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Incentive design, iterative planning and local knowledge in a maturing socialist economy

David Laibman

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Libertarians, mainstream neoclassical economists, and even most socialists accept the idea that there are only two ways to organize economic activity: the market, and top-down command. A third alternative, however, is multi-level iterative coordination: enterprises create their own plans, and coordination occurs through repeated aggregation and fine-tuning. Enterprises must have incentives to plan ambitiously, but also realistically. The Western incentive design literature suggests that this is impossible: No incentive structure will prevent the enterprise from manipulating its plan and activity in ways that distort information and cause inefficiencies. However, when enterprise performance requires principled and knowledgeable participation of all of its members, reward maximization leads to convergence between plan and true potential, and between plan and actual result. This possibility theorem helps us to envision, and create, socialist institutions that combine the advantages of central coordination with the autonomy of enterprises and their ability to use local and tacit knowledge.

Keywords: Socialism; incentive design; planning; local knowledge

1. Introduction

At mid-twentieth century, the great libertarian economist and philosopher Friedrich Hayek (1944, 1945) issued his famous challenge: any attempt to circumvent the spontaneous, unregulated competitive market would lead to serfdom. And, from a much more eclectic perspective, in 1983 the doyen of comparative economists, Alec Nove (1983, 44), essentially concurred, insisting that there are, at bottom, only two possible economic coordinating principles: the market, and top-down command. It would not be unfair to say that this rigid bipolarity has been the axis on which thinking about socialism has turned, even up to the present. Even at the more socialism-friendly end of the spectrum – as in the discussion in Cuba today, and among proponents of ‘market socialism with Chinese characteristics’ (Yang 2009) – the bipolarity as such is not questioned; reforms are sought which somehow balance and combine the positive and negative features of markets and ‘planning’, but the essential dichotomy itself remains in place.

The experience of the Soviet Union, however, suggests the possibility of a third alternative: A system of multi-level iterative coordination. In this conception, economic activity is always undertaken within the framework of a plan, which serves both as guide and as the normative foundation for evaluation and reward. The plan, however, is not imposed from a ‘top’ or ‘center’, but instead emerges from the planning activity that takes place at each local site of production (and, perhaps also, consumption). The local units – I will call these units ‘enterprises’ in what follows.

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– create their own plans, which are then transmitted to a central site or authority, aggregated, adjusted for overall consistency and to address any macro imbalances that could not have been known or foreseen at the enterprise level, and returned to the enterprises. The enterprises then re-adjust (‘fine-tune’) their local plans accordingly, and send them back ‘up’ (or ‘in’) to the center. The iterations – in theory! – lead to sufficiently rapid convergence to a consistent overall grid for the plan to become effective. In the Soviet system, which was a very early precursor of what becomes possible after the electronic revolution, there was a rigid separation between the planning phase and the execution phase, and the planning phase tended to hyperextend into the period for which the plan was intended. In a modern information technology (IT) conception, we may imagine planning (coordination) and execution to be continuous and simultaneous. The result, however (again, in theory!) is quite different from whatever might emerge from unplanned, atomistic market interaction.¹ First, enterprises are able to situate their own activity within the larger macro process of which they are a part, which thus becomes a stable foundation on which rational calculation may be projected. (Contrast this to the random noise and uncertainty of markets, especially financialized markets.) Second – and herein lies an essential socialist response to the Hayek critique – the plan, as envisaged in the iterative conception, incorporates local (and even tacit or non-propositional) knowledge that by definition exists only at the enterprise level and cannot be transmitted to or held by the center.²

Now if enterprises are to be given the responsibility to create their own plans, the question arises: since local knowledge is truly local, how can the center provide incentives to enterprises to plan both ambitiously and realistically? The enterprise’s reward should be maximized when it achieves its full actual potential – for level, quality and assortment of output; for output and productivity growth, technical change, social, educational and technical development of the workforce; and perhaps other goals. There are really two questions here. First, given the desire of the center – which acts as the representative of society as a whole in this – to reward the enterprise both for planning ambitiously, and for planning realistically (i.e., for fulfilling the plan once announced), can this in fact be achieved? Second, can an incentive structure be created that encourages the enterprise to reveal its true possibilities? Put negatively, can the enterprise be prevented from manipulating its announced plan and/or its subsequent behavior, and distorting its true position, in order to maximize its reward?

These questions were addressed in the Soviet economic reform literature of the 1960s and beyond (for a summary, see Ellman 1979). They are also at the heart of the incentive design literature in Western economics (Hurwicz 1972; Campbell 1995), which addresses the question of incentives for non-distorting behavior on the part of agents, in cases in which principals cannot have good knowledge of agents’ preferences or possibilities.

The incentive design models suggest that there are indeed mechanisms that may induce agents, in their own interest, to reveal their true positions and act accordingly. An example of such a mechanism is the proposal for a second-price, sealed-bid auction (Vickrey 1961; cf. Campbell 1995, 140ff) which induces participants to state (in their bids) the true reservation value they attach to the object being auctioned, and then forces the winner of the auction to pay the true social cost of her victory (namely, the reservation value of the second-highest bidder). The upshot of this literature, however, is that – apart from special instances – it is not possible in general to find a mechanism that a) induces truthfulness;

¹It should go without saying that markets in their specifically capitalist form are not at issue here.
and b) produces Pareto-optimal outcomes. This result, the Impossibility Theorem (Hurwicz 1972; Roberts 1979; cf. Campbell 1995, 294ff), suggests, in our context, that there is no way for a socialist society (as represented by its center, or central planning authority) to provide a motivational structure for enterprises that would preclude strategic behavior on their part: concealing reserves, falsifying achievement possibilities, and so on. If this is correct, it would essentially vindicate the Nove binary of market vs. command: Hayek’s draconian vision and ‘free market’ fundamentalism aside, we would have to settle for some ‘compromise’ between spontaneous, horizontal markets with their propensity toward cyclical instability, polarization, and promotion of egoistic and short-sighted behavior, on the one hand; and central, top-down command, with its propensity toward authoritarianism and inefficiency, on the other. I will note, for the present, that the Impossibility Theorem of the incentive design literature assumes implicitly (but crucially) that there is no link between enterprise planning behavior (manipulative or otherwise), the actual outcomes of its activity, and its true possibilities.

In the sections that follow, I will present a canonical model of a typical reward function for an enterprise in a decentralized, iterative planning system of the sort under consideration (section 2). I will then study two preliminary simple models of reward maximization (section 3), leading to the heart of my proposal to form an answer to the Impossibility Theorem: a Collective Morale Function linking an enterprise’s plan, its actual locally-known possibility, and the outcome of its efforts (section 4). The final section concludes, with some wider implications.

2. The reward function

2.1. Material incentives

A prominent strain of socialist thought resists any attempt to use income incentives as a stimulus to exertion, diligence, productivity and self- or collective improvement; it sees all such attempts as catering to ‘defects’ or otherwise contrary to Marx’s conception of communist society (see, e.g., Lebowitz 2010; Ollman 1998). In contradistinction to this way of thinking, I see the determination of rewards by some measure of an enterprise’s or individual’s performance as essential for successful development of both production and consciousness in a socialist context (see also Laibman 2007, ch. 6). Marx’s *Critique of the Gotha Programme* (Marx 1966) makes clear his commitment to the need for material reward in relation to work performed, in the lower stage of communism as defined by him. I would, however, affirm this even if Marx could be read to the contrary, on the basis of the entire experience of the twentieth century up to the present and of a materialist philosophical platform for thinking about the dialectic of changing conditions and consciousness. It is a matter of greatest principle that every working collective in a socialist society consider itself responsible to the entire society for its guardianship over a particular set of productive resources, and for the efficiency with which that guardianship is exercised. Differential rewards, both to collectives and to individuals, follow from this, throughout a long period in which consumption is still constrained by labor income. The system of rewards must be ‘gotten right’, precisely so that it can eventually be transcended. The ideas that material incentives can simply be denied, or that people’s consciousness and behavior can be transformed through a simple act of revolutionary political will, are what give socialism a sense of unreality, and drive people toward advocates of ‘the market’; concern for the hard reality of motivation and incentives makes it seem as though it is only the anti-socialists who have their feet on the ground. From this point of view, artificially accelerating equalization of incomes among different classes of labor (so long as such classes exist), and among different individuals within each class (so long as levels of consciousness, effort, skill and educational attainment vary), may have the opposite of the intended effect, causing resentment, demoralization, and cynicism and obstructing, rather than promoting, socialist consciousness (cf. Laibman 2007, ch. 6).
In the present context, this means that the reward function for enterprise performance is of crucial importance. It must provide a clear message concerning how the society, through a democratic mandate embodied in the reward structure, values different aspects of group and individual activity – including (as we will see) both plan development and plan execution. If that mandate dictates that a given group or individual receive a differential for their valued contribution, it is important for that differential to be accepted by the group or individual in question, as that validates the standards that apply to all. (If any individual feels strongly that she does not need the differential, she can always donate it to solidarity funds set up to eliminate still-existing social inequalities.)

The reward function, of course, does not replace a certain basic support wage, and associated social consumption systems in areas such as health, education, and retirement. The generalized access to security and dignity afforded by this basic level of equality is a fundamental value of socialist society, and the viability of socialism as such rests on the premise that technical and social levels have been reached at which this basic platform is a requirement for, not a hindrance to, further development. The reward is distributed, on top of the basic wage, as individual and collective bonuses, within each tier of the enterprise’s structure (again, assuming such tiers – managerial, creative, skilled, unskilled – still exist, at an early stage in which the stratifications inherited from the capitalist past have not yet been overcome). It may also be used to support various investment and collective consumption funds that are at an enterprise’s disposal. I therefore prefer the more general term ‘reward’ to the (perhaps more common) ‘bonus’ to designate this concept.

2.2. The performance measure

Before a canonical reward function can be formulated, care must be taken in establishing the nature of the performance criterion to be used in determining the reward. Central to the superiority of socialist over capitalist (or any spontaneous market) coordination is the proposition that the achievement of the enterprise is in fact a vector of outcomes in different dimensions. There are, first, various measures of enterprise performance in the narrow sense of ‘production’, and many of these appeared in the twentieth-century socialist experiments: level, assortment and quality of output; output sold (as distinct from merely produced); productivity; productivity growth; ‘profit’ (net income generated for the public sector); targets for control over costs; etc. Note, first, that here there is a unique problem for socialist economic theory: does the enterprise count as its ‘value added’ all of the net income, including wage income, or just the surplus over costs that accountants in a capitalist context will recognize as ‘profit’? Definition of the surplus from enterprise activity clearly embodies a tension, in which workers-as-owners must treat themselves as a means to their own ends! What is the appropriate measure of the rate of return to the resources (capital stocks) entrusted to the enterprise: is it realized net income divided by capital value, or just the portion of net income not paid out as wages? Capitalists (or state enterprise managers who act as an alien presence in relation to the workforce and treat that workforce as a means to their own ends) would have no problem with this issue. By contrast, a socialist enterprise director and her staff, in consultation with committees and representatives of the production, creative and engineering workforce, must wrestle with it.

But even if we assume that some scalar measure of operational efficiency in this narrow sense can be identified, that still does not exhaust the content of the performance criterion. In a socialist context, society ‘tasks’ the enterprise with various social goals, including (but perhaps not limited to): establishing and promoting relations with the geographic community in which the enterprise is sited; meeting targets for ecologically sound and sustainable activity; developing educational and other programs for children and young people in the community; pursuing goals of eliminating
inherited discriminatory distinctions among workers, on the basis of color, gender, nationality, or culture, by means of affirmative action programs; and implementing projects to disseminate its knowledge and capacities to other regions and enterprises, to overcome developmental lags.

In what follows I will leap over this huge area for socialist theoretical study, by simply assuming that the standard ‘economic’ measures of performance can be combined with some way of rating the enterprise in all of these other areas. The vector of ‘scores’ achieved (or planned) must be weighted, in order to reduce the multidimensional complexity of social evaluation to a single scalar measure, $x$. I ask the reader to remember that this $x$ is not a simple measure of ‘output’ in some natural units (as it indeed often was in early Soviet planning), but can be as multi-dimensional and complex as necessary, given the task of evaluating enterprise performance in increasingly sophisticated technical and cultural environments.

Some may question whether reducing performance to a single scalar is possible at all; I will only justify this on the grounds that it makes possible the following analysis. Certainly, at a minimum I must be assuming that the weights used to determine the scalar $x$ are independent of the resulting behavior.\(^3\)

The final step in this sequence will be to distinguish between the planned level of the performance indicator, $x_p$, and the actual achieved level, $x_a$. The planned level, $x_p$, is announced by the enterprise, and therefore generally known. The achieved level, $x_a$, depends on accurate reporting of observable information, and also, crucially, on the political process whereby the enterprise’s work in the various qualitative dimensions of performance enumerated above is rated. It should go without saying that this rating process could be subject to unprincipled manipulation! That is to say nothing more than that we are here dealing with a very human situation, and that there are no simple fixes.

### 2.3. The reward function

The reward, $R$, can now be specified, in a way that encourages the enterprise to both plan ambitiously and to fulfill the plan, once announced. There are various ways in which this can be done, but one specification seems to have canonical status, in that it treats upward and downward deviations of result from plan in a symmetrical manner, and is a continuous function and therefore amenable to optimization analysis (Pickersgill and Pickersgill 1973; cf. Ellman 1973).

The reward, $R$, is related to both plan and achievement, as follows:

$$ R = ax_p - b(x_p - x_a)^2 $$

(1)

$R$ has two terms, and the coefficients $a$ and $b$ determine the relative weights assigned to each. The first term on the right has the level of the plan, $x_p$, contributing positively to the reward, in a simple linear manner; this is a straightforward incentive for ambitious planning. The second term, however, reduces $R$ to the extent that the actual result, $x_a$, deviates from $x_p$, in either direction. It places a value on plan fulfillment, treating overfulfillment as just as disruptive as underfulfillment (due to unplanned burdens on storage and transportation capacities, but perhaps more to the distortions imposed upon knowledge for aggregation and macro planning by significant deviations of $x_a$ from $x_p$).

\(^3\)The idea of $x$ as a linear weighted sum of performance indicators, $x = \sum \eta_i q_i$, suggests that the enterprise can choose among different ways of achieving a given value of $x$, in which case the center might wish to impose some side constraints establishing minimum requirements in all areas. The linear scheme could be replaced by a more interactive (multiplicative) formulation. All this awaits further study.
The reward function (1) will be used in later sections. It may, however, be seen as a development arising from several simpler reward models. First, we may imagine a managerial incentive in which planning is not valued or rewarded explicitly at all; the reward depends only on the achieved level. In this case, \( R = ax_a \). Second, we posit a simple model of central planning, in which the plan is given to the enterprise by the center:

\[
R = \bar{R} - b(\bar{x}_p - x_a) = (\bar{R} - b\bar{x}_p) + bx_a
\]

Here \( \bar{R} \) is a constant, the reward ‘base’ that does not depend on the enterprise’s activity. In this case, deviation from plan is formally penalized, but since the plan \( \bar{x}_p \) is not under the control of the enterprise, it amounts (as the last expression on the right side shows) to a ‘more is better’ incentive, to get \( x_a \) to the highest possible level. This becomes a variant of the first, managerial, case. A further development of central planning is to place a value on plan fulfillment and penalize deviations in either direction, in which case the formula becomes

\[
R = \bar{R} - b(\bar{x}_p - x_a)^2
\]

When, crucially, the planning function itself is devolved to the enterprise, central planning evolves into multilevel coordination, and the reward function becomes Equation (1).

3. Preliminary models

3.1. Properties of the reward function

To see how the reward function works, treat \( R \) as constant, for the moment – \( R = \bar{R} \) – and solve Equation (1) for \( x_a \) in terms of \( x_p \). We have, in effect, all of the possible combinations, for a given reward, of actual achievement with the announced plan:

\[
x_a = x_p \pm \sqrt{\frac{ax_p - \bar{R}}{b}}
\]

(2)

Figure 1. Combinations of plan and achievement for a given level of reward.
This is shown in Figure 1. Real values of $x_a$ require $x_p \geq \bar{R}/a$. We have

$$
\frac{\partial x_a}{\partial x_p} = 1 \pm \frac{a/2}{\sqrt{b(ax_p - R)}}
$$

(3)

This derivative approaches 1 as $x_p$ approaches infinity. On the lower branch, it becomes 0 (the function is minimized) at $\bar{x}_p = \bar{R}/a + a/4b$. In practice, the lower branch, associated with the minus sign, will be effective, since all reasonable constraints will place the upper branch out of the feasible range.

3.2. **Constant achievement level**

Suppose the workforce is impervious to incentives, and completely insensitive to the plan: $x_a = \bar{x}_a$. The reward function then becomes a straightforward maximization problem:

$$
\frac{\partial R}{\partial x_p} = a - 2b(x_p - \bar{x}_a) = 0 \Rightarrow x_p^* = \bar{x}_a + \frac{a}{2b}
$$

(4)

and putting this result back into Equation (1),

$$
R^* = a\bar{x}_a + \frac{a^2}{4b}
$$

(5)

This simple result is graphed in Figure 2. The $x_a$-$x_p$ curve (2) begins at a point on the 45° line; increasing $R$ means increasing $R/a$, equivalent to moving up along that line. For $R/a$ less than $R^*/a$, the curve will cross $\bar{x}_a$, identifying two ($x_p$, $x_a$) strategies that will produce that value of $R$. For $R/a$ greater than $R^*/a$, no plan $x_p$ is consistent with the given $\bar{x}_a$. $R^*$ is therefore shown to be the maximum achievable reward.

Figure 2. Reward maximization with a constant achievement level.
Note that, in this case, $R$-maximization requires the enterprise to set a plan, $x_p^*$, that is greater than the (constant) achievable level, $\bar{x}_a$. While in this ultra-simple case we may imagine that $\bar{x}_a$ is known by the center, so that the local-knowledge problem does not arise, we still find a deviation between what the enterprise can do and what it declares to be ‘the truth’ about what it can do.

3.3. Material incentives

The constant achievement level can be replaced by one that responds to the reward received by the enterprise. The amount of effort and diligence that the enterprise can draw out of itself may reasonably be thought to depend on the size of the bonus and other funds achieved, or at least anticipated. We imagine a base level of $x_a$, $\alpha$, that would result when $R = 0$, and enterprise income is restricted to the maintenance described in section 2.1. $x_a$ then rises with $R$, but with diminishing returns, to an asymptote at $\alpha + \beta$. This is captured by the hyperbola

$$x_a = \alpha + \frac{\beta R}{\gamma + R}$$

This can be re-written to make it compatible with the space of Figures 1 and 2, by dividing the numerator and denominator of the second term on the right side by $a$:

$$x_a = \alpha + \frac{\beta R}{\gamma a + R}$$

Since the objective $R$ is now also an argument determining one of the choice variables, we have, instead of a straightforward maximization problem, a simultaneous solution, and this is shown in Figure 3. Since the object is to maximize $R/a$, and since $R/a$ is the ‘starting point’ of the $x_a$-$x_p$ curve (2), which lies on the $45^\circ$ line, that point must be consistent with the material incentive function (6a). The solution is therefore at the intersection of the $45^\circ$ line and (6a), at point A in Figure 3.

---

4This proposition of the socialist economic literature bears a major resemblance to the efficiency wage literature in Western labor economics and macroeconomics; see Akerlof and Yellen (1986), Mankiw and Romer (1991).
The solution is found from 

\[ \frac{R}{a} = \alpha + \frac{\beta(R/a)}{\gamma/a + R/a}, \]

which results in 

\[ R^* = -\frac{\gamma - a\alpha - a\beta}{2} + \frac{1}{2} \sqrt{(\gamma - a\alpha - a\beta)^2 + 4a\alpha\gamma} \]  

(7)

Both roots are real, but the plus sign corresponds to the only positive (and therefore relevant) root. The optimal plan and achievement levels, equal in this case, are 

\[ x_p^* = x_a^* = \alpha + \frac{\beta R^*}{\gamma + R^*} \]  

(8)

Notice that the coefficient \( b \), which measures the center’s relative distaste for discrepancies between the enterprise’s plan and its result, plays no role in this solution, as we would expect: since \( x_a \) coincides with \( x_p \) in the solution, no discrepancy arises, and it does not matter at what rate discrepancies are penalized.

It should be noted that point B in Figure 3, which looks like a solution of some sort, is not: at B, the \( x_p-x_a \) combination indeed produces \( R^* \), but this value of \( x_a \) is higher than \( x_a^* \), which is the only value that is consistent with \( R^* \) along the material incentive function. Point B, despite appearances, is not feasible.

4. The collective morale function

The preceding analysis of the constant achievement and material incentive cases lays the groundwork for the core proposal of this paper: a model in which both the enterprise’s plan and the result of its activity are closely related to its actual level of possibility – something that is in principle known only to the collectives of the enterprise and in fact only discoverable via the process of plan formation at the enterprise level. This section discusses and describes this proposal in detail.

4.1. Logic and properties

The Collective Morale Function (CMF) will only become operational when social and technical conditions have developed to a point at which production – not to speak of production development – cannot take place without significant discretionary involvement and rational understanding among the members of the enterprise. In socialist conditions, there is a strong culturally reinforced political commitment to full employment and social security; moreover, production requires high and rising educational levels, analytical and communicative skills and creative responsibility on the part of individual workers. In this context, the only managerial strategy consistent with broadly successful results is a highly devolved strategy: one that relies on dissemination and cultivation of management and creative responsibilities throughout the workforce, and on sophisticated incentives. We are envisioning here a world in which morale matters; in which all essential contracts are highly and increasingly implicit and incomplete; and in which instrumental levers alone are not sufficiently motivating. Jobs in this sort of environment are not ‘just jobs”; they are ‘worth doing well’. Job satisfaction, broadly defined to include satisfying interpersonal
relations in all phases of work (planning, creating, executing), is not just a valued side-product, but rather an essential prerequisite for collective success and growth.

This raises the question whether the technical, material and cultural prerequisites for positing this sort of advanced enterprise environment existed to a significant degree in the experiments in early socialism of the twentieth century; indeed, whether they can reasonably be expected to be present, except in limited circumstances, in the foreseeable future. My sense is that they can, and must, be present, but this may be regarded as an open question. The purpose of this paper is to explore the implications of the assumption that they are present, for a model of decentralized, iterative planning under conditions of significant locality of knowledge.

The key assumption, then, is as follows: there is a level of the achievement variable \( x \) – we will call this level \( z \) – which is actually attainable by an enterprise, given its best (not greatest) effort. As noted above, we may assume that \( z \) is known only to the enterprise; the center cannot have information on it. We may further assume, in fact, that this knowledge of \( z \) does not exist even at the enterprise level ‘ready to hand’, so to speak; rather, it is the outcome of the enterprise’s concerted deliberations, study, debate, and consensus formation. In a word: the enterprise – and this means not just the people holding central management posts but all members, with (perhaps) varying levels of detail and sophistication – comes to know what it can accomplish, if it sets out to do that. Moreover, the enterprise – again, all of it – experiences the morale effects of divergence between \( z \) and \( x_p \).

More precisely, we are looking for a specification of the CMF with these properties: if, and to the extent that, \( x_p < z \), so that enterprise leadership has failed to incorporate what everyone knows is possible into the official plans of the enterprise, morale will suffer, and \( x_a \) will fall short of \( x_p \). And, conversely, if, and to the extent that, \( x_p > z \) – the plan is unrealistically ambitious – again morale will suffer, and \( x_a \) will again turn out to be \(< x_p \). If the plan, as announced by the enterprise, coincides with (locally known) reality, then it will turn out that \( x_a = x_p = z \).

In constructing some representation of this, I began with the geometry. The result was Figure 4. It should be easy to verify, visually, that the CMF drawn has the stated properties, and that tangency with the 45° line at \( z \) is necessary for that to be the case. Assuming, reasonably, that the CMF has the form of a parabola, its specification emerges straightforwardly:

\[
x_a = -cz^2 + (1 + 2cz)x_p - cx_p^2
\]

(9)

Figure 4. The collective morale function.
The requisite properties can be easily verified: writing the right side of Equation (9) as \( f(x_p) \), we have \( f(z) = z, f'(x_p) = 0 \) at \( x_p = z + 1/2c, f'(z) = 1 \), and so on. The parameter \( c \) regulates the ‘tightness’ of the parabola; when \( c = 0, f(x_p) \) coincides with the 45° line \( (x_a = x_p) \); the morale factor disappears, and the enterprise planners have complete, unconstrained control over the achievement level. As \( c \) increases, the parabola becomes narrower; its maximum approaches \( z \), as do its ‘zeros’ (crossings of the horizontal axis). This represents the maturing of the enterprise, as expressed in increasing effectiveness and importance of the morale factor: deviations of \( x_p \) from \( z \) have immediate and drastic effects on achievement levels.

### 4.2. Maximizing reward subject to the CMF

We can now proceed directly to the formal optimization result. The enterprise maximizes the reward function, subject to the Collective Morale Function as constraint:

\[
\text{MAX: } R = ax_p - b(x_p - x_a)^2 \\
\text{S.T. } x_a = -cz^2 + (1 + 2cz)x_p - cx_p^2
\] (10)

The Lagrangian, and first-order conditions, are:

\[
Z = ax_p - b(x_p - x_a)^2 + \lambda [cz^2 - (1 + 2cz)x_p + cx_p^2 + x_a]
\] (11)

\[
\frac{\partial Z}{\partial x_p} = a - 2b(x_p - x_a) - (1 + 2cz)\lambda + 2c\lambda x_p = 0 \tag{i}
\]

\[
\frac{\partial Z}{\partial x_a} = 2b(x_p - x_a) + \lambda = 0 \tag{ii}
\]

\[
\frac{\partial Z}{\partial \lambda} = cz^2 - (1 + 2cz)x_p + cx_p^2 + x_a = 0 \tag{iii}
\]

A closed-form solution can be teased out of the system (i)–(iii), as follows. First, Equation (iii) can be used to form

\[
x_p - x_a = c(z^2 - 2zx_p + x_p^2) = c(z - x_p)^2
\]

and inserting this into Equation (ii),

\[
\lambda = -2bc(z - x_p)^2 \tag{A}
\]

Next, use Equations (ii) and (i) to find

\[
a - 2c(z - x_p)\lambda = 0, \text{ or } \lambda = \frac{a}{2c(z - x_p)} \tag{B}
\]
Combining Equations A and B,
\[
\frac{a}{2c(z - x_p)} = -2bc(z - x_p)^2, \text{ or } (z - x_p)^3 = -\frac{a}{4bc^2}
\]
This cubic equation in \(x_p\) has only one real root, easily found from
\[
z - x_p = \left(-\frac{a}{4bc^2}\right)^{1/3} = \left(-\frac{a}{4bc^2}\right)^{1/3}
\]
so that the optimal plan is revealed as
\[
x^*_p = z + \left(\frac{a}{4bc^2}\right)^{1/3} \tag{12}
\]
Substituting \(x^*_p\) into the constraint – Equation (9), or Equation (iii) – we find the achievement level predicted from reward maximization:
\[
x^*_a = z + \left(\frac{a}{4bc^2}\right)^{1/3} - c\left(\frac{a}{4bc^2}\right)^{2/3} = x^*_p - c\left(\frac{a}{4bc^2}\right)^{2/3} \tag{13}
\]
The most striking observation one can make about this result is twofold. First, the reward-maximizing enterprise will systematically find the best-effort outcome, \(z\), and then announce a plan that exceeds it, by some amount equivalent to the right-most term in Equation (12)! To this extent, then, the incentive embodied in this model leads only to approximate ‘truth-telling’, since the underlying ‘truth’, known only to the enterprise, is associated with \(z\). Second, however, reasonable hypotheses about the dynamic evolution of the enterprise suggest that \(x^*_p\) may converge toward \(z\) over time, so that the model has efficient properties from the standpoint of society as a whole over the long run. First, the ratio \(a/b\) measures the relative importance attached by the center to ambition over precision in planning, and we may well imagine that this ratio will fall over time, as the inherent drive to realize its full potential increasingly kicks in for the enterprise, making additional reward less necessary, and as increasing complexity in inter-enterprise relations places an ever-greater premium on exactitude in plan fulfillment. Second, maturation of the enterprise means that the morale factor acquires greater importance over time, causing \(c\) to increase. Both of these evolutionary tendencies push the plan ever closer to the underlying true best-effort position, \(z\), and, from Equation (13), push the optimal achievement level, \(x^*_a\), ever closer to \(x^*_p\).

The impact of the parameters \(z\), \(a\), \(b\), and \(c\) on the maximized reward, \(R^*\), can be studied by substituting \(x^*_p\) and \(x^*_a\) into the reward function, and simplifying, to obtain:
\[
R^* = az + (4^{-1/3} - 4^{-4/3})a^{4/3}b^{-1/3}c^{-2/3} = az + (0.47247 \ldots)a^{4/3}b^{-1/3}c^{-2/3} \tag{14}
\]
from which we observe the qualitative results:
\[
\frac{\partial R^*}{\partial z}, \frac{\partial R^*}{\partial a} > 0, \ \frac{\partial R^*}{\partial b}, \frac{\partial R^*}{\partial c} < 0 \tag{15}
\]
An increase in the objective (‘true’) possibilities of the enterprise increases the maximized reward (this is hardly surprising), as does an increase in the value assigned to ambitious planning. But
increases either in the value assigned to precise plan fulfillment or in the morale factor; the maximized reward; the last result is interesting, because it is slightly counter-intuitive. On reflection, however, we can see that a rising morale factor pushes the announced plan closer to and therefore lessens the extent to which the enterprise can ‘play’ the reward function by taking advantage of the $ax_p$ term.

The second-order conditions confirm the stationary value $R^*$ as a maximum. Writing the reward function as $f(x_p, x_a)$ and the CMF as $g(x_p, x_a)$, the single relevant bordered Hessian is

$$|\mathcal{H}| = \begin{vmatrix} 0 & g_p & g_a \\ g_p & f_{aa} & f_{ap} \\ g_a & f_{pa} & f_{pp} \end{vmatrix},$$

which, after considerable simplification, can be shown to be

$$|\mathcal{H}| = 12bc^2(z - x_p)^2 > 0,$$

confirming the stationary position of Equations (12)–(14) as maximizing $z$, and $R$.

### 4.3. The geometry

The model developed in this section, in which the reward function is maximized subject to the CMF as constraint, can be visualized easily using the geometry of earlier sections of this paper. Refer to Figure 5.

The maximand is represented by $R/a$, the initial point of each $x_a-x_p$ curve for a given value of $R$, on the $45^\circ$ line. The curve beginning at $R_0/a$ represents all levels of reward less than the maximum, $R^*$, for which there are two feasible combinations of $x_p$ and $x_a$ that generate the given $R$, shown as points A and B in Figure 5. The curve beginning at $R^*/a$ is optimal, as can be seen from the unique point of tangency to the CMF at $x_p^*$. For any higher levels of $R$ the reward curve does not touch the CMF at all, and those levels are not feasible.

Figure 5. Reward maximization subject to the collective morale function.
4.4. The model as guide to policy

In addition to affirming dynamic convergence of enterprise behavior to its true potential (truth-telling in plan formation), the model of this section can be used to formulate an approach to systematic thinking about the policy choice controlled by the center: setting the proportions between parameters \(a\) and \(b\). Recall that this amounts to setting the importance of precision in plan fulfillment, in relation to that of ambitious (‘taut’) planning. We will see that a degree of freedom remains in setting the scale of the two parameters; the relative size of \(R\) as such in relation to the base income of the enterprise affects both the share of the income generated by enterprise activity that goes to the center for general (society-wide) investment and social expenditure, and the degree of income security of the enterprises’ members, as measured by the share of income that does not depend on enterprise planning or results in any way. These are, of course, complex social decisions, and we will not pursue them in detail in this paper. The \(a/b\) ratio, however, is of interest in relation to our overall concern with incentives for planning and execution, and their result.

There are, apparently, two possible goals of policy: achieving an outcome of enterprise activity, \(x^a_\ast\) that is exactly equal to \(z\), the enterprise’s own estimate of its actual potential; and maximizing \(x^a_\ast\). We begin with the former.

From Equation (13), it becomes apparent that \(x^a_\ast = z\) is equivalent to

\[
\left(\frac{a}{4bc^2}\right)^{1/3} = c\left[\left(\frac{a}{4bc^2}\right)^{1/3}\right]^2,
\]

and this reduces to

\[
k = \frac{4}{c}, \text{ where } k = \frac{a}{b} \tag{18}
\]

We can get the enterprise to ‘produce’ \(z\), by setting this value for \(k\). Note that if, as previously hypothesized, \(c\) rises over time, this would mean a steadily falling \(a/b\) ratio: greater relative emphasis on precision over ambition. Everything, however, depends on the size of \(c\).

To get a sense of how \(c\) might be estimated, we can interpret \(x_p, x_a,\) and \(z\) dynamically, as growth factors: ratios of levels in the present period to those in the immediately prior period. A \(z\) of 1.04, for example, means that the locally-known best-practice outcome is an improvement in the achievement index of four percent over the preceding period (year, or quarter, or some other unit of time). We will need one other benchmark value to determine \(c\), and that can be the maximum value of \(x_a\). Suppose this is 1.08. Consulting Figure 4, we see that this implies

\[
z + \frac{1}{2c} = 1.04 + \frac{1}{2c} = 1.08,
\]

from which we find \(c = 12.5\). Using Equation (18), we have \(k = a/b = 0.32\). The \(x^a_\ast = z\) outcome, under the conditions assumed, suggests a value for \(a\) that is approximately one-third of the value of \(b\) (caution is needed here: the dimensionality of \(a\) differs from that of \(b\), and we cannot say whether any given ratio is inherently ‘large’ or ‘small’).
In the case under consideration, the enterprise’s plan can be found by plugging Equation (18) into Equation (12), which, after simplification, results in

\[ x_p^* = z + \frac{1}{c} \]  

(19)

Rising \( c \) over time causes \( x_p^* \) to converge to \( z \). At any moment in time, of course, the tension remains: the reward structure induces the enterprise to set a plan that is greater than its own internally known best-practice position. Under the present assumptions, from Equation (19) the plan would be \( 1.04 + 0.08 = 1.12 \), a rather draconian eight percentage points above the best-practice \( z \).

The \( x_p^* = z \) outcome, therefore, cannot be regarded as socially optimal. A look at Figure 6 reveals that \( x_a^* \) is equated to \( z \) on the far side of the CMF, not at the tangency point (which, in the given story, is not attainable). We get the enterprise’s own best result, therefore, by pushing it far beyond any range consistent with a high-morale regime.

The alternative policy option, to maximize the enterprise’s actually achieved level, is straightforward. In this case, we must have Equation (12) and \( x_p = z + 1/2c \) simultaneously, which suggests

\[ \left( \frac{k}{4c^2} \right)^{1/3} = \frac{1}{2c}, \text{ or } k = \frac{1}{2c} \]  

(20)

Using the same assumptions as before – \( z = 1.04 \) and \( x_p (x_a = \text{max}) = 1.08 \), and therefore \( c = 12.5 - k \) turns out to be \( 1/25 = 0.04 \), a seemingly much more extreme emphasis on exact plan fulfillment over plan ambitiousness. In this case, \( x_p^* \) (of course) = 1.08, and \( x_a^* = -12.5 (1.04)^2 + 1 + 2(12.5)(1.04)(1.08) - 12.5(1.08)^2 = 1.06 \). Figure 7 illustrates.

Figure 6. Forcing the enterprise to overachieve its local optimum.
4.5. A synthesis of the collective morale and material incentive cases

During the long period of Marx’s lower phase of communism – which I have called ‘maturing socialism’ – it seems reasonable to suppose that the Collective Morale Function is operative (c has attained a significant positive value), but that material incentives are also at work. In section 3.3 the effect of the reward $R$ on the enterprise’s activity result was studied. Can this effect be combined with the analysis of the CMF in this section?

In the context of the CMF, the obvious approach will be to transfer the effect of $R$ from the result, $x_a$, to the locally-known best-practice outcome, $z$. Adapting Equation (6), therefore, we have

$$z = \alpha + \frac{\beta R^*}{\gamma + R^*},$$

(21)

using $R^*$ instead of $R$ since we assume the enterprise is maximizing the reward under the CMF constraint, as in earlier subsections. From that analysis, we also have the additional relation between $R^*$ and $z$ (Equation (14) above):

$$R^* = az + (0.47247)a^{4/3}b^{1/3}c^{-2/3} = az + \varphi$$

(22)

The two $R^*$-$z$ relations, Equations (21) and (22), are graphed in Figure 8. The solution is simultaneous, determining $z^*$ and $R^{**}$ at point C, where the double-starring of $R$ suggests equilibrium in two senses: maximizing in relation to the CMF, and consistency with the material-incentive effect in Equation (21). The causal directions associated with each curve are indicated in Figure 8. Paths from arbitrary points A and B are therefore stable, converging on C.

Figure 7. Forcing the enterprise to maximize the achievement level.
Substituting Equation (21) into Equation (22) and solving for $R^{**}$ yields

$$R^{**} = \frac{1}{2} \left( \gamma - \varphi - a \alpha - ab \right) + \frac{1}{2} \sqrt{\left( \gamma - \varphi - a \alpha - ab \right)^2 + 4 (a\alpha + \varphi)} \tag{23}$$

Both roots are real; the positive sign alone yields a positive value for $R^{**}$. The full equilibrium can be characterized as follows:

$$z^* = \alpha + \frac{\beta R^{**}}{\gamma + R^{**}} \tag{24}$$

$$x^*_p = z^* + \left( \frac{a}{4bc^2} \right)^{1/3}$$

$$x^*_a = x^*_p - c \left( \frac{a}{4bc^2} \right)^{2/3}$$

The parameter $b$ does not play a role in determining $R^{**}$ and $z^*$, but it does enter into the enterprise’s optimal plan and the eventual result. All of the qualitative conclusions concerning the equilibrium continue to hold in this final case in which there is a material-incentive feedback from the optimal reward $R^{**}$ to the best-practice position $z^*$.

5. Some wider implications

5.1. Five great fears

Following the fall of the Soviet Union at the end of the last century, and the financial–structural crisis gripping the capitalist world economy at the beginning of the new one, there are (re)awakenings of socialist thought, and interest in the possibility of significant alternatives to capitalist polarization and crisis. Along the road to a vibrant new socialism, however, we encounter roadblocks, in the form of specific preoccupations, or fears, which prevent or problematize the required synthesis. I identify five such fears, enumerated below.
In speaking of ‘fears’, I do not mean to ridicule or belittle these perceptions; they point to legitimate and important concerns. I will suggest, however, that raised to a position of dominance over socialist thinking, each of them becomes a barrier, to be transcended.

5.1.1. **Fear of planning as such: market socialism**

In periods of transition from relative technological and social underdevelopment, many countries may experience a more-or-less stable standoff between a core state sector under socialist leadership, on the one hand, and a surrounding environment of spontaneous market relations and private property, on the other. This is Lenin’s New Economic Policy, projected forward to conditions in which the preparatory phase may last for quite a long time. It is entirely understandable that this experience should give rise to a theoretical perspective combining social ownership with ‘markets’, conceived as simple instruments for accomplishing production and consumption coordination ‘automatically’, without a need for planning (for which the human and material prerequisites are still scarce) (Roemer 1994; Schweickart 1996; Weisskopf 1992; Yang 2009).

Those prerequisites, however, can only emerge if planning institutions and procedures are themselves increasing in relative and absolute importance. The fear of planning results, first, in a situation in which the society must succumb to the ‘inevitability’ of polarization, new or re-emergent capitalist forms of property ownership, cyclical instability, helplessness in the face of external capitalist market pressures, and so on. Perhaps even more serious, this would amount to renunciation of the movement to build the environment of democratic planning, with its need to foster communication, principled behavior, awareness of social interconnections, consciousness leading to a consensus in favor of greater equality – in short, all of the foundations for an intentional, participatory and non-alienating social and economic system. These foundations are necessarily undermined by uncontrolled marketization.

5.1.2. **Fear of authority: horizontal iterations**

Understandably, given the perversion and hyperextension of authority within recent post-capitalist experience, and the degeneration of legitimate lines of authority into bureaucracy and despotism, some recent authors (Albert and Hahnel 1991, 2002; Hahnel 2002) have envisioned a non-market system of ‘horizontal’ iterations between producer collectives and consumer collectives, aided by an ‘Iteration Facilitation Board’ which, however, has no power to shape or alter the process. Collectives on both sides make proposals, and other collectives either accept or reject these, until a fit is achieved. In the process, something like a consistent set of outputs, exchanges, prices, and consumptions emerges; the authors of this model indeed draw heavily upon Walrasian tâtonnement for their formalizations.

Without a center, however, the model projects an economy running very much as an automatic mechanism, independent of human will and democracy, which operate only at the micro level of the collectives. It thus bears an uneasy resemblance to the very competitive market equilibrium that it proposes to replace. The center in a socialist planning system enables people society-wide to grasp the unity and consistency of the whole, and of their place within that whole. It is the site of decisive interventions concerning the overall direction of investment and growth, and the shaping of the built environment, and these cannot be the simple post facto sum of individual and ground-level collective choices, but rather must emerge out of a process of discussion and debate establishing the mandate and controlling the activity of the center itself. In short, democracy is vitally important, both for the micro-units and for the center.
5.1.3. **Fear of calculation: negotiated coordination**

In a model of ‘participatory planning through negotiated coordination’, Pat Devine (1988, 2002) envisions a system in which collectives, made up of ‘stakeholders’ (workers, community members, consumers), engage in a continuous process of discussion and bargaining leading to economic outcomes. Market ‘relations’, although not market ‘forces’, are included in this scheme. The division of labor into separate managerial, creative, skilled, unskilled, and caring categories is to be rapidly transcended, with all workers participating in all of these categories. Very little is said about prices, or about quantitative issues in income formation.

While ‘voice’ relationships and discussions among collectives and representative bodies undoubtedly play a role in socialist coordination, it is hard to see how the complex problems of assigning activities and positions in networks and production chains could be left to a process of open-ended negotiation. This would appear to lead to endless talk, raising concern over hyper-extension of meetings and over-politicization of coordination. Socialists have traditionally placed a high value on participation, and indeed the opportunity to participate in economic management is an expression of economic democracy, certainly important for overcoming the alienation from the worklife experience so common in capitalist societies. Participatory democracy, however, should be optimized, not maximized, and this seems impossible without use of *parametric forms*, such as planned prices and reward formulae (the latter being the object of study in this paper).

5.1.4. **Fear of delegation: the new central planning**

In their contribution to a 2002 symposium on socialist theory (*Science & Society* 2002), Paul Cockshott and Allin Cottrell (2002, 55n) noted: ‘In systems of the Soviet type the implementation of material balances was only partial. The information processing techniques needed to fully implement material balances did not exist. They do now’ (see also Cockshott and Cottrell 1993; cf. Jablonowski 2011). These authors refer to modern computer technology – an instance of the development of the forces of production – to ‘re-invent’ central planning: determination of enterprise plans through inversion of huge matrices containing all the necessary coefficients. This approach tackles the ‘millions of equations’ critique head-on.

The point is certainly important. The unending growth of firms in capitalist economies relies on electronic networks (intranets) to manage entities that are in some cases the size of small countries. There is every reason to believe that these information technologies can be transferred to and further developed within a socialist context, in which the obstacles to their full potential application created by the anarchy and antagonism of capitalism would be left behind.

Modern IT, however, also contains huge potentials for *delegation* – for coordinating and aggregating local plan formation and execution activities in ways that both provide those activities with a stable and comprehensible macro frame, and enable localities (enterprises) to act creatively, autonomously and meaningfully in terms of local possibilities. The new central planning model simply fails to address the valid core of the Hayek critique: the existence of local (and, especially given dynamics of change, tacit) knowledge that defies all attempts at quantitative standardization and transmission.

5.1.5. **Fear of the Soviet experience**

This final fear runs through almost the entire Western left. Even many Communist parties have concluded that 1989–1991 signifies rejection, by the peoples of the former Soviet Union and Eastern Europe, of the ‘Soviet model’ of socialism. I have argued (Laibman 2001, 2002, 2005,
2007) that the entire Soviet project, from the late 1920s until the demise, contains unique and vital contributions to the theoretical and empirical basis for socialist renewal. This applies especially to the experience with system-wide material balances; the reforms of the 1960s; and the much more profound transformations initiated in 1979 and known to the world as Mikhail Gorbachev’s ‘perestroika’. These developments, by and large, had no counterpart in Eastern Europe, or Asia, or Cuba. Their potential was thwarted due to the political instability arising from the authoritarian distortion of political, scientific and cultural life associated with the cult of Stalin from the 1930s forward. The peoples of the Soviet Union rejected, not socialism as it existed there, but the profound authoritarianism and repressiveness that took root in the early decades, and was not adequately confronted in the immediate postwar decades.

5.2. A possibility theorem

The fears must be transcended. As indicated, none of them is unfounded: Soviet planning was associated with serious – ultimately fatal – degeneration into misuse of bureaucratic power and authoritarian violations of the norms of human conduct. Delegation carries dangers of loss of overall coordination and stability; hierarchical structures, on the other hand, entail risks of abuse of power. Prices and incentives must be used in ways that transcend the alienating quality they had in the past, when ‘the market’ enforced an invisible fist of coercion and domination. Finally, the continuing ignorance and wholesale rejection of the Soviet experiment, so widespread due to the cumulative weight of Cold War ideology, deprives the world left of vital resources for overcoming one-sidedness in thinking about socialist forms – incorporating the best of previous experience while learning from defects and weaknesses. The slogan ‘Socialism for the Twenty-first Century’ embodies, implicitly or explicitly, this ‘rejectionism’. It must be said: there is only one road leading from the nineteenth century to the twenty-first, and that one passes through the twentieth.

Multi-level, iterative, democratic coordination seems to offer a perspective that combines the positive features of central plan formation, local autonomy, and sophisticated evaluation. In particular, the model developed in this paper suggests that overall central coordination can be combined with local autonomy and initiative, and that it is possible for incentives to be structured in a way that encourages enterprises to report and act upon their own true possibilities – at least, as the outcome of a dynamic maturation process in which the morale factor becomes ever-increasing in importance. If further development of the productive forces of an advanced socialist society requires autonomy and principled behavior on the part of its ground-level collectives, this provides the objective foundation for a rising c, and therefore for tendential convergence of plans to locally-known possibilities, and of results to plans. Plans become ever-better indicators of reality, enhancing the efficacy of central coordination. They also become less and less subject to possible manipulation in an unprincipled direction, since plan fulfillment requires the knowledgeable participation of all individuals making up the collective. Rational understanding on the part of all agents in this process means that the very separation between principal and agents is progressively overcome: we incentivize ourselves, and in so doing we lay the foundations for full internalization of incentives, transcending the need for their existence in the explicit form of the labor–income connection.

This is the socialist possibility theorem. Unlike impossibility theorems, possibility theorems are just that: they only point to agendas for further research, and for action.

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