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A Predator-Prey Model of Unemployment and W-shaped Recession in the COVID-19 Pandemic

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Abstract

The paper presents a predator-prey model which captures the interactions between unemployment rate and COVID-19 infection rate. The model shows that lockdown measures can effectively reduce the infection rate, but at the cost of higher unemployment rate. The solution of the system makes the case for an endemic equilibrium of COVID-19 infections, hence producing waves in the unemployment rate in the absence of widespread immunity and/or vaccination. Furthermore, we simulate the model, calibrating it for the US. The simulation shows the dramatic effects on unemployment and on overall economic activity produced by potential recurrent waves of COVID-19, leading to a series of W-shaped recessions that - in absence of adequate policy response - jeopardize the coming back to the normal trend in the medium run.

Keywords: COVID-19, unemployment rate, jobless recovery, W-shaped recession

JEL classification: E24, E60, H51, I18
1 Introduction

On May 13, 2020, Dr Michael J Ryan, Senior Advisor and emergencies director for the World Health Organization (WHO) declared during one of the three WHO’s weekly virtual press conferences that SARS-CoV-2 "may become just another endemic virus in our communities, and this virus may never go away"\(^1\). Even though this prospect may be prevented in the long run by widespread vaccination and immunity, the words of the WHO representative point to the eventuality that, up to that point, the COVID-19 pandemic may evolve in a series of epidemic waves before stabilizing around some endemic level. This scenario would necessarily be accompanied by waves of more or less stringent lockdown measures. The adoption of more restrictive policies could be successful in slowing down the spreading of the virus, but it may at the same time harm the economy, raising the unemployment rate. Conversely, relaxing lockdown measures would have the effect of reflating the economy on one hand, but at the cost of greater circulation of the virus and thus more infections, on the other.

In this short contribution, we seek to capture the dynamic interaction between COVID-19 infection and the unemployment rate. We do that by building a stylized predator-prey model (Lotka, 1926; Volterra, 1928) that captures the development of the pandemic through the weekly infection rate and unemployment rate, as well as their interdependence due to containment measures (Section 2). The model shows that lockdown measures can effectively reduce the infection rate, but at the cost of a higher unemployment rate. The solution of the system makes the case for an endemic equilibrium of COVID-19 infections, thus producing waves in the unemployment rate in the absence of widespread immunity and/or vaccination. Besides the endemic one, the system can settle down in a second equilibrium for which the virus is eradicated. As it will be discussed, this equilibrium requires particularly strict conditions in terms of optimal lockdown policies, which at present seem to have been refuted by the evolution of the epidemic.

Section 3 solves the model using the method of numerical integration. The model is calibrated in light of the most recent data on contagion and unemployment in the US. The simulation shows the dramatic effects on unemployment and on overall economic activity produced by potential recurrent waves of COVID-19, leading to a series of W-shaped recessions that jeopardize the coming back to the normal trend in the medium run. Last, Section 4 concludes, summarizing our results.

2 The Model

This Section develops a predator-prey model (Lotka, 1926; Volterra, 1928) for the analysis of the interactions between the epidemiological evolution of COVID-19 and its effect on

\(^1\)See Appendix A.
the economy, in absence of widespread vaccination. More specifically, we model the effect of lockdown policies on the infection rate and the unemployment rate.

First, we define the infection rate $i(t)$ as the ratio of new COVID-19 cases over population, under the simplifying assumption that all individuals are equally at risk of contracting the virus\(^2\). The infection rate increases over time if new individuals get infected, whilst it is reduced by cure, death and lockdown policies. The parameters $r, c$ and $d$ describe the contact, cure and death rate, respectively\(^3\), whereas $\gamma$ captures the effect of the lockdown in reducing infections.

The unemployment rate $u(t)$ is conventionally defined as the ratio of unemployed workers over the labor force. We assume that $u(t)$ responds to a generically defined normal rate of unemployment\(^4\) $u_n$ via a reaction coefficient $\alpha$. Moreover, in the presence of lockdown policies, the unemployment is reduced through the interaction term $\rho$.

Therefore, the resulting system of 2 differential equations in two variables is given by:

\[
\frac{du(t)}{dt} = -\alpha [u(t) - u_n] + \rho u(t) i(t)
\]

\[
\frac{di(t)}{dt} = i(t) [r - c - d - \gamma u(t)]
\]

where all parameters are assumed to be positive.

Solving for the steady-state ($du(t)/dt \equiv 0, di(t)/dt \equiv 0$) and denoting the fixed points as $\bar{u}(t) \equiv u^*$ and $\bar{i}(t) \equiv i^*$, it follows that:

\[-\alpha [u^* - u_n] + \rho u^* i^* = 0 \quad (3)\]

\[i^* [r - c - d - \gamma u^*] = 0 \quad (4)\]

The algebraic solution of equations (3) and (4) yields the following two equilibrium solu-

\(^2\)Furthermore, it is worth stressing that we assume away population dynamics; for the sake of simplicity, COVID-19 deaths have a negligible effect on total population. Moreover, we exclude the effects of net immigration of infected individuals from abroad.

\(^3\)In a similar fashion as in compartmental epidemiological models, e.g. the SIR model, our model defines the basic reproduction number $R_0$ as the ratio of the contact rate over the cure plus death rates (capturing what compartmental models call removed population): $R_0 = r/(c + d)$. The number captures the expected number of new infections in a population in which everyone is susceptible, which is true ex definitione in our model.

\(^4\)It is worth stressing that, at this level of abstraction, the normal rate of unemployment may be interpreted in multiple ways. Generally speaking, it could be conceived as the rate of unemployment around which the economy gravitates in normal times. In more theoretical terms, it may be given by what is commonly understood as the NAIRU, i.e. the non-accelerating inflation rate of unemployment. However, as it will be described below, in our analysis we favor the definition of $u_n$ as the degree of involuntary unemployment needed to maintain a given balance of power between labor and capital, in line with Goodwin (1967). For an overview of the distinction between this notion and the NAIRU, see Shaikh (2016, p. 721-723).
tions:

\[(u^*_1, i^*_1) = \left( \frac{r - c - d}{\gamma}, \frac{\alpha (r-c-d) - u_n}{\beta r - c - d} \right) \]  

(5)

\[(u^*_2, i^*_2) = (u_n, 0) \]  

(6)

The equilibrium presented in equation (5) is an endemic one, i.e. one in which the infection rate is equal to a positive value at any time. More specifically, this holds true in our model whenever \((r - c - d)/\gamma > u_n\), i.e. \(u^*_1 > u_n\) (Appendix B).

The second equilibrium (equation 6) corresponds to a non-endemic one, that is one in which the epidemic dies out. We will denote it as a 'shamrock' equilibrium, as it would require particularly strict conditions to be achieved\(^5\), which unfortunately have already been refuted by the evolution of the epidemic.

In the next Section, the method of numerical integration will be adopted to solve for both steady-states, showing as the second one does not match the evolution neither of the infection nor of the unemployment rate.

3 Numerical Solution

Since an analytical solution to the two systems of differential equations cannot be found explicitly, the method of numerical integration is adopted. Accordingly, the first challenge is to provide a sound calibration of the models’ structural parameters and initial values.

3.1 Parameter Calibration and Initial Values

The initial values are set in accordance with the evolution of the pandemic in the US. More specifically, the initial value of the infection rate \((i_0)\) is given by the sum of new cases in the week prior to the adoption of lockdown measures - from March 10 to March 16 - divided by the US population (CDC data, see Appendix A). The initial unemployment rate \((u_0)\) is simply set equal to the observation of the unemployment rate on March 2020 (FRED data, see Appendix A). The two initial conditions are reported in Table (1).

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\(^5\) More specifically, for this equilibrium to take place, the lockdown measures would have to be sufficiently strong to extinguish the disease (which depends on the parameter \(\gamma\)). Given that all parameters are positive, this equilibrium point would be a sink if \(u_n = \frac{r-c-d}{\gamma}\), a spiral sink if \(u_n > \frac{r-c-d}{\gamma}\) or a saddle if \(u_n < \frac{r-c-d}{\gamma}\).
Table 1: Initial conditions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0$</td>
<td>Unemployment Rate</td>
<td>0.035</td>
</tr>
<tr>
<td>$i_0$</td>
<td>Infection Rate</td>
<td>1.56^{-5}</td>
</tr>
</tbody>
</table>

Source: authors’ calculation, various sources (see Appendix A)

The fatality rate $d$ is set to 0.06, in line with the US data as of May 12th, 2020 (see Appendix A). The cure rate $c$ is simply assumed to be the complement of $d$ (i.e. $c = 1 - d = 0.94$). Consequently, the contact rate $r$ is set to match a basic reproduction number ($R_0$) - in absence of any form of lockdown and/or immunization - of 2.2, as found by Li et al. (2020) in the early stage of the contagion in the region of Wuhan. Since $c + d = 1$ and $R_0 = 1/(c + d)$, it follows that $r = 2.2$. The sensitivity of the weekly infection rate to lockdown measures ($\gamma$) is set based on estimates of how much the latter reduce $R_0$, i.e. $\gamma u(t) = R_t - R_0$ where $R_t$ is the effective reproduction number. More specifically, $\gamma$ is set on the basis of the estimated effect of non-pharmaceutical interventions (lockdown, quarantine, closure of schools and universities) in reducing total deaths and peak hospital ICU bed demand, as computed by Ferguson et al. (2020) in the case of $R_0 = 2.2^6$. The sensitivity of the unemployment rate to lockdown policies ($\rho$) is simply calibrated to match the evolution of COVID-19 in terms of job losses and weekly infection rates, based on the latest data on unemployment rate and new cases (Appendix A). Lastly, we set the normal rate of unemployment to 5% and $\alpha$ to 0.15, implying a fairly strong weekly adjustment of the actual rate to its normal value (in absence of the pandemic). The parameter calibration is reported in Table (2).

Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Sensitivity of the the actual to the normal rate of unemployment</td>
<td>0.15</td>
</tr>
<tr>
<td>$u_n$</td>
<td>Normal rate of unemployment</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Sensitivity of the unemployment rate to lockdown policies</td>
<td>440</td>
</tr>
<tr>
<td>$r$</td>
<td>Basic reproduction number of COVID-19 (no lockdown)</td>
<td>2.2</td>
</tr>
<tr>
<td>$c$</td>
<td>COVID-19 cure rate</td>
<td>0.94</td>
</tr>
<tr>
<td>$d$</td>
<td>COVID-19 death rate</td>
<td>0.06</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Sensitivity of the infection rate to lockdown policies</td>
<td>12.2</td>
</tr>
</tbody>
</table>

Source: authors’ calculation, various sources (see Appendix A)

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6In particular, our $\gamma u(t)$ contributes to a decrease of the effective reproduction number of about 20% at the beginning of the lockdown and up to around 90% at the end of the first peak.
3.2 Simulation Results

This section discusses the numerical solution of the system of differential equations given by equations (1) and (2).

The parameter calibration discussed in Subsection (3.1) implies that \((r - c - d)/\gamma > u_n\). Under this condition, the system settles down in an endemic equilibrium\(^7\), i.e. the equilibrium value \(i^*\) is positive. In light of the initial conditions and calibration discussed above, the system exhibits the dynamics shown in Figure (1).

In absence of a vaccine, the economy would fluctuate around an equilibrium unemployment rate of about 10%, well above the initial and normal rates. Conversely, the new weekly COVID-19 cases over population would stabilize around a value of \(i^* = 1.6 \times 10^{-4}\) corresponding to about 5.5 cases per 10,000 individuals.

Figure 1: The endemic equilibrium

![Figure 1](image-url)

Source: authors’ representation

Figure (2) represents the unrealistic case of a sufficiently strong\(^8\) \(\gamma\) in the beginning of the epidemic which would lead the disease to die out before it spreads. Accordingly, the peak of the infection rate is at a significantly lower level, without stronger negative effects in the unemployment rate. In this case - contemplated by the equilibrium point \((u_2^*, i_2^*)\) in equation (6) - the lockdown measures would require an impracticable efficiency of policy making in terms of prediction (needed to contain the virus before it spreads too

\(^7\)See Appendix B.

\(^8\)Ceteris paribus the calibration discussed in Subsection (3.1), we set \(\gamma\) such that \((r - c - d)/\gamma = u_n\), i.e. \(\gamma = 24\).
much), timing and strength. Both magnitudes of the dissemination of the virus and the increase in the unemployment rate have been proved wrong by real data already in March 2020. For these reasons, it will not be further discussed.

Figure 2: The 'shamrock’ equilibrium

![Figure 2: The 'shamrock’ equilibrium](image)

Source: authors’ representation

Furthermore, we simulate the effect of the unemployment shock caused by the pandemic on the overall level of economic activity in the case that COVID-19 becomes endemic. Accordingly, we model the growth process in the most general form as follows:

\[
\ln Y_t = \ln Y_0 + \eta t + \nu_t \quad \text{where} \quad \nu_t = \sum_{i=1}^{t} \psi_i
\]

(7)

As discussed by Shaikh (2016, p. 632), if the error term \( \psi_t \) was a pure noise, the path in equation (7) would be a random walk with a drift, i.e. \( \ln Y_t \) would fluctuate around its deterministic trend (the dashed line in Figure 3). However, to account for the unemployment shock triggered by the COVID-19 pandemic, we model the error term as \( \psi_t = -\theta(u_t - u_n) + \epsilon_t \), where \( \epsilon_t \) is a pure white noise. As a consequence, the error term will be serially correlated, i.e. every \( \eta_t \) will depend on itself at time \( t - 1 \) and on the white noise \( \epsilon_t \). The stochastic component will be negligible "over long time spans" (ibid.), as the deterministic trend will dominate the overall time path. Accordingly, the growth rate of actual output would fluctuate around a normal rate in the long run, when the deterministic component prevails over the stochastic one.
Figure 3: The dynamics of the unemployment rate and of new COVID-19 cases over population

Setting \( \theta \) equal to 1 (the unemployment shock translates entirely to output) we could appreciate the simulated path of output over time. As reported in Figure (3), the system described in equations (1) and (2) with an endemic equilibrium produces a W-shaped recession\(^9\). Moreover, in absence of widespread immunity and/or vaccination, this could determine a L-shaped path in the medium run. This would hold at least as long as the stochastic component is not absorbed or reverted by another shock of opposite sign, e.g. stimulus policies. More specifically, public injections of purchasing power would be particularly needed to sustain demand. Moreover, a coordination of fiscal and labour market policies would be required in order to reduce the variability of employment and avoid hysteresis in the unemployment rate.

Overall, our simulation produces cycles of COVID-19 infections and economic turmoils; more precisely, the re-introduction of lockdown measures raise the unemployment rate, thus preventing economic activity to come back to its long-run normal path. In this sense, there will be hysteresis in path levels over the short to medium run; in the absence of adequate stimulus policies, the economy would stabilize to a new lower path in the medium run.

\(^9\)It is worth stressing that the trajectory of actual output is above the linear trend before the COVID-19 crisis, as the actual rate of unemployment was below the normal one (\( u_n = 0.05 \)). See Appendix A for data sources.
Before concluding, a disclaimer is in order. Our results should not be taken as accurate predictions both in epidemic as well as economic terms. Although we parametrize the model in light of current data (Appendix A), we acknowledge that modeling and simulation techniques are surrounded by a great deal of uncertainty. In this sense, our model captures only one of the many possible scenarios. In addition, we recognize that alternative ways to model the interaction of the pandemic and economic activity are possible. For instance, a higher dimensional model with delays and interactions between output gap and the spread of the pandemic can be found in Maurer and Semmler (2020).

4 Conclusion

In this paper we have build a system of two differential equations in line with the Lotka-Volterra model. The contribution investigates the dynamic interaction between the COVID-19 infection rate and the unemployment rate. The model shows that lockdown measures can effectively reduce the infection rate, but at the cost of a higher unemployment rate. The solution of the system makes the case for two equilibria.

One of the two possible equilibria is a non-endemic one, with COVID-19 dying out after its first wave. We denoted it as the ‘shamrock’ equilibrium, as the conditions to reach it are particularly strict. More specifically, for this equilibrium to take place, the lockdown measures would have to be exactly as strong as the force of infection. At the present stage, this efficiency of policy making in terms of prediction, timing and strength has already been proven wrong by real data on the evolution of the pandemic.

The other - more plausible - equilibrium is an endemic one, in which COVID-19 infections stabilize around a positive level. The endemic equilibrium produces waves in the unemployment rate in the absence of widespread immunity and/or vaccination. Furthermore, simulating the effect of the unemployment shock caused by COVID-19 on the overall level of economic activity, we find that the endemic equilibrium produces a W-shaped recession. In absence of an adequate policy response, this could determine a L-shaped path in the medium run. In this sense, public injections of purchasing power would be particularly needed to sustain demand. Moreover, a coordination of fiscal and labour market policies would be needed to reduce the variability of employment and avoid hysteresis in the unemployment rate in the longer period.
References


A Appendix: Data Sources

- Number of new positive COVID-19 cases in the US: https://www.cdc.gov/coronavirus/2019-ncov/cases-updates/cases-in-us.html
- Observed case-fatality rate: https://coronavirus.jhu.edu/data/mortality
- Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted [GDPC1]: https://fred.stlouisfed.org/series/GDPC1
- Unemployment rate, Percent, Monthly, Seasonally Adjusted [UNRATE]: https://fred.stlouisfed.org/graph/?g=qVvQ

All weblinks last accessed on May 12th, 2020
B Appendix: Stability Analysis

Let us consider the Jacobian matrix of the system of differential equations described in equations (1) and (2).

\[
J[i(t), u(t)] = \begin{bmatrix}
    r - c - d - \gamma u(t) & -\gamma i(t) \\
    \rho u(t) & \rho i(t) - \alpha
\end{bmatrix}
\]

When evaluated at the endemic equilibrium, the Jacobian matrix becomes:

\[
J[i^*, u^*] = \begin{bmatrix}
    0 & -\frac{\alpha \gamma (\frac{r-c-d}{\gamma} - u_n)}{\rho(r-c-d)} \\
    \frac{\rho(r-c-d)}{\gamma} & -\frac{\alpha \gamma u_n}{r-c-d}
\end{bmatrix}
\]

Therefore, the trace is \( \text{Tr}(J) = -\frac{\alpha \gamma u_n}{r-c-d} \). Given that all parameters are assumed to be positive, the numerator is always positive. Therefore, under the condition \( r > c + d \), the trace of our Jacobian matrix will be negative.

The determinant is given by: \( \text{Det}(J) = \alpha \gamma \left( \frac{r-c-d}{\gamma} - u_n \right) \), which is positive whenever the numerator is positive, i.e. \( \frac{r-c-d}{\gamma} > u_n \Rightarrow u^* > u_n \), provided that the above mentioned condition holds \( r > c + d \) - the denominator is positive.

Under these conditions, the fixed points \( u^* \) and \( i^* \) are stable: our \( 2 \times 2 \) system of differential equations is hence a spiral sink, as represented in Figure (4).
Figure 4: Phase diagram of the system

Source: authors’ representation