansionary in the short run and slightly contractionary in the medium run. On impact, the real wage increases by roughly 0.3%. This translates into an expansion of consumption which drives output up by 0.15%. In terms of the relationship between wage and output expansion, a 1%-point acceleration of wage inflation would increase output by about 0.4%, close to the steady state. Investment is depressed persistently which causes output to fall below the steady state after 10 quarters. Hence, firms substitute labor for capital despite an increasing real wage.

The DSDE-VAR seems to support these results in general. Other than the DSDE model, however, it suggests an expansion of investment on impact and a more hump-shaped and less
Table 3: Conditional variance decomposition (in percent) in period 1

<table>
<thead>
<tr>
<th></th>
<th>DSDE</th>
<th>DSGE</th>
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<tr>
<td></td>
<td>$\varepsilon_g$</td>
<td>$\varepsilon_a$</td>
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<tr>
<td>$\Delta \ln \tilde{Y}_t$</td>
<td>51.07</td>
<td>22.38</td>
</tr>
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<td>$\Delta \ln \tilde{C}_t$</td>
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<td>53.30</td>
</tr>
<tr>
<td>$\Delta \ln \tilde{I}_t$</td>
<td>.56</td>
<td>.39</td>
</tr>
<tr>
<td>$R_t - R$</td>
<td>14.78</td>
<td>22.93</td>
</tr>
<tr>
<td>$\Pi_{p,t} - \Pi_p$</td>
<td>13.21</td>
<td>46.92</td>
</tr>
<tr>
<td>$\Pi_{w,t} - \Pi_w$</td>
<td>8.79</td>
<td>20.79</td>
</tr>
<tr>
<td>$u_t - u$</td>
<td>22.70</td>
<td>53.71</td>
</tr>
</tbody>
</table>

pronounced response of unemployment.

In the DSGE model, a rise in the workers’ bargaining power slightly contracts output persistently. This is consistent with Groshenny (2009) who also finds a negative effect of the workers’ bargaining power on output. The rate of nominal wage inflation as well as the real wage increase by the same amount as in the DSDE model relative to the steady state. Yet, this does not translate into a rise in consumption as firms choose to not substitute labor for capital. Investment decreases only moderately. Because firms do not hire more labor, consumption does not increase. Overall output is slightly depressed due to decreasing investment. In the DSGE model, the strengthening of the workers’ position dampens output.

5 Variance decomposition

The conditional forecast error variance decomposition for the first period is documented in Table 3. It reports, for each of the observed variables, to what extent the seven structural shocks drive the variation observed in the data. In the DSDE model, around 70% of the variation in output is explained by shocks to government spending or investment. Productivity shocks explain only about 20%. This is in stark contrast to the DSGE model where productivity shocks explain 95% of the variation in output. The variation in consumption is explained by productivity shocks to 55% in both models. The similarity is due to the fact that both models feature rule-of-thumb households. Moreover, the variation in investment is rather exogenous, explained by exogenous disturbances to Tobin’s $q$ to around 98% in both models. In the DSDE model, monetary policy shocks explain around 45% of the variation in the interest rate. In the DSGE model with a monetary policy that responds aggressively to inflation, monetary policy shocks play a minor role. In explaining the
variation in price inflation, both models give most weight to productivity and demand shocks. In
the DSDE model, also bargaining power shocks drive price inflation due to the cost push effect.
Bargaining power shocks are also the primary source of variations in wage inflation in the DSDE
model. In the DSGE model, however, bargaining power shocks do not seem to drive wages, but
mark-up shocks. Most importantly, the explanation of unemployment variation differs considerably
between the two models. Overall, wages and prices are strongly affected by the labor market in the
DSDE model but not so much in the DSGE model. In the DSDE model, it is mainly productivity
shocks that, apart from demand shocks, that drive the variation of unemployment as it affects how
much labor is needed to serve a given product demand. In stark contrast to that, the DSGE model
assigns no role to productivity shocks. However, almost 60% are explained by labor supply shocks.
In the DSDE model, it is only around 10%.

To summarize, the main findings are the following: (i) Output is mainly driven by demand
shocks in the DSDE model, while it is supply shocks in the DSGE model. The emphasis on
demand shocks in the DSDE model confirms the findings of Farmer (2008) and Mian and Sufi
(2014). (ii) Shocks to the bargaining power explain a part of price inflation in the DSDE model
due to a cost push mechanism. (iii) While wage inflation is determined on the labor market in the
DSDE model, it is determined on the goods market in the DSGE model. (iv) The DSDE model
explains unemployment variation to a considerable extent by productivity and demand shocks.
The DSGE model traces back unemployment variation primarily to labor supply shocks. Demand
shocks are not seen as important.

6 Concluding remarks

In the present paper, I have estimated an empirical but still tractable variant of the Dynamic
Stochastic Disequilibrium (DSDE) model proposed by Schoder (2017a). It features involuntary
disequilibrium unemployment arising from insufficient aggregate demand. The two core differences
of the estimated model with respect to Schoder (2017a) are the assumption of an exogenously grow-
ing labor supply and the introduction of rule-of-thumb households which receive all wage income
and do not save. I estimate the DSDE model and, for the sake of evaluation and comparison,
the corresponding DSDE-VAR without theory implied cross-coefficient restrictions as well as the
corresponding Dynamic Stochastic Disequilibrium (DSDE) model with search and matching un-
employment. I exploit a data set for the Euro Area from 1980Q1 to 2016Q4. Standard Bayesian
estimation techniques introduced to the macroeconomic literature by An and Schorfheide (2007)
and Smets and Wouters (2003) have been employed.

(i) The data interpreted through the DSDE model assign an important role to fiscal policy
for economic stabilization. In the DSGE model, stabilization is achieved by monetary policy. The
fiscal multiplier is considerably higher in the former than in the latter. This is because a fiscal
stimulus raises the real wage in the DSDE model. It originates from a labor market tightening and
it stimulates consumption. (ii) In the DSDE model, the estimated shock persistence is lower than
in the DSGE model. Hence, the former requires less persistence in the shocks than the latter in
order to capture the persistence in the data. (iii) The DSDE model predicts the real wage to move
pro-cyclically with a lag as observed empirically. In the DSGE model, the real wage moves counter-
cyclically. (iv) The results for the DSDE model suggest that a productivity shock is contractionary
in the short run and slightly expansionary in the medium run. Rising productivity primarily lowers
the demand for labor in the short run. This increases unemployment and cuts into consumption.
At the same time it triggers and investment boom that drives output above the steady state after 8 quarters. (v) Strengthening the worker’s bargaining power is expansionary in the short run and contractionary in the medium run. This holds only for the DSDE model and not for the DSGE model where output decreases persistently. The expansionary effects in the DSDE model are driven by the rise in the real wage which stimulates consumption. However, it also depresses investment which starts dominating after 8 quarters. (vi) The forecast error variance decomposition reveals that, in the DSDE model, it is mainly demand shocks that drive the variation in output. In particular more than 70% of the output variation in the Euro Area can be explained by demand shocks. In the DSGE model, it is primarily productivity shocks. While wage inflation is determined on the labor market and driven by bargaining power shocks in the DSDE model, it is determined on the goods market and driven by mark-up shocks in the DSGE model. Finally, the variation of unemployment, in the DSDE model, is primarily caused by demand and productivity shocks that sum up to roughly 85%. In the DSGE model, 60% of the unemployment variation is explained by labor supply shocks. Demand shocks are not important.

These results suggest Keynesian policies: (i) Fiscal policy could be used as an effective tool to stabilize output even without a monetary policy that is constrained by the zero lower bound. This is because Ricardian equivalence which limits fiscal policy transmission in most DSGE models does not prevail in the DSDE model. (ii) The institutional framework of wage bargaining may affect wage inflation and, through a pronounced cost push effect, also price inflation. (iii) Strengthening the bargaining position of trade unions during times of slow growth may help to boost growth at least in the short run. This is perfectly consistent with the Bundesbank’s recommendation from July 2014 that German wages should increase.

Despite rather robust results, policy conclusions have to be drawn with caution. This is because at least three strong assumptions that keep the model tractable but should be relaxed in future research: (i) the assumption that there is no saving out of wage income; (ii) the assumptions of log utility and exogenous labor supply; (iii) the assumption that the Euro Area is a homogenous economy with a centralized wage bargaining process. Future research could consider an open economy model of a central Euro Area country trading with the rest of the Euro Area.

Finally, future research should address the identified weaknesses of the DSDE model in terms of impulse-response analysis. As indicated by the results of an unconstrained DSDE-VAR, the DSDE model fails to predict pro-cyclical investment, hump-shaped responses of unemployment and the demand components, and a cyclical return to the steady state.
References


A DSDE model appendix

This appendix derives all aggregated equations characterizing the DSDE model. It will be convenient to let $\Gamma \equiv \Gamma_n \Gamma_e$ denote the deterministic gross growth rate of the economy, respectively. $\Gamma_n$ is population growth and $\Gamma_e$ is labor-embodied productivity growth. We use the following notation: $\tilde{X}_t \equiv X_t / \Gamma_t$, $\bar{X}_t \equiv X_t / \Gamma^n_t$ and $\hat{X}_t \equiv X_t / \Gamma^n_e$ for any aggregated variable $X_t$ with $X$ denoting the steady state value. Note that a description of the variables is provided in Appendix A.

A.1 Households

Individuals are born into generations. Each household is born into the labor force and supplies labor hours which the firm will employ to some extent. This type of household is referred to as active. In each period, the active household faces the risk of a permanent income loss, i.e. becoming inactive, which it cannot insure against. Once the household is inactive it cannot return to the active state. It faces the risk of death with a constant probability. While active, the rational household will accumulate precautionary savings as a buffer for the time when inactive.

Let $\Theta_{a,t}$, $\Theta_{i,t}$, and $\Theta_{r,t}$ denote the beginning-of-period population sizes of active, inactive and rule-of-thumb households, respectively. Active households are born into generations growing by $\Gamma_n$ and face a per-period risk $U$ of becoming inactive. The law of motion of the active population size then is

$$\Theta_{a,t} = \Theta_{a,t-1} + \Theta_{a,0} \Gamma^n_t - U \Theta_{a,t-1}$$

where $\Theta_{a,0}$ is the size of the initial generation of active households. Inactive households face a per-period risk $D$ of dying. Hence, the law of motion of the inactive population size is

$$\Theta_{i,t} = \Theta_{i,t-1} + U \Theta_{a,t-1} - D \Theta_{i,t-1}.$$ 

The rule-of-thumb households are simply assumed to grow by $\Gamma_n$. Solving these difference equations, we obtain the following growth paths for each household type:

$$\Theta_{a,t} = \frac{\Theta_{a,0}}{1 - \frac{1-U}{\Gamma_n}} \Gamma^n_t = \theta_a \Gamma^n_t$$

$$\Theta_{i,t} = \frac{U \Theta_{a,0}}{(1 - \frac{1-U}{\Gamma_n})(1 - \frac{1-D}{\Gamma_n})} \Gamma^n_t = \theta_i \Gamma^n_t$$

$$\Theta_{r,t} = \theta_r \Gamma^n_t,$$

where we define $\theta_a \equiv \Theta_{a,0}/(1 - (1 - U)/\Gamma_n)$ and $\theta_i \equiv (U \Theta_{a,0})/((1 - (1 - U)/\Gamma_n)(1 - (1 - D)/\Gamma_n))$. Since all household types grow by the same rate, $\theta_a$, $\theta_i$, and $\theta_r$ can be interpreted as shares if we assume $\theta_a + \theta_i + \theta_r = 1$. Hence, for a given share of rule-of-thumb households $\theta_r$, we calibrate $\Theta_{a,0}$ such that $\theta_a + \theta_i = 1 - \theta_r$.

Inactive households. Let us consider, in period $t$, a representative inactive household $i$ which became inactive at the beginning of period $t - s$ with $s \in \{0, 1, \ldots, \infty\}$. Inactive households do not obtain labor or profit income and face a per-period probability of death. While we will make an assumption that ensures wealth to be homogeneous across active households, this is not the case
for inactive households. Households entering the inactive state at the same time \( t - s \) will have the same wealth, but households entering this state at different times \( t - s \) will have different stocks of wealth. To facilitate aggregation over inactive households, I adapt the perpetual youth framework proposed by Blanchard (1985). Inactive households have access to an insurance market to hedge against the risk of accidental bequests: The household sells, to an insurance company, the right to inherit its wealth \( R_t P_t b_{i,t-s,t} \) at the beginning of period \( t + 1 \) in case of dying between \( t \) and \( t + 1 \). \( R_t \) is the nominal interest rate, \( P_t \) the price level and \( b_{i,t-s,t} \) the end-of-period \( t \) real wealth held by the household which became inactive in \( t - s \). In turn, it receives the flow value of inheriting this wealth at the beginning of \( t + 1 \), i.e., \( D/(1 - D) R_t P_t b_{i,t-s,t} \) in \( t \) in case it is still alive. Its problem is to choose optimal paths for consumption \( c_{i,t-s,t} \) and wealth \( b_{i,t-s,t} \) conditional on being alive and subject to the per-period budget constraint

\[
P_t c_{i,t-s,t} + P_t b_{i,t-s,t} = R_{t-1} P_{t-1} b_{a,t-1}
\]

for the newly inactive household \((s = 0)\) and

\[
P_t c_{i,t-s,t} + P_t b_{i,t-s,t} = R_{t-1} P_{t-1} b_{i,t-s,t-1} + \frac{D}{1 - D} R_{t-1} P_{t-1} b_{i,t-s,t-1}
\]

for continuing inactive households \((s \in \{1, \ldots, \infty\})\). Note that newly inactive households do not get a payout from the insurance company.

**Continuing inactive households.** Let us start with the solution of the continuing inactive household. Assuming log-utility, the household’s problem can be represented by the following dynamic program:

\[
V_i(b_{i,t-s,t-1}) = \max_{c_{i,t-s,t}} \left\{ \ln c_{i,t-s,t} + \beta (1 - D) E_t V_i(b_{i,t-s,t}) \right\}
\]

\[
s.t. \quad c_{i,t-s,t} + b_{i,t-s,t} = \frac{1}{1 - D} R_{t-1} P_{t-1} b_{i,t-s,t-1}
\]

where \( \beta \) is the discount factor and \( \Pi_{p,t} \) is the gross rate of price inflation from \( t - 1 \) to \( t \). \( V_i(b_{i,t-s,t-1}) \) is the value function in \( t \) and \( b_{i,t-s,t-1} \) the state variable. Before solving this problem let us substitute out \( b_{i,t-s,t} \). We get

\[
V_i(b_{i,t-s,t-1}) = \max_{c_{i,t-s,t}} \left\{ \ln c_{i,t-s,t} + \beta (1 - D) E_t V_i \left( \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} b_{i,t-s,t-1} - c_{i,t-s,t} \right) \right\}.
\]

The FOC w.r.t. consumption implies

\[
\frac{1}{c_{i,t-s,t}} = \beta (1 - D) E_t V_i \left( \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} b_{i,t-s,t-1} - c_{i,t-s,t} \right)
\]

\[
= \beta (1 - D) E_t V_i \left( b_{i,t-s,t} \right).
\]

This equation implicitly gives us the optimal consumption for a given state, \( b_{i,t-s,t-1} \). I represent this dependence by the function \( c_i^*(b_{i,t-s,t-1}) \). Substituting this function back into the objective, we obtain

\[
V_i(b_{i,t-s,t-1}) = \ln c_i^*(b_{i,t-s,t-1}) + \beta (1 - D) E_t V_i \left( \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} b_{i,t-s,t-1} - c_i^*(b_{i,t-s,t-1}) \right)
\]
Applying Benveniste and Scheinkman (1979), i.e. taking the derivative w.r.t. $b_{i,t-s,t-1}$, noting that $c_i'(b_{i,t-s,t-1}) = 0$, and using the FOC w.r.t. consumption yields

$$V_i'(b_{i,t-s,t-1}) = \frac{1}{c_i'(b_{i,t-s,t-1})} c_i''(b_{i,t-s,t-1}) + \beta E_t V_i'(b_{i,t-s,t}) \left( \frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}} - c_i'(b_{i,t-s,t-1}) \right)$$

$$= \beta (1-D) E_t V_i'(b_{i,t-s,t}) \frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}}$$

$$= \frac{1}{c_{i,t-s,t}} \frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}}.$$ 

Iterating forward this result by one period and substituting into the FOC w.r.t. consumption yields

$$\frac{1}{c_{i,t-s,t}} = \beta E_t \frac{R_t}{\Pi_{p,t+1}} \frac{1}{c_{i,t-s,t+1}}$$

which is the well-known Euler equation for consumption. Iteratively substituting forward leads to

$$\frac{1}{c_{i,t-s,t}} = \beta^n E_t \prod_{k=1}^{n} \frac{R_{t+k-1}}{\Pi_{p,t+k}} \frac{1}{c_{i,t-s,t+n}}.$$ 

Having obtained the FOCs of the continuing inactive household, we can now derive a relation between consumption and wealth. To achieve this, we consider the beginning-of-period wealth in $t$, i.e. $1/(1-D)R_{t-1}/\Pi_{p,t}b_{i,t-s,t-1}$ for $s = 1, 2, \ldots, \infty$. Solving the budget constraint for $b_{i,t-s,t-1}$ yields

$$b_{i,t-s,t-1} = \left( \frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}} \right)^{-1} b_{i,t-s,t} + \left( \frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}} \right)^{-1} c_{i,t-s,t}.$$

Iterating this expression forward, recursively substituting into the budget constraint, and using the FOC w.r.t. consumption we can show that

$$\frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}} b_{i,t-s,t-1} = b_{i,t-s,t} + c_{i,t-s,t}$$

$$= \left( \frac{1}{1-D} \frac{R_{t}}{\Pi_{p,t+1}} \right)^{-1} b_{i,t-s,t+1} + \left( \frac{1}{1-D} \frac{R_{t}}{\Pi_{p,t+1}} \right)^{-1} c_{i,t-s,t+1} + c_{i,t-s,t}$$

$$= \sum_{n=0}^{\infty} \prod_{k=1}^{n} \left( \frac{1}{1-D} \frac{R_{t+k-1}}{\Pi_{p,t+k}} \right)^{-1} c_{i,t,t+n}$$

$$= \sum_{n=0}^{\infty} (1-D)^n \prod_{k=1}^{n} \left( \frac{R_{t+k-1}}{\Pi_{p,t+k}} \right)^{-1} c_{i,t-s,t+n} \beta^n E_t \prod_{k=1}^{n} \frac{R_{t+k-1}}{\Pi_{p,t+k}} \frac{1}{c_{i,t,t+n}} c_{i,t-s,t}$$

$$= \sum_{n=0}^{\infty} (\beta(1-D))^n c_{i,t-s,t}$$

$$= \frac{1}{1-\beta(1-D)} c_{i,t-s,t}$$

$$c_{i,t-s,t} = \kappa \frac{1}{1-D} \frac{R_{t-1}}{\Pi_{p,t}} b_{i,t-s,t-1}$$

where $\kappa = (1 - \beta(1 - D))$. Hence, the continuing inactive household’s choice of consumption is proportional to its previous wealth.
Newly inactive households. Assuming log-utility, the newly inactive household's ($s = 0$) problem can be represented by the following dynamic program:

$$V_i(b_{i,t,t-1}) = \max_{c_{i,t,t}} \{ \ln c_{i,t,t} + \beta (1 - D) E_t V_i(b_{i,t,t}) \}$$

subject to

$$c_{i,t,t} + b_{i,t,t} = \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1}$$

for $t = 1$

$$c_{i,t,t} + b_{i,t,t} = \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} b_{i,t,t-1}$$

for $t = 2, 3, \ldots$

Before solving this problem let us substitute out $b_{i,t,t}$. We get

$$V_i(b_{i,t,t-1}) = \max_{c_{i,t,t}} \{ \ln c_{i,t,t} + \beta (1 - D) E_t V_i \left( \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_{i,t,t} \right) \}.$$

The FOC w.r.t. consumption implies

$$\frac{1}{c_{i,t,t}} = \beta (1 - D) E_t V_i'(b_{i,t,t}).$$

This equation implicitly gives us the optimal consumption for a given state, $b_{i,t,s,t-1}$. I represent this dependence by the function $c_i^*(b_{i,t,s,t-1})$. Substituting this function back into the objective, we obtain

$$V_i(b_{i,t,t-1}) = \ln c_i^*(b_{i,t,t-1}) + \beta (1 - D) E_t V_i \left( \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_i^*(b_{i,t,t-1}) \right).$$

Applying Benveniste and Scheinkman (1979), i.e. taking the derivative w.r.t. $b_{i,t,s,t-1}$, noting that $c_i''(b_{i,t,t-1}) = 0$, and using the FOC w.r.t. consumption yields

$$V_i'(b_{i,t,t-1}) = \frac{1}{c_i^*(b_{i,t,t-1})} c_i''(b_{i,t,t-1}) + \beta E_t V_i'(b_{i,t,t}) \left( \frac{R_{t-1}}{\Pi_{p,t}} - c_i''(b_{i,t,t-1}) \right)$$

$$= \beta (1 - D) E_t V_i'(b_{i,t,t}) \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}}$$

$$= \frac{1}{c_{i,t,t}} \frac{R_{t-1}}{\Pi_{p,t}}.$$

Iterating forward this result by one period and substituting into the FOC w.r.t. consumption yields

$$\frac{1}{c_{i,t,s,t}} = \beta E_t \frac{R_t}{\Pi_{p,t+1}} \frac{1}{c_{i,t-s,t+1}}$$

which is the well-known Euler equation for consumption. Iteratively substituting forward leads to

$$\frac{1}{c_{i,t-s,t}} = \beta^n E_t \prod_{k=1}^{n} \frac{R_{t+k-1}}{\Pi_{p,t+k}} \frac{1}{c_{i,t-s,t+n}}.$$

Having obtained the FOCs of the continuing inactive household, we can now derive a relation between consumption and wealth. To achieve this, we consider the beginning-of-period wealth in
\[ t, \text{ i.e.} \frac{1}{(1 - D)}R_{t-1}/\Pi_{p,t}b_{i,t-s,t-1} \text{ for } s = 1, 2, \ldots, \infty. \] Solving the budget constraint for \( b_{i,t-s,t-1} \) yields
\[
b_{i,t-s,t-1} = \left( \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} \right)^{-1} b_{i,t-s,t} + \left( \frac{1}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} \right)^{-1} c_{i,t-s,t}.
\]
Iterating this expression forward, recursively substituting into the budget constraint, and using the FOC w.r.t. consumption we can show that
\[
\frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} = b_{i,t,t} + c_{i,t,t}
\]
\[
= \sum_{n=0}^{\infty} \prod_{k=1}^{n} \left( \frac{1}{1 - D} \frac{R_{t+k-1}}{\Pi_{p,t+k}} \right)^{-1} c_{i,t,t+n}
\]
\[
= \sum_{n=0}^{\infty} (1 - D)^n \prod_{k=1}^{n} \left( \frac{R_{t+k-1}}{\Pi_{p,t+k}} \right)^{-1} c_{i,t,t+n} \beta^n E_t \prod_{k=1}^{n} \frac{R_{t+k-1}}{\Pi_{p,t+k}} c_{i,t,t+n}
\]
\[
= \sum_{n=0}^{\infty} (\beta (1 - D))^n c_{i,t,t}
\]
\[
= \frac{1}{1 - \beta (1 - D)} c_{i,t,t}
\]
\[
c_{i,t,t} = \kappa \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1}
\]
where \( \kappa = (1 - \beta (1 - D)) \). Hence, the continuing inactive household’s choice of consumption is proportional to its previous wealth.

**Active households.** The active household faces a risk \( U \) of permanent income loss. We assume that households cannot lose income and die in the same period. Since we will assume a transfer ensuring that all active households have the same wealth independent from when they were born, we drop the \( t-s \) subscript for active households. It is convenient to set up the household’s problem of choosing consumption \( c_{a,t} \), labor supply \( n_t \) and wealth \( b_{a,t} \) as a dynamic program:
\[
V_a(b_{a,t-1}) = \max_{c_{a,t}} \left\{ V_c \ln c_{a,t} + \beta (1 - U) E_t V_a(b_{a,t}) + \beta U E_t V_i(b_{a,t}) \right\}
\]
\[
\text{s.t.} \quad c_{a,t} + b_{a,t} = h_t - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1}
\]
where \( h_t \) and \( t_t \) denote the distributed profits and a lump-sum government tax, respectively. \( V_c \) is a preference scaling parameter. The variable \( \tau_t \) needs some clarification (cf. Carroll and Jeanne 2009): A natural consequence of the population dynamics assumed is that the stock of wealth increases with the age of the household. To simplify aggregation, we assume a transfer from non-newborn to newborn household which ensures that wealth is distributed equally across active households at any point in time. A newborn household receives a transfer of \( R_{t-1}/\Pi_{p,t}b_{a,t-1} - \tau_t \). The non-newborn household must give away \( \tau_t \) of its wealth. The crucial implication of this transfer is that both
newborn and non-newborn households face the same budget constraint after the transfer. Assuming that the transfer is financed by a tax on beginning-of-period wealth, i.e. \( \tau_t = \tau R_{t-1}/\Pi_{p,t} b_{a,t-1} \), what is the tax rate \( \tau \)? The payments aggregated over all non-newborn active households with mass \((1/(1 - (1 - U)/\Gamma_n)) - 1)\Theta_{a,0}\Gamma_n\) must equal the receipts aggregated over all newborn active households with mass \(\Theta_{a,0}\Gamma_n\), i.e.

\[
\left( \frac{1}{1 - (1 - U)/\Gamma_n} - 1 \right) \Theta_{a,0}\Gamma_n \tau R_{t-1} \Pi_{p,t} b_{a,t-1} = \Theta_{a,0}\Gamma_n \frac{R_{t-1}}{\Pi_{p,t}} (b_{a,t-1} - \tau b_{a,t-1}).
\]

From this, it is easy to see that the required tax rate is \( \tau = 1 - (1 - U)/\Gamma_n \).

Note that \( V_a(b_{a,t-1}) \) is the value function in \( t \) and \( b_{a,t-1} \) the state variable. Note further that \( V_i(b_{a,t}) \) is the \( (t+1) \) value function of a household that became inactive at the beginning of \( t+1 \). Before solving this problem let us substitute out \( b_{a,t} \). We get

\[
V_a(b_{a,t-1}) = \max_{c_{a,t}} \left\{ +\beta (1 - U) E_t V_a \left( h_t - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_{a,t} \right) + \right\}
\]

The FOC w.r.t. consumption implies

\[
\frac{V_c}{c_{a,t}} = \beta (1 - U) E_t V_a' \left( h_t - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_{a,t} \right) + \left. + \beta U E_t V_i' \left( h_t - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_{a,t} \right) \right.
\]

\[
= \beta (1 - U) E_t V_a' (b_{a,t}) + \beta U E_t V_i' (b_{a,t}).
\]

This equation implicitly gives us the optimal consumption for a given state, \( b_{a,t-1} \). We represent this dependence by the function \( c_a^*(b_{a,t-1}) \). Substituting this function back into the objective, we obtain

\[
V_a(b_{a,t-1}) = \max_{c_{a,t}} \left\{ V_c \ln c_{a,t} + +\beta (1 - U) E_t V_a \left( h_t - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_{a,t} \right) + \right\}
\]

\[
+ \beta U E_t V_i' \left( h_t - t_t - \tau_t + \frac{R_{t-1}}{\Pi_{p,t}} b_{a,t-1} - c_{a,t} \right)
\]

Following Benveniste and Scheinkman (1979), i.e. taking the derivative w.r.t. \( b_{a,t-1} \), noting that \( c_a'(b_{a,t-1}) = 0 \), and using the FOC w.r.t. consumption yields

\[
V_a'(b_{a,t-1}) = \frac{V_c}{c_a^*(b_{a,t-1})} c_a''(b_{a,t-1}) + + \beta (1 - U) E_t V_a' (b_{a,t}) \left( \frac{R_{t-1}}{\Pi_{p,t}} - c_a'(b_{a,t-1}) \right) + \beta U E_t V_i' (b_{a,t}) \left( \frac{R_{t-1}}{\Pi_{p,t}} - c_a'(b_{a,t-1}) \right)
\]

\[
= \beta (1 - U) E_t V_a' (b_{a,t}) \frac{R_{t-1}}{\Pi_{p,t}} + \beta U E_t V_i' (b_{a,t}) \frac{R_{t-1}}{\Pi_{p,t}}
\]

\[
= \frac{V_c R_{t-1}}{c_a, t \Pi_{p,t}}
\]

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Iterating forward this result as well as the previous result for the inactive household $V_i'(b_{i,t,t-1}) = \frac{1}{c_{i,t}} \frac{R_{t-1}}{\Pi_{p,t}}$ by one period and substituting into the FOC w.r.t. consumption yields

$$V_c\frac{c_{a,t}}{c_{a,t}} = \beta E_t \frac{R_t}{\Pi_{p,t+1}} \left( (1 - U) \frac{V_c}{c_{a,t+1}} + U \frac{1}{c_{a,t+1}} \right)$$
$$V_c\frac{c_{a,t}}{c_{a,t}} = \beta (1 - U) E_t \frac{R_t}{\Pi_{p,t+1}} \frac{V_c}{c_{a,t+1}} + \beta U \frac{1}{\kappa b_{a,t}}$$

where the last line uses the FOC of the newly inactive household. The expected marginal utility of consumption in $t+1$ takes into account the risk of income loss.

**Rule-of-thumb households** Worker households or rule-of-thumb households are excluded from financial markets and do not pay taxes. All worker households are assumed to be equally affected by unemployment which, for simplicity, occurs on the intensive margin. Since worker households do not save, their problem is static and reads

$$\max_{c_r,t} \ln c_r,t - \frac{\theta_r}{V_{n,t}}n_t$$
$$s.t. \quad c_r,t = \omega_t(1 - u_t)n_t$$

where $n_t$, $\omega_t$, $u_t$, and $V_{n,t}$ are labor supply, the real wage, the unemployment rate, and a labor supply shock process. Multiplying the disutility of labor by $\theta_r$ will facilitate aggregation. The FOCs w.r.t. consumption and labor supply are,

$$c_r,t = \omega_t(1 - u_t)n_t$$

and The FOC w.r.t. labor supply implies, after substituting in the FOC w.r.t. consumption,

$$\frac{\theta_r}{V_{n,t}} = \frac{1}{c_r,t} \omega_t(1 - u_t)$$
$$\theta_r n_t = V_{n,t}$$

respectively.

**Aggregation.** The inactive household’s wealth $b_{i,t-s,t}$ depends on its age $s$ and on how much it brought over from the active state. Due to the FOC w.r.t. consumption, consumption will vary across households. Aggregation of a stock or flow variable $x_{i,t-s,t}$ of the inactive household is summing over all households. Noting that inactive households of age $s$ are a homogeneous cohort of size $(1 - D)^s U \Theta_{a,t-1-s}$, we can define the aggregated stock or flow as $X_{i,t} \equiv \sum_{s=0}^{\infty} (1 - D)^s U \Theta_{a,t-1-s} x_{i,t-s,t}$.

Using these definitions let us first aggregate the inactive households’ budget constraints for period $t$. I first aggregate each cohort $s$ separately and then sum over the cohorts, as illustrated in Table 4. Note that, unlike households that have been inactive before, newly inactive households do not receive a payment from the insurance company. Since $B_{a,t-1} = \Theta_{a,t-1} b_{a,t-1}$, their initial wealth carried over from the active state is simply $UR_{t-1}/\Pi_{p,t} B_{a,t-1}$. Note further that the beginning-of-period $t$ wealth of all continuing inactive households is the end-of-period $t - 1$ wealth of all inactive households in $t - 1$ plus the end-of-period $t - 1$ wealth of the inactive households who died at the
Table 4: Aggregation over inactive households in period $t$.

\[
\begin{array}{lll}
(1 - D)^0 U \Theta_{a,t-1} c_{t-0,t} + (1 - D)^1 U \Theta_{a,t-1} b_{t-0,t} &=& (1 - D)^0 U \Theta_{a,t-1} \frac{R_{t-1}}{\Pi_{p,t}} b_{t-0,t-1} + (1 - D)^1 U \Theta_{a,t-1} \frac{R_{t-1}}{\Pi_{p,t}} b_{t-0,t-1} \\
(1 - D)^1 U \Theta_{a,t-2} c_{t-1,t} + (1 - D)^2 U \Theta_{a,t-2} b_{t-1,t} &=& (1 - D)^1 U \Theta_{a,t-2} \frac{R_{t-1}}{\Pi_{p,t}} b_{t-1,t-1} + (1 - D)^1 U \Theta_{a,t-2} \frac{R_{t-1}}{\Pi_{p,t}} b_{t-1,t-1} \\
\vdots & \vdots & \vdots \\
(1 - D)^\infty U \Theta_{a,t-\infty} c_{t-\infty,t} + (1 - D)^\infty U \Theta_{a,t-\infty} b_{t-\infty,t} &=& (1 - D)^\infty U \Theta_{a,t-\infty} \frac{R_{t-1}}{\Pi_{p,t}} b_{t-\infty,t-1} + (1 - D)^\infty U \Theta_{a,t-\infty} \frac{D}{1 - D} \frac{R_{t-1}}{\Pi_{p,t}} b_{t-\infty,t-1} \\
\end{array}
\]

\[
C_{i,t} + B_{i,t} = \frac{R_{t-1}}{\Pi_{p,t}} B_{i,t-1} + U \frac{R_{t-1}}{\Pi_{p,t}} B_{i,t-1} + \frac{R_{t-1}}{\Pi_{p,t}} B_{a,t-1}
\]
Table 5: Aggregation over active households in period $t$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - U)^{0} \Theta_{a,t} \Gamma_{n}^{t-0} c_{a,t}$</td>
<td>$(1 - U)^{0} \Theta_{a,t} \Gamma_{n}^{t-0} b_{a,t}$</td>
</tr>
<tr>
<td>$(1 - U)^{1} \Theta_{a,t} \Gamma_{n}^{t-1} c_{a,t}$</td>
<td>$(1 - U)^{1} \Theta_{a,t} \Gamma_{n}^{t-1} b_{a,t}$</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>$(1 - U)^{\infty} \Theta_{a,t} \Gamma_{n}^{t-\infty} c_{a,t}$</td>
<td>$(1 - U)^{\infty} \Theta_{a,t} \Gamma_{n}^{t-\infty} b_{a,t}$</td>
</tr>
<tr>
<td>$\sum_{s=0}^{\infty} (1 - U)^{s} \Theta_{a,t} \Gamma_{n}^{t-s} c_{a,t}$</td>
<td>$\sum_{s=0}^{\infty} (1 - U)^{s} \Theta_{a,t} \Gamma_{n}^{t-s} b_{a,t}$</td>
</tr>
<tr>
<td>Using the solution of geometric series, we get:</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{1 - \frac{1}{\Gamma_{n}}} \Theta_{a,t} \Gamma_{n}^{t} c_{a,t}$</td>
<td></td>
</tr>
<tr>
<td>$+ \frac{1}{1 - \frac{1}{\Gamma_{n}}} \Theta_{a,t} \Gamma_{n}^{t} b_{a,t}$</td>
<td></td>
</tr>
<tr>
<td>$+ \left( \frac{1}{1 - \frac{1}{\Gamma_{n}}} - 1 \right) \Theta_{a,t} \Gamma_{n}^{t} \frac{R_{t-1}}{\Pi_{p,t}} \tau b_{a,t-1}$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{1}{1 - \frac{1}{\Gamma_{n}}} \Theta_{a,t} \Gamma_{n}^{t} t$</td>
<td>$+ \frac{1}{1 - \frac{1}{\Gamma_{n}}} \Theta_{a,t} \Gamma_{n}^{t} \frac{R_{t-1}}{\Pi_{p,t}} \tau b_{a,t-1}$</td>
</tr>
<tr>
<td>Using the previous result that $\tau = 1 - (1 - U)/\Gamma_{n}$</td>
<td></td>
</tr>
<tr>
<td>$C_{a,t}$ + $B_{a,t}$ + $(\Gamma_{n} - (1 - U)) \frac{R_{t-1}}{\Pi_{p,t}} B_{a,t-1} - \Theta_{a,t} \Gamma_{n}^{t} \frac{R_{t-1}}{\Pi_{p,t}} \tau b_{a,t-1}$</td>
<td>$Z_{t}$ + $\frac{R_{t-1}}{\Pi_{p,t}} B_{a,t-1} - \Theta_{a,t} \Gamma_{n}^{t} \frac{R_{t-1}}{\Pi_{p,t}} \tau b_{a,t-1}$</td>
</tr>
</tbody>
</table>
beginning of \( t \). This wealth of the latter group (inactive in \( t - 1 \) and died in \( t \)) is transferred to the former group (inactive in \( t - 1 \) and still inactive in \( t \)) through the insurance company. After normalizing by the deterministic productivity trend \( \Gamma^t \), we obtain the aggregated budget constraint of the inactive households as

\[
C_{i,t} + B_{i,t} = \frac{R_{t-1}}{\Pi_{p,t}} (B_{i,t-1} + UB_{a,t-1})
\]

\[
\frac{C_{i,t}}{\Gamma^t} + \frac{B_{i,t}}{\Gamma^t} = \frac{R_{t-1}}{\Pi_{p,t}} \left( \frac{B_{i,t-1}}{\Gamma^t} + UB_{a,t-1} \right)
\]

\[
\tilde{C}_{i,t} + \tilde{B}_{i,t} = \frac{1}{\Gamma} \frac{R_{t-1}}{\Pi_{p,t}} (\tilde{B}_{i,t-1} + UB_{a,t-1})
\]

(A.1)

The inactive household’s FOC tells us that its period \( t \) consumption will be proportional to its beginning-of-period wealth. As seen in Table 4, the continuing inactive household’s wealth \( 1/(1-D)R_{t-1}/\Pi_{p,t}B_{i,t-s,t-1} \) aggregates to \( R_{t-1}/\Pi_{p,t}B_{i,t-1} \). The newly inactive household’s wealth \( R_{t-1}/\Pi_{p,t}B_{a,t-1} \) aggregates to \( UR_{t-1}/\Pi_{p,t}B_{a,t-1} \). Since the proportionality factor \( \kappa \) is constant for both types of inactive households, we aggregate the FOC w.r.t. consumption as

\[
\tilde{C}_{i,t} = \kappa \frac{1}{\Gamma} \frac{R_{t-1}}{\Pi_{p,t}} (\tilde{B}_{i,t-1} + UB_{a,t-1})
\]

(A.2)

Since active households hold identical stocks of wealth \( b_{a,t} \) in any period \( t \) independent of their age, aggregation of stocks and flows is simply \( \theta_a \Gamma^t_n x_{a,t} \) for any variable \( x_{a,t} \) of the active household in \( t \), recalling that \( \theta_a \Gamma^t_n \) is the mass of active households in period \( t \). Recall further that beginning-of-period \( t \) active wealth is end-of-period \( t - 1 \) active wealth minus the wealth that became inactive at the beginning of \( t \). The aggregation is illustrated step-by-step in Table 5 where \( z_t = h_t - t_t \). The active households aggregated and detrended budget constraints reads

\[
\tilde{C}_{a,t} + \tilde{B}_{a,t} + \tilde{T}_t = \tilde{H}_t + (1 - U) \frac{1}{\Gamma} \frac{R_{t-1}}{\Pi_{p,t}} \tilde{B}_{a,t-1}
\]

(A.3)

where \( \tilde{H}_t \) are distributed profits by the intermediate good firms to be specified below. Aggregating the FOC w.r.t. consumption for active households is simple as they are identical. Hence, it is easy to show that

\[
\frac{V_c}{C_{a,t}} = \beta (1 - U) \frac{1}{\Gamma} \frac{R_t}{\Pi_{p,t+1}} \frac{V_c}{C_{a,t+1}} + \beta U \frac{1}{\kappa B_{a,t}}
\]

(A.4)

Aggregate consumption of the rule-of-thumb households can be derived as

\[
\theta_r \Gamma^t_n c_{r,t} = \theta_r \Gamma^t_n \omega_t (1 - u_t) n_t
\]

\[
C_{r,t} = \omega_t (1 - u_t) N_t
\]

\[
\frac{C_{r,t}}{\Gamma^t} = \frac{\omega_t (1 - u_t) N_t}{\Gamma^t}
\]

\[
\tilde{C}_{r,t} = \omega_t (1 - u_t) \tilde{N}_t
\]

(A.5)

where unemployment rate is implicitly defined by

\[
1 - u_t = \frac{L_t}{N_t}
\]

(A.6)
Aggregation of the FOC w.r.t. labor supply leads to
\[ \theta_r n_t = V_{n,t} \]
\[ \theta_r \Gamma_n n_t = V_{n,t} \]
\[ \hat{N}_t = V_{n,t}. \]  
\( (A.7) \)

A.2 Firms

**Final good firms.** Taking as given the price \( p_{i,t} \), the final good firm’s demand for the intermediate good \( y_{i,t} \) supplied by an intermediate good firm \( i \) of a continuum of mass one can be obtained from the following cost minimization problem:

\[
\min_{y_{i,t}} \int_0^1 p_{i,t} y_{i,t} di \\
\text{s.t. } Y_t = \int_0^1 (y_{i,t} + \frac{1}{V_{p,t}} di)^{1+V_{p,t}},
\]

where \( Y_t \) is total output and \( 1+1/V_{p,t} > 1 \) is the elasticity of substitution. \( V_{p,t} \) has the interpretation of the mark-up over marginal costs in the absence of price adjustment costs. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index, \( P_t \), one can show the FOC to read

\[
y_{i,t} = \left( \frac{p_{i,t}}{P_t} \right)^{1+\frac{1}{V_{p,t}}} Y_t.
\]

**Intermediate good firms.** Taking as given total output \( Y_t \), the overall price level \( P_t \), and the wage rate \( \omega_t \) as well as the law of motion of capital, the production function, the demand function for intermediate goods, and the requirement to maintain a debt-capital ratio \( \lambda \), the firm \( i \) chooses \( \{p_{i,t}, l_{i,t}, i_{i,t}, k_{i,t}, d_{i,t}\}_{t=0}^{\infty} \) to maximize discounted inter-temporal distributed profits. Dropping the firm index for convenience and evaluating at period \( t = 0 \), the optimization problem reads

\[
\max_{\{p_t, l_t, i_t, k_t, d_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \frac{P_0}{P_t} \Lambda_0,t \left[ \begin{array}{c}
 p_t y_{t} - \omega_t l_{t} - P_t d_{t} - P_t \frac{\tau_p}{2} \Gamma^t \left( \frac{p_t}{P_{t-1}} - \Pi_p \right)^2 + P_t d_{t} - R_{t-1} P_{t-1} d_{t-1} \\
 + \end{array} \right]
\]

\text{s.t. } k_t = \left( 1 - \frac{\tau_i}{2} \left( \frac{i_t}{\Gamma_{t-1}^i} - 1 \right)^2 \right) i_t + (1 - \delta) k_{t-1}

\[ y_t = V_{a,t} \left( \Gamma k_{t-1} \right)^{\alpha} \left( \Gamma_{t}^l l_t \right)^{1-\alpha} \]

\[ y_t = \left( \frac{p_t}{P_t} \right)^{1+\frac{1}{V_{p,t}}} Y_t \]

\[ d_t = \lambda_{t} q_{t} k_{t} \]

where \( i_t, k_t, d_t, V_{a,t} \) and \( q_t \) are investment, the capital stock, outstanding bonds, total factor productivity, and the shadow price of capital, respectively. \( \tau_i, \delta, \tau_p, \) and \( \alpha \) denote the capital adjustment costs scaling parameter, the rate of capital depreciation, the price adjustment costs
scaling parameter, and the capital elasticity of production. \( \Lambda_{t,t+1} \) is the stochastic discount factor which expresses the value of a unit real profit in time \( t+j \) in terms of the value of a unit real profit in time \( t \). Note that the definition of the stochastic discount factor implies that \( E_t \frac{R_t}{\Pi_{p,t+1}} \Lambda_{t,t+1} = 1 \). Then, the FOC w.r.t. \( d_t \) is

\[
\frac{P_t}{P_t} \Lambda_{t,t} P_t + \frac{P_t}{P_t} \Lambda_{t,t} P_t \mu_t - E_t \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} R_t P_t = 0 \\
1 + \mu_t - E_t \frac{R_t}{\Pi_{p,t+1}} \Lambda_{t,t+1} = 0 \\
\mu_t = 0.
\]

The financial structure of the firm is irrelevant from the household’s perspective. Regarding the price decision, note that all firms charge the same price, \( p_t = P_t \), and, hence \( y_t = Y_t \) with a mass one of firms. The FOC w.r.t. \( p_t \) then implies

\[
\left[ -\frac{1}{V_{p,t}} \Psi - P_t \Gamma p \left( \frac{\rho_t}{\rho_{t-1}} - \Pi_t \right) \frac{1}{\rho_{t-1}} + \left( 1 + \frac{1}{V_{p,t}} \right) P_t \varphi_t \frac{\rho_t}{\rho_{t-1}} \right] = 0 \\
\left( \frac{1}{V_{p,t}} - \left( 1 + \frac{1}{V_{p,t}} \right) \varphi_t \right) \dot{Y_t} + \tau_p (\Pi_{p,t} - \Pi_p) \Pi_{p,t} - E_t \Lambda_{t,t+1} \Gamma \tau_p (\Pi_{p,t+1} - \Pi_p) \Pi_{p,t+1} = 0 \quad (A.8)
\]

where

\[
\Lambda_{t-1,t} = \beta \left( \frac{(1-U)c_{a,t}^{-1} + U(1+D)c_{a,t-1}^{-1}}{c_{a,t-1}^{-1}} \right) = \left( \frac{R_t^{-1}}{\Pi_{p,t}} \right)^{-1} \quad (A.9)
\]

To derive the implications of the FOC w.r.t. \( l_t \) first note that the production function can be rewritten as \( \left( \frac{\Gamma k_{t-1}}{\Gamma k_{t-1}} \right)^{-\alpha} = \left( \frac{y_t}{\Gamma k_{t-1}} \frac{1}{V_{a,t}} \right)^{\frac{\alpha}{1-\alpha}} \). Then,

\[
-w_t + P_t \varphi_t V_{a,t} \Gamma e^{(1-\alpha)(1-\alpha)l_t^{-\alpha} (\Gamma k_{t-1})^\alpha} = 0 \\
\varphi_t = \tilde{\omega}_t \frac{1}{1-\alpha} \left( \frac{1}{V_{a,t}} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{\dot{Y}_t}{\dot{K}_{t-1}} \right)^{\frac{\alpha}{1-\alpha}} \quad (A.10)
\]

The FOC w.r.t. to \( i_t \) implies

\[
1 = \left[ q_t \left( 1 - \frac{\tau_i}{\Gamma_{t-1}} \left( \frac{i_t}{\Gamma_{t-1}} - 1 \right)^2 - \tau_i \left( \frac{i_t}{\Gamma_{t-1}} - 1 \right) \frac{i_t}{\Gamma_{t-1}} \right) + \right] \\
+ E_t \Lambda_{t,t+1} q_{t+1} \Gamma \tau_i \left( \frac{i_{t+1}}{\Gamma_{t+1}} - 1 \right) \left( \frac{i_{t+1}}{\Gamma_{t+1}} \right)^2 \\
to which we add a shock process \( V_{i,t} \) to obtain

\[
V_{i,t} = \left[ q_t \left( 1 - \frac{\tau_i}{\Gamma_{t-1}} \left( \frac{i_t}{\Gamma_{t-1}} - 1 \right)^2 - \tau_i \left( \frac{i_t}{\Gamma_{t-1}} - 1 \right) \frac{i_t}{\Gamma_{t-1}} \right) + \right] \\
+ E_t \Lambda_{t,t+1} q_{t+1} \Gamma \tau_i \left( \frac{i_{t+1}}{\Gamma_{t+1}} - 1 \right) \left( \frac{i_{t+1}}{\Gamma_{t+1}} \right)^2 \quad (A.11)
\]
Recalling that $\mu_t = 0$ and noting that $V_{a,t} \left( \frac{\Gamma_{k_t-1}^{r_t-1}}{\Gamma_{k_t}^{r_t}} \right)^{\alpha-1} = \frac{y_t}{\Gamma_{k_t-1}^{r_t-1}}$, the FOC w.r.t. $k_t$ implies

$$
P_t q_t = \frac{P_t}{P_{t+1}} \Lambda_{t,t+1} \left( P_{t+1} \varphi_{t+1} V_{a,t+1} \left( \frac{r_{t+1}^{r_{t+1}-1}}{\Gamma_{r_{t+1}}} \right)^{1-\alpha} \left( \Gamma_{k_t} \right)^{\alpha} \frac{1}{k_t} + P_{t+1} q_{t+1} (1 - \delta) \right)$$

$$
q_t = \Lambda_{t,t+1} \left( \varphi_{t+1} \frac{y_{t+1}}{\Gamma_{k_t}} + q_{t+1} (1 - \delta) \right)
$$

$$
q_t = \Lambda_{t,t+1} \left( \varphi_{t+1} \frac{\tilde{Y}_{t+1}}{K_{t+1}} + q_{t+1} (1 - \delta) \right) \tag{A.12}
$$

The aggregate law of motion of the capital stock normalized by trend growth is

$$
\Gamma \tilde{K}_t = (1 - \delta) \tilde{K}_{t-1} + \left( 1 - \frac{\tau_t}{2} \left( \frac{\bar{I}_t}{I_{t-1}} - 1 \right) \right)^2 \tilde{I}_t \tag{A.13}
$$

The production function can be aggregated as follows. Note that all firms set the same price, $p_t = P_t$.

$$
\tilde{Y}_t = V_{a,t} \tilde{K}_{t-1}^{\alpha} L_t^{1-\alpha} \tag{A.14}
$$

Recalling that firms maintain a debt-capital ratio of $\lambda$, the aggregated detrended real distributed profits are

$$
\tilde{H}_t = \tilde{Y}_t - \bar{\omega}_t \bar{L}_t - (1 - \lambda) \bar{I}_t - \frac{7p}{2} (\Pi_{p,t} - \Pi_p)^2 \tag{A.15}
$$

The growth rate of the real wage is linked to wage and price inflation according to

$$
\frac{\bar{\omega}_t}{\bar{\omega}_{t-1}} - 1 = \Pi_{w,t} - \Pi_{p,t} \tag{A.16}
$$

### A.3 Wage, fiscal and monetary policy

We assume that the rate of wage inflation is subject to a bargaining process between a workers’ and a firms’ representative. The respective return functions are crucial for the bargaining game. We take the steady-state real wage $\bar{\omega}$ which can be computed for a given wage inflation $\Pi_{w,t}$, i.e., $\bar{\omega}(\Pi_{w,t})$, as the worker’s return and the steady-state profit rate, $r(\Pi_{w,t})$, as the firm’s return. Due to the presence of price adjustment costs, the former can be shown to increase and the latter to decrease in the rate of wage inflation. Hence, we suggest that the bargaining parties are concerned with the long-run implications of the bargaining. By assuming that the state of the labor market affects the relative bargaining power, however, the rate of wage inflation will be cyclical. The bargaining game reads

$$
\max_{\Pi_{w,t}} (\bar{\omega}(\Pi_{w,t}))^{\nu_t} (r(\Pi_{w,t}))^{1-\nu_t}
$$

where $\bar{\omega}(\cdot)$ and $r(\cdot)$ are the steady states of the real wage and the profit rate, respectively, as functions of the wage inflation and $nu_t$ is the worker’s relative bargaining power. The FOC of this problem determines the rate of nominal wage inflation, $\Pi_{w,t}$, and reads

$$
1 = (1 - 1/\nu_t) \bar{\omega}(\Pi_{w,t}) \frac{r'(\Pi_{w,t})}{r(\Pi_{w,t})} \bar{\omega}'(\Pi_{w,t}) \tag{A.17}
$$
where

\[
\frac{\nu_t}{\nu} = \left( \frac{\nu_{t-1}}{\nu} \right)^{\rho_{\nu}} \left( \frac{1 - u_t}{1 - u} \right)^{\phi_{\nu u}(1-\rho_{\nu})} \exp(\varepsilon_{\nu, t}) \tag{A.18}
\]

Note that there is no feedback of the labor market to wage formation if \( \phi_{\nu u} = 0 \). In this case, the rate of wage inflation is constant.

We assume taxes to be a-cyclical with the structural budget to be balanced, i.e.

\[ \tilde{T}_t = \tilde{G}. \tag{A.19} \]

The budget deficit follows the rule

\[
\frac{\tilde{G}_t}{G} = \left( \frac{\tilde{Y}_t}{\bar{Y}} \right)^{-\phi_{gy}} V_{g,t} \tag{A.20}
\]

Monetary policy follows a Taylor rule

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_r} \left( \frac{\Pi_{p,t}}{\Pi_p} \right)^{\phi_{ry}(1-\rho_r)} \left( \frac{\tilde{Y}_t}{\bar{Y}} \right)^{\phi_{ry}(1-\rho_r)} \exp(\varepsilon_{r,t}) \tag{A.21}
\]

### A.4 Market clearing

Aggregating over the households budget constraint and the firms profits yields the macroeconomic balance condition

\[
\tilde{Y}_t = \tilde{C}_{a,t} + \tilde{C}_{i,t} + \tilde{C}_{r,t} + \tilde{I}_t + \tilde{G}_t + \frac{\tau_p}{2} (\Pi_{p,t} - \Pi_p)^2. \tag{A.22}
\]

### A.5 Remaining exogenous processes

\[
V_{g,t} = V_{g,t-1}^{\rho_g} \exp(\varepsilon_{g,t}) \tag{A.23}
\]

\[
V_{a,t} = V_{a,t-1}^{\rho_a} \exp(\varepsilon_{a,t}) \tag{A.24}
\]

\[
V_{i,t} = V_{i,t-1}^{\rho_i} \exp(\varepsilon_{i,t}) \tag{A.25}
\]

\[
V_{n,t} = \left( \frac{V_{n,t-1}}{V_n} \right)^{\rho_n} \exp(\varepsilon_{n,t}) \tag{A.26}
\]

\[
V_{p,t} = \left( \frac{V_{p,t-1}}{V_p} \right)^{\rho_p} \exp(\varepsilon_{p,t}) \tag{A.27}
\]


**B DSGE model appendix**

The DSGE model differs from the DSDE model in only two but essential aspects. First, the risk of permanent income loss is zero, \( U = 0 \). Second, the labor market exhibits search and matching frictions causing equilibrium unemployment rather than disequilibrium unemployment.

Without the risk of permanent income loss, the household’s problem yields the standard FOC,

\[
\frac{1}{\bar{C}_{a,t}} = \beta \frac{1}{\Gamma} \frac{E_t}{\Pi_{p,t+1}} \frac{1}{\bar{C}_{a,t+1}}
\]

(B.1)

where \( \bar{C}_{a,t} \), \( R_t \), and \( \Pi_p \) denote aggregate and detrended consumption of the active households, the gross interest rate and the gross price inflation rate. \( \beta \) and \( \Gamma \) are the discount rate and the gross growth rate of the economy, respectively.

The labor market is a simplified version of Groshenny (2009), Christoffel et al. (2009), and Stähler and Thomas (2012). The household’s labor is differentiated and sold to a labor firm which produces labor services to be sold to the intermediate good firm. Each labor firm can hire only one unit of labor to produce one unit of labor services sold to the intermediate firms on a competitive labor market. Matches between unemployed labor units and open vacancies are determined by the following technology,

\[
\hat{M}_t = \kappa_m (\hat{u}_t \hat{N}_t)^{\kappa_e} \hat{V}_t^{1-\kappa_e}
\]

(B.2)

where \( \kappa_m \), \( \kappa_e \), \( \hat{M}_t \), \( \hat{V}_t \), \( \hat{u}_t \), \( \hat{N}_t \) denote the matching efficiency, the matching elasticity, the number of matches, the number of vacancies, the beginning of period unemployment rate, and the supply of labor units, respectively. The beginning-of-period and end-of-period unemployment rates are defined as

\[
\hat{u}_t = \frac{\hat{N}_{t-1} - (1-s)\hat{L}_{t-1}}{\hat{N}_t \Gamma_n}
\]

(B.3)

and

\[
u_t = 1 - \frac{\hat{L}_t}{\hat{N}_t}
\]

(B.4)

respectively, where \( s \) is the exogenous separation rate, or labor firm destruction rate. The law of motion of labor services is

\[
\hat{L}_t = (1-s)/\Gamma_n \hat{L}_{t-1} + \hat{M}_t.
\]

(B.5)

Every period a fraction \( 1 - \xi \) of both newly created and continuing labor firms can renegotiate the nominal wage with the labor units. Expressed in real terms the evolution of the real wage is

\[
\bar{\omega}_t = (1-\xi)\bar{\omega}_t^\# + \xi \frac{\bar{\omega}_{t-1}}{\Gamma_e \Pi_{p,t}}
\]

(B.6)

The probabilities to fill a vacancy and to find a job are

\[
p_{v,t} = \frac{\hat{M}_t}{\hat{V}_t}
\]

(B.7)
and

\[ p_{u,t} = \frac{\dot{M}_t}{\dot{u}_t N_t}, \tag{B.8} \]

respectively. The value function of a negotiating new or continuing labor firm, i.e. a matching, and the value function of a non-negotiating new or continuing labor firm are

\[ \tilde{J}^\#_t = \tilde{A}_{1,t} - A_{2,r,t} \tilde{\omega}^\#_t + E_t \Lambda_{t,t+1} (1 - \xi)(1 - s) \Gamma_e \tilde{A}_{3,t+1} \tag{B.9} \]

and

\[ \tilde{J}_t = \tilde{A}_{1,t} - A_{2,r,t} \frac{\tilde{\omega}_t - 1}{\Gamma_e \tilde{P}_{p,t}} + E_t \Lambda_{t,t+1} (1 - \xi)(1 - s) \Gamma_e \tilde{A}_{3,t+1} \tag{B.10} \]

respectively, where

\[ \tilde{A}_{1,t} = \tilde{x}_t + E_t \Lambda_{t,t+1} \xi(1 - s) \Gamma_e \tilde{A}_{1,t+1} \tag{B.11} \]

\[ A_{2,r,t} = 1 + E_t \Lambda_{t,t+1} \xi(1 - s)/\Pi_{p,t+1} A_{2,r,t+1} \tag{B.12} \]

\[ \tilde{A}_{3,t} = \tilde{J}^\#_t + E_t \Lambda_{t,t+1} \xi(1 - s) \Gamma_e \tilde{A}_{3,t+1} \tag{B.13} \]

In equilibrium, the costs of a new match are equal to its value, i.e.

\[ \frac{c_v}{p_{v,t}} + c_e = (1 - \xi) \tilde{J}^\#_t + \xi \tilde{J}_t \tag{B.14} \]

where \( c_v \Gamma_e^t \) and \( c_e \Gamma_e^t \) are the costs of opening a vacancy and training newly employed workers, respectively. The Nash bargaining sharing rule implies

\[ \bar{H}^\#_t = \frac{\nu}{1 - \nu} \frac{A_{2,w,t}}{A_{2,r,t}} \tilde{J}^\#_t \tag{B.15} \]

where \( \nu \) is the worker’s bargaining power and where

\[ \bar{H}^\#_t = A_{2,w,t} \tilde{\omega}^\#_t + \tilde{A}_{4,t} \tag{B.16} \]

\[ A_{2,w,t} = 1 + E_t \Lambda_{w,t+1} \xi(1 - s)/\Pi_{p,t+1} A_{2,w,t+1} \tag{B.17} \]

\[ \tilde{A}_{4,t} = \tilde{A}_{5,t} + E_t \Lambda_{w,t+1} \xi(1 - s) \Gamma_e \tilde{A}_{4,t+1} \tag{B.18} \]

\[ \tilde{A}_{5,t} = E_t \Lambda_{w,t+1} (1 - s) \Gamma_e \left( ((1 - \xi) - (1 - \xi)p_{u,t+1}) \tilde{H}^\#_{t+1} - p_{u,t+1} \xi \tilde{H}_{t+1} \right) \tag{B.19} \]

\[ \tilde{H}_t = A_{2,w,t} \tilde{\omega}_{t-1} / \Pi_{p,t} + \tilde{A}_{4,t} \tag{B.20} \]

and

\[ \Lambda_{w,t-1} = \beta \frac{C_{w,t}^{-1}}{(1/\Gamma C_{w,t-1})^{-1}}. \tag{B.21} \]

The worker’s bargaining power evolves exogenously according to

\[ \nu_t = \nu_{t-1}^{\rho_v} \nu^{1-\rho_v} \exp(\varepsilon_{v,t}). \tag{B.22} \]

The DSGE model is fully characterized by equations (A.5) to (A.14), (A.16), and (A.19) to (A.27) which it shares with the DSDE model as well as equations (B.1) to (B.22).