How Short is the Short Run in the Neo-Kaleckian Growth Model?
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Ettore Gallo

Abstract

The paper provides an analytical solution to the differential equation that regulates the motion of the neo-Kaleckian model in the short run. After presenting a simple open economy neo-Kaleckian model with government activity, the paper analytically derives an expression for the time of adjustment, defined as the time required for the system to make a $k$ percent adjustment from one steady-state to another. The solution shows that there is an inverse relationship between the strength of the Keynesian stability condition and the time of adjustment. Last, the model is calibrated for the US, showing that vicinity of the new equilibrium is reached after a period of about 4 quarters. By formally analyzing the out-of-equilibrium trajectory of the neo-Kaleckian model, this contribution moves away from the method of comparative dynamics and provides a historical-time representation of the model’s traverse.

Keywords: neo-Kaleckian Model; Time; Adjustment Period; Traverse; Effective Demand; Growth; Distribution

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Because of the domination of the equilibrium mode of thought, most economists unknowingly evacuate time from their analysis, exactly like Mr. Jourdain spoke prose: equilibrium economics is really timeless economics."

Henry (1987, p.472)

1 Introduction

The neo-Kaleckian growth model has been mainly criticized because of its failure to provide a long-run convergence of the rate of capacity utilization to the normal one (Skott, 2012; Dávila-Fernández et al., 2019; Girardi and Pariboni, 2019). A partial admission of the difficulties of neo-Kaleckian models in explaining long-run phenomena has also been recently recognized by Lavoie (2018, p.9): “Maybe the mistake was to speak of long-run equilibria; perhaps there would have been no controversy if from the beginning we had called them medium-run equilibria.”

While the Kaleckian literature and its critiques have focused on issues related to the stability of the Neo- and Post-Kaleckian models of growth and distribution (Del Monte, 1975; Lavoie, 2010; Skott, 2010; Franke, 2017), little to no attention has been paid to the formal analysis of the traverse from one steady-state position to another. As a consequence, even if we admit that the neo-Kaleckian model ought to be restricted to short or medium-run analysis, it is still left to know what the short and medium runs actually are. More specifically, what needs to be proven is that the neo-Kaleckian model moves between steady-state positions in a time span that the existing literature identifies as either short or medium run.

Accordingly, the first research goal of this paper is to seek an analytical solution to the differential equation that describes the short-run adjustment mechanism of a simple open economy neo-Kaleckian model with government activity. Second, the paper aims to explicitly find a solution of the system in terms of the time of adjustment, thus exploring how short is the short run in the neo-Kaleckian model by means of model calibration. Methodologically, the paper follows the line of research pioneered by Sato (1963, 1964, 1980) in analyzing the adjustment period in Neoclassical growth models.

The remainder of the paper is organized as follows. Section 2 presents a simple open
economy neo-Kaleckian model with government activity, characterized by the endogeneity of the rate of capacity utilization in the short run. Section 3 discusses the ordinary differential equation that explains the motion of the neo-Kaleckian system in the short run, providing a general solution to it. Subsequently, the resulting equation is then expressed in terms of the adjustment period $t_k$ required for a $k$ percent adjustment from one steady-state position to another second one. Section 4 calibrates the model for the US in line with existing studies and BEA data, showing that the neo-Kaleckian model provides for a very fast pace of adjustment of saving to investment. Last, Section 5 concludes, summarizing the findings of the paper.

2 A Simple Open Economy neo-Kaleckian Model with Government Activity

This Section presents a simple version of an open economy neo-Kaleckian model with government activity for the analysis of short-run dynamics.

In order to derive the growth model, let us first start with the output equation of an open economy with government activity:

$$Y_t = C_t + I_t + G_t + (X_t - M_t)$$  (1)

where the current level of aggregate output ($Y_t$) is defined as the sum of aggregate consumption ($C_t$), private investment ($I_t$), public expenditures ($G_t$) and net exports ($X_t - M_t$). Consumption, investment, government spending, exports and imports can be modelled as follows:

$$C_t = C_{0t} + c(1 - t)Y_t$$  (2)
$$I_t = [\alpha_t + \beta u_t]K_t$$  (3)
$$G_t = \overline{C}_t$$  (4)
$$X_t = \overline{X}_t$$  (5)
$$M_t = mY_t$$  (6)
Equation (2) assumes that aggregate consumption is partly induced - via the tax-adjusted propensity to consume \( c(1 - t) \) - and partly autonomous from the current level of income \( (C_0t) \). Investment (Equation 3) is modeled in line with the neo-Kaleckian treatment of capital formation as (linearly) dependent on the rate of capacity utilization \( (u_t = Y_t/Y^p) \), as postulated by Steindl (1952) and formalized in the 80s by Rowthorn (1981); Dutt (1984); Taylor (1983) and Amadeo (1986). More specifically, the parameter \( \alpha \) reflects “the animal spirits of firms, for instance expectations about the future trend rate of sales growth” (Lavoie, 2014, p. 361), while the parameter \( \beta \) represents the sensitivity of the investment rate to changes in the actual rate of capacity utilization \( (u_t) \). Both \( \alpha \) and \( \beta \) are assumed to be positive.\(^1\) Government spending (Equation 4) and exports (Equation 5) are both treated as autonomous expenditures, the first because public consumption and investment depend on the arbitrary decisions of the general government, the second because exports depend on foreign demand, which depends in turn on foreign income. For the sake of simplicity, imports of goods and services are assumed to be linearly dependent on the level of income, via the propensity to import \( m \) (Equation 6).

Given Equations (2) and (4), and considering that \( s = 1 - c(1 - t) \) is the tax-adjusted propensity to save, we can write the domestic saving equation as follows:

\[
S_t = Y_t - C_t - G_t = Y_t - C_{0t} - c(1 - t)Y_t - G_t = sY_t - C_{0t} - G_t
\]  

(7)

Dividing Equation (7) by the capital stock \( (K_t) \), we can obtain the saving rate \( (\sigma_t) \), with \( v \) denoting the capital-capacity ratio.:

\[
\sigma_t = \frac{S_t}{K_t} = s \frac{Y_t}{K_t} - \frac{C_{0t}}{K_t} - \frac{G_t}{K_t} = s \frac{Y_t}{Y^p} \frac{Y^p}{K_t} - \frac{C_{0t}}{K_t} - \frac{G_t}{K_t} = \frac{su}{v} - \frac{C_{0t}}{K_t} - \frac{G_t}{K_t}
\]  

(8)

The accumulation rate \( (g_t) \) is obtained by dividing Equation (3) by the capital stock \( (K_t) \):

\[
g_t = \frac{I_t}{K_t} = \alpha + \beta u_t
\]  

(9)

\(^1\) Since the analysis is restricted to short-run dynamics, the paper abstains from the consideration of a normal degree of utilization, in line with the original vision of Steindl (1952) and Kalecki (1954). Therefore, the model does not provide for a return to a normal degree of capacity utilization, under the assumption - widely acknowledged by Kaleckian authors - that the rate of capacity utilization is an endogenous variable, at least in the short run. For a more-in-depth discussion, see Hein (2014), Lavoie (2014) and Blecker and Setterfield (2019).
Lastly, given Equations (5) and (6), we can obtain the net export rate ($b_t$):

$$b_t = \frac{X_t - M_t}{K_t} = \frac{\bar{X}_t}{K_t} - \frac{m}{Y^p} \frac{Y_t}{K_t} = \frac{\bar{X}_t}{K_t} - \frac{m u_t}{v}$$  \hspace{1cm} (10)

As discussed by Blecker and Setterfield (2019, p. 192), the goods market equilibrium condition requires that the saving rate has to be equal to the sum of the accumulation and net export rates:

$$\sigma_t = g_t + b_t$$  \hspace{1cm} (11)

Therefore, after equating and rearranging Equations (9), (10) and (11), we can obtain the short-run goods market equilibrium as follows:

$$\frac{(s + m) u^*}{v} - z = \alpha + \beta u^*$$  \hspace{1cm} (12)

where $z$ denotes the ratio of autonomous expenditures to the capital stock. Similarly to Lavoie (2016), the ratio is assumed to be constant in the short run:

$$z = \frac{\bar{Z}_t}{K_t} = \frac{C^0_t}{K_t} + \frac{\bar{G}_t}{K_t} + \frac{\bar{X}_t}{K_t}$$  \hspace{1cm} (13)

Last, let us solve the model for the equilibrium rate of capacity utilization ($u^*$):

$$u^* = \frac{\alpha + z}{(s + m)/v - \beta} = \frac{(\alpha + z)v}{s + m - \beta v}$$  \hspace{1cm} (14)

The model leads to a stable equilibrium if and only if the denominator in equation (14) is positive. This implies that the short-run stability condition is met if saving adjusts faster than investment and the trade balance to changes in the rate of utilization, as discussed by Hein (2014, p.290).

The simple open economy version of the neo-Kaleckian model presented here maintains all the fundamental properties of Kaleckian analysis:\footnote{It is worth noting that the paper relies on the consideration of a unique economy-wide tax-adjusted propensity to save. The main reason is to move beyond the traditional Cambridge assumption that wage earners do not save, thus making the analysis in Sections 3 and 4 more consistent with economic reality (Barbieri Góes, 2020). This way, however, issues related to shifts in the functional distribution of income take a back seat. In}
1. Growth is demand-led through the investment channel;

2. The rate of capacity utilization is endogenous in the short-run, bearing the brunt of the adjustment of saving to investment and the trade balance;

3. A positive change in the animal spirits parameter ($\alpha$) boosts accumulation (Equation 3);

4. The paradox of thrift holds in the short run: an increase in the economy-wide tax-adjusted propensity to save ($s$) lowers the equilibrium utilization and accumulation rates;

Having sketched the basics of the model and its steady-state, let us now move to the consideration of out-of-equilibrium dynamics, formally analyzing the characteristics of the short-run traverse.

3 Analysis of the Adjustment Period

In the short-run steady-state, $\frac{du}{dt} \equiv 0$ per definitionem. Rewriting equation (14), it follows that:

$$(\alpha + z)v - (s + m - \beta v)u^* = 0 \equiv \frac{du^*}{dt}$$

(15)

Therefore, when considered outside the steady-state, the general form of equation (15) becomes:

$$\frac{du}{dt} = (\alpha + z)v - (s + m - \beta v)u_t$$

(16)

Equation (16) is of key importance, as it constitutes the first-order linear differential equation that explains the motion of the neo-Kaleckian model in the short run. It postulates that entrepreneurs adjust the utilization of productive capacity on the basis of goods market conditions. More specifically, whenever investment demand and the trade balance fall short of

order to bring them back, the analysis should be extended by modeling the economy-wide propensity to save as equal to the average of the propensities to save out of wages and out of profits weighted by the respective factor shares and assuming the former to be greater than the latter, in line with the Kaleckian and Post-Keynesian literature. For the sake of analytical tractability, the paper abstains from this further step, that would however permit to recover two further postulates of Kaleckian analysis, i.e. the ideas that demand and growth are wage-led and that the paradox of cost holds in the short run. For a more extensive discussion, see Hein (2014, Sec. 7.2).
(exceeds) the supply of savings, the rate of capacity utilization will decrease (increase) to match the new equilibrium in the goods market, making possible the *ex-post* adjustment of saving to investment and net exports.\(^3\) Moreover, the equation captures all the fundamental properties of the neo-Kaleckian model moving towards its new steady-state, postulating that changes in the rate of capacity utilization are positively related to changes in the animal spirits parameter \((\alpha)\) and the autonomous demand-capital ratio \((z)\), and negatively related with changes in the tax-adjusted propensity to save \((s)\), in line with the paradoxes of thrift. The general solution\(^4\) of equation (16) is given by:

\[
 u_t = \frac{(\alpha + z)v - C \exp[-t(s + m - \beta v)]}{s + m - \beta v} \tag{17}
\]

Let us consider the case of an increase in the parameter capturing animal spirits \((\alpha)\).\(^5\) Accordingly, from equation (14), it follows that the old and new steady-state values of the capacity utilization rate are, respectively:

\[
 u_0^* = \frac{(\alpha_0 + z)v}{s + m - \beta v} \quad \text{and} \quad u_1^* = \frac{(\alpha_1 + z)v}{s + m - \beta v} \tag{18}
\]

Since \(\alpha_1 > \alpha_0\), the new equilibrium rate of capacity utilization \((u_1^*)\) will be greater than the initial one \((u_0^*)\), i.e. \(u_1^* > u_0^*\).

Furthermore, as the increase from \(\alpha_0\) to \(\alpha_1\) takes place at time \(t = 0\), equation (17) becomes

\[
 u_0 = \frac{(\alpha_1 + z)v - C}{s + m - \beta v} \quad \text{and, by construction, it must be equal to} \quad u_0^* \quad \text{in equation (18). Therefore, it follows that:}
\]

\[
 \frac{(\alpha_1 + z)v - C}{s + m - \beta v} = \frac{(\alpha_0 + z)v}{s + m - \beta v} \tag{19}
\]

---

\(^3\) It ought to be noted that equation (16) can be easily rewritten as \(du_t/dt = v(g_t + b_t - \sigma_t)\).

\(^4\) The ordinary differential equation in equation (16) can be easily solved with most statistical softwares.

For a formal proof, see Appendix A.

\(^5\) It is worth stressing that the mathematical derivation would yield the same result for the time of adjustment \(t_k\) even if the initial change would be in \(s, m, \beta\) or \(v\). The analysis starts with a change in the parameter \(\alpha\) because the mathematical derivation becomes more straightforward.
Simplifying and rearranging, we have:

\[ C = (\alpha_1 - \alpha_0)v \quad (20) \]

Therefore, Equation (17) can be rewritten as follows:

\[ u_t = \frac{(\alpha_1 + z)v - (\alpha_1 - \alpha_0)v\exp[-t(s + m - \beta v)]}{s + m - \beta v} \quad (21) \]

At this stage, we ought to consider the difference between the two steady-states in equation (18):

\[ \Delta u^* = u_1^* - u_0^* = \frac{(\alpha_1 - \alpha_0)v}{s + m - \beta v} \quad (22) \]

Let us now denote with \( t_k \) the time period corresponding to a \( k \) (percent) adjustment to the new steady-state value \( u_1^* \). Accordingly, the amount of the adjustment in capacity utilization at time \( t_k \) is given by \( k\Delta u^* = u_k - u_0^* \), implying that:

\[ u_k = u_0^* + k\Delta u^* = \frac{(\alpha_0 + z)v + kv(\alpha_1 - \alpha_0)}{s + m - \beta v} \quad (23) \]

where \( u_k \) is the value of \( u_t \) at time \( t_k \). Therefore, \( u_k \) must be equal to \( u_t \) in equation (21) with \( t = t_k \). Equating the former with equation (23), it follows that:

\[ \frac{(\alpha_1 + z)v - (\alpha_1 - \alpha_0)v\exp[-t_k(s + m - \beta v)]}{s + m - \beta v} = \frac{(\alpha_0 + z)v + kv(\alpha_1 - \alpha_0)}{s + m - \beta v} \quad (24) \]

Simplifying and rearranging, we can explicitly solve equation (24) in terms of the adjustment period \( t_k \), as follows:

\[ t_k = -\frac{\ln(1 - k)}{s + m - \beta v} \quad (25) \]

Equation (25) provides an analytical relation between the adjustment period (more specifically, a \( k \) percent of the adjustment) and the other relevant parameters of the neo-Kaleckian model presented in Section 2. At first glance, it can be easily noted that there is an inverse relationship between the strength of the Keynesian stability condition and the the time of adjustment, i.e. the greater the denominator \( s + m - \beta v \), the smaller the \( k \) percent adjustment.
period $t_k$. More precisely, the inspection of the above equation allows to state the following fundamental results:

1. The adjustment period does not depend neither on the initial nor on the new value of animal spirits ($\alpha$);
2. The adjustment period does not depend neither on the initial nor on the new value of the autonomous demand-capital ratio ($z$);
3. The greater the propensity to save ($s$), the shorter the adjustment period;
4. The greater the propensity to import ($m$), the shorter the adjustment period;
5. The greater the capital-capacity ratio ($v$), the longer the adjustment period;
6. The greater the sensitivity of accumulation to changes in the rate of capacity utilization ($\beta$), the longer the adjustment period;
7. The greater the percentage of adjustment ($k$), the longer the adjustment period.

4 Parameter Values and Adjustment Time

This section provides a parameter calibration of the neo-Kaleckian model, in order to find an approximate time length for a given percentage of the adjustment to a new steady-state. By relying on existing studies and BEA data, the calibration is carried out in light of the empirical evidence for the US economy in the period between 2002 and 2019, i.e. the years encompassing the Great Moderation and the Global Financial Crisis, before the COVID-19 Recession.

In order to be able to coherently interpret the results in calendar time, it is important to point out that we need to assume a priori that the adjustment of saving to investment does not occur faster than the unit period inherent in the data (Gandolfo, 2012). In other terms, if we were to use an annual calibration (as most of the existing literature does), we would need to assume that the adjustment does not take place within a year. In the opposite case, it would be difficult to derive a plausible discrete-time representation of the adjustment process, as showed by Gandolfo (2012). For this reason, using an annual calibration is somewhat problematic in
the case of fast processes. Accordingly, the model is calibrated at a quarterly frequency, under
the more realistic assumption that the adjustment does not occur at higher frequencies (daily,
weekly or monthly). Calibrating the accumulation rate and all other relevant parameters to
account for quarter-on-quarter growth ensures that the unit period can be interpreted as a
single quarter. Therefore, assuming that a quarter is a sufficiently small time step, we can then
coherently provide a continuous-time representation of a discrete process.

In order to calibrate the quarterly capital-capacity ratio \((v)\), let us decompose it as follows:

\[
v = \frac{K}{Y_p} = \frac{K}{I} \frac{I}{Y} \frac{Y}{Y_p} = \frac{h_t u_t}{g_t}
\]

Therefore, the capital-capacity ratio depends positively on the investment share \((h_t)\) and on the
rate of capacity utilization \((u_t)\) and negatively on the accumulation rate \((g_t)\). The benchmark
value of the ratio is obtained from the analysis of capital dynamics in the US, in line with Fazzari
et al. (2020, Supplementary Appendix). The authors abstain from the complicated matter of
measuring capital and the problem of aggregating heterogeneous capital goods, thus not relying
on BEA fixed assets data. Instead, they make use of national accounts and investment data
to calibrate the capital-actual output ratio. In particular, they do so by starting from the
empirical observation of the average investment share from 2002 to 2016 (equal to 12.5\%) and
of the annual gross capital accumulation rate (10.9\%) - obtained as the sum of a yearly growth
rate of 2.5\% and a 8.4\% depreciation rate. In quarterly frequency, the latter observation implies
an accumulation rate of 2.62\%.6 With a private non-residential investment share of 12.65\% and
a rate of capacity utilization of 77\% - equal to the average measure of utilization from 2002 to
2019 - Equation (26) yields a quarterly capital-capacity ratio of 3.72.7

The value of the economy-wide propensity to save \((s)\) is set to 0.5, in line with the empirical

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6 The quarterly growth rate is obtained using the formula \(g_{qtr} = (1 + g_{yr})^{1/4} - 1\).
7 The adopted value of the investment share is just slightly above the one used by Fazzari et al. (2020), as
the data is extended until the last quarter of 2019. In order to measure capacity utilization, the paper makes
use of the average value of the Federal Reserve Board (FRB) measure of utilization from 2002 to 2019 (for data
sources, see Appendix B). It should be noted that there is no definite consensus on whether the FRB index is
the most appropriate measure of the degree of capacity utilization. For a critical discussion, the reader should
refer to Nikiforos (2016) and Gahn and González (2020). However, the empirical controversies on the use of
FRB data are centered on the discussion of the stationarity of the series and thus on the opportunity of using
it to properly measure long-run variations of utilization. The purpose of the current exercise is rather different,
as the average value of the rate of capacity utilization is used as a mere benchmark; the adoption of a different
measure of utilization to calibrate the model would have no effect on the overall results.
estimation of Blecker et al. (2020) and the recent evidences and calibration exercise by Fazzari et al. (2020, Supplementary Appendix). The value of the propensity to import \( (m) \) is obtained by calculating imports of goods and services in percent of GDP from 2002Q1 to 2019Q4 and averaging the time series; the result yields \( m = 17\% \).

The expected growth rate of sales \( (\alpha) \) is calibrated using quarterly real GDP growth as a proxy of expected revenues, yielding an average growth rate of 0.51\% at quarterly rates from 2002 to 2019. Lastly, the parameter that captures the impact of the rate of capacity utilization on accumulation \( (\beta) \) and the autonomous demand-capital ratio \( (z) \) are both set to to match the above mentioned steady-state values of the degree of capacity utilization and the quarterly accumulation rate.\(^8\)

The parameter values are summarized in Table 1.

<table>
<thead>
<tr>
<th>Par.</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>Capital-capacity ratio (quarterly)</td>
<td>3.7204</td>
<td>Author’s calculation, based on Fazzari et al. (2020)</td>
</tr>
<tr>
<td>( s )</td>
<td>Propensity to save</td>
<td>0.5</td>
<td>Fazzari et al. (2020); Blecker et al. (2020)</td>
</tr>
<tr>
<td>( m )</td>
<td>Propensity to import</td>
<td>0.17</td>
<td>Author’s calculation, based on BEA data (See Appendix B)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Animal spirits</td>
<td>0.0051</td>
<td>Author’s calculation, based on BEA data (See Appendix B)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Impact of ( u_t ) on the accumulation rate</td>
<td>0.0274</td>
<td>Author’s calculation</td>
</tr>
<tr>
<td>( z )</td>
<td>Autonomous demand-capital ratio</td>
<td>0.1126</td>
<td>Author’s calculation</td>
</tr>
<tr>
<td>( k )</td>
<td>Percentage of the adjustment</td>
<td>0.90 (0.99)</td>
<td>-</td>
</tr>
</tbody>
</table>

Under this parameter constellation, we can now explicitly compute the adjustment period.\(^9\) Defining vicinity to the new steady-state position as 90\% of the total adjustment, it follows that:

\[
t_{0.90} = \frac{-\ln(1 - 0.90)}{0.5 + 0.17 - 0.0274 \times 3.72} \approx 4 \text{ quarters} \approx 1 \text{ year} \quad (27)
\]

Therefore, the model approaches the new steady-state in about 1 year, reaching it almost

\(^8\) It is worth noting that since neither \( \alpha \) nor \( z \) have an effect on the length of the adjustment period, their calibration is merely carried out for expositive purposes.

\(^9\) Since the main scope of the paper is analytical rather than empirical, it does not include a sensitivity analysis, thus deriving the qualitative results from the benchmark values reported above. However, the interest reader may easily perform a re-parameterization of the neo-Kaleckian model using the resource reported in the Online Appendix C.
entirely (99% of the total adjustment) in about 2 years:

$$t_{0.99} = \frac{-\ln(1 - 0.99)}{0.5 + 0.17 - 0.0274 \times 3.72} \approx 8 \text{ quarters} \approx 2 \text{ year}$$  \hspace{1cm} (28)

Figure 1 provides a graphical illustration of the adjustment process under the parameter calibration described above, following an initial increase in the expected growth rate of sales ($\alpha$). The two dotted lines match the time needed for a 90% and 99% adjustment to the new equilibrium $u^*_1$.

Figure 1: The adjustment of the rate of capacity utilization to an increase in $\alpha$ at $t = 0$

Therefore, under a reasonable parameter calibration, vicinity (90%) of the new equilibrium in the model is reached after a period of about 4 quarters (8 quarters for 99% of the adjustment). This consideration implies that, in historical time, the neo-Kaleckian model presented here is characterized by a very fast pace of adjustment of saving to investment.

Before drawing conclusions from the analysis conducted above, two important remarks are in order. First, the analysis of the short-run traverse in the neo-Kaleckian model rests on a framework that, although simple, embeds an open economy with government activity and au-
tonomous consumption spending. Conducting the same calibration exercise on a simpler model that does not account for foreign trade, government activity and/or autonomous consumption may lead to misleading conclusions regarding the time of adjustment needed for the transition between steady states.\textsuperscript{10} Second, even though the calibration exercise is conducted in light of empirical evidences for the US economy, this does not imply that economic reality follows the same adjustment path postulated by the model. In other terms, the analysis does not provide any empirical support whatsoever to the Kaleckian claim that the rate of capacity utilization is endogenous in the short run, nor to the implication of a stable convergence of saving to investment. Rigorous econometric analysis aimed at supporting or disproving Kaleckian investment and output theory is therefore still needed, leaving space to further research on the matter.

5 Concluding Remarks

The paper presents a simple open economy neo-Kaleckian model with autonomous components of aggregate demand. Most importantly, it finds an analytical solution to the differential equation that regulates the motion of the neo-Kaleckian model in the short run. In line with the methodology introduced by Sato (1963, 1964, 1980), the analysis provides and discusses a general solution to the ordinary differential equation that explains out-of-equilibrium dynamics in the model. Subsequently, the effect of an increase in animal spirits is considered, rewriting the general solution of the neo-Kaleckian model in terms of the time of adjustment $t_k$, i.e. the time required for the system to make a $k$ percent adjustment to the new steady-state.

The explicit analysis of the short-run traverse in the neo-Kaleckian model yields few fundamental results. First, the time of adjustment is not affected by changes neither in the animal spirits parameter ($\alpha$) nor in the autonomous demand-capital ratio ($z$). Second, the adjustment period depends negatively on the propensity to save ($s$) and on the propensity to import ($m$). Third, $t_k$ is in a direct relation with the capital-capacity ratio ($v$) and with the sensitivity of accumulation to changes in the rate of capacity utilization ($\beta$). Therefore, the paper has proven that the time of adjustment depends entirely on the determinants of the Keynesian stability condition.

\textsuperscript{10} I wish to thank Robert Blecker for pointing this out to me.
Last, the paper performs a parameterization of the neo-Kaleckian model in line with empirical evidences and recent Post-Keynesian literature. The calibration exercise shows that, under a reasonable parameter constellation, vicinity of the new equilibrium - defined as 90% of the total adjustment - is reached after a period of about 4 quarters, and the model almost settles in the new steady state (99% of the adjustment) after about 8 quarters. This result, implying a very fast pace of adjustment, provides more solid foundation to Lavoie’s (2018, p. 9) claim - reported in the Introduction - that the neo-Kaleckian model is better suited for short and medium-run analysis rather than for giving a proper representation of long-run macrodynamics.

While the investment theory upon which the neo-Kaleckian model rests needs to be further assessed empirically, the analysis of the short-run traverse conducted in the present contribution calls for a closer connection between the neo-Kaleckian model of growth and distribution and Kalecki’s original business cycle theory. As the neo-Kaleckian model appears to be moving between steady-state positions at business cycle frequencies (Angeletos et al., 2020), the former is consistent with Kalecki’s idea that the short run is characterized by damped oscillations perturbed continuously by stochastic shocks that generate semi-regular cyclical movements (Kalecki, 1971, p. 134-135).

On a more general level, the analysis conducted in the paper points to the importance of explicitly taking into account the time scale of steady-state growth models when describing their comparative dynamic effects and policy implications, thus coherently combine logical-time analysis and real-world historical time, as advocated by Joan Robinson (1980). In this respect, the paper has analytically showed the validity of the line of argument put forward by Henry (1987), Park (1995) and Lavoie (2016, p.183-184) on the importance of paying more attention to the values that the relevant variables of a system take during the traverse rather than to their potential steady-state values. Whilst the ultimate assessment of the validity of the neo-Kaleckian model for policy analysis ought to rest on rigorous empirical investigation, this contribution wishes to set the ground for a new agenda for Kaleckian authors and demand-led growth theorists, suggesting to move away from the comfortable but limited realm of comparative dynamics and think more carefully about the properties exhibited by economic models during the traverse. The comparison between steady-state positions is undoubtedly useful to
grasp the logic of a model as it moves from one equilibrium to another, but it needs to be coupled with a precise description of the model’s out-of-equilibrium trajectory if we want to provide a valid representation of a real-world economy operating in historical time on human time scales.

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A Appendix: Proof of the General Solution in Equation 17

1. In order to prove that equation (17) is the general solution of the ordinary differential equation (16), let us first conveniently simplify the notation. In particular, let us denote with the term $K$ the Keynesian stability condition, i.e. $K = s + m - \beta v > 0$. Therefore, equation (16) becomes:

$$\frac{du}{dt} = (\alpha + z)v - Ku$$

(29)

2. Rewrite equation (29) in the form $dy/dt + p_t y_t = q$, as follows:

$$\frac{du}{dt} + Ku = (\alpha + z)v$$

(30)

which implies that $p_t = Ku_t$ and $q = (\alpha + z)v$.

3. Let us find the integrating factor, i.e. the continuous function that satisfies the condition $\mu_t p_t = \mu'_t$, as follows:

$$\mu_t = e^{\int K dt} = e^{Kt}$$

(31)

4. Let us now multiply all the terms in the differential equation (30) by the integrating factor:

$$e^{Kt}\frac{du}{dt} + Ke^{Kt}u_t = (\alpha + z)v e^{Kt}$$

(32)

$$(e^{Kt}u_t)' = (\alpha + z)v e^{Kt}$$

(33)

5. Integrating both sides of equation (33), it follows that:

$$\int (e^{Kt}u_t)' dt = \int (\alpha + z)v e^{Kt} dt$$

$$e^{Kt}u_t + k = \frac{(\alpha + z)v}{K} e^{Kt} + c$$

(35)
6. Subtracting $k$ from both sides, we get:

$$e^{Kt}u_t = \frac{(\alpha + z)v}{K}e^{Kt} + c - k$$

(36)

7. Both $c$ and $k$ are unknown constants and so the difference is also an unknown constant. Therefore, we can write the difference as $c_1 = c - k$:

$$e^{Kt}u_t = \frac{(\alpha + z)v}{K}e^{Kt} + c_1$$

(37)

8. We now have only one constant of integration $c_1$. It should be noted that the constant $c_1$ is negative for economically meaningful initial values of the rate of capacity utilization (if $u_0 > 0$, then $c_1 < 0$). For convenience, let us then define another constant $C$ as $C = -c_1/K$. Therefore, equation (37) becomes:

$$e^{Kt}u_t = \frac{(\alpha + z)v}{K}e^{Kt} - C$$

(38)

9. Multiplying both sides by $e^{-Kt}$, we can obtain the general solution to the ODE that regulates out-of-equilibrium dynamics in the model, as follows:

$$u_t = \frac{(\alpha + z)v - C e^{-Kt}}{K}$$

(39)

10. Last, substituting $K = s + m - \beta v$, we can write $u_t$ as follows:

$$u_t = \frac{(\alpha + z)v - C \exp[-t(s + m - \beta v)]}{s + m - \beta v}$$

(40)
B Appendix: Data sources

- Capacity Utilization, Rate, All industry, SA, Federal Reserve Board (FRB), https://fred.stlouisfed.org/series/TCU


All weblinks last accessed on September 26, 2021.

C Online Appendix: Sensitivity analysis

The interested reader could easily perform a re-parameterization of the neo-Kaleckian model under scrutiny through the following interactive Web App - created with Shiny R: http://ettoregallo.shinyapps.io/Short_run_NKM