An Extended Integrated Assessment Model for Mitigation and Adaptation Policies on Climate Change

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Overview: Mitigation and Adaption

- Builds on previous Models with Alfred Greiner, Lars Gruene and Helmut Maurer
- Integrated Assessment Model of Mitigation and Adaptation to Climate Change
- Optimal Control Problem with 5 State Variables and up to 6 Control Variables
- Model and Numerical Explorations of Uncertain Parameters
- We use Homotopic Solutions for Essential Parameters

Model of Mitigation and Adaptation to Climate Change

State variables:

K	:	private capital per capita,
g	:	public capital per capita,
b	:	country's level of debt,
R	:	non-renewable resource ,
M	:	GHG (Green House Gas) concentration in the atmosphere.

Control variables:

C	:	per capita consumption,
e_P	:	government's net tax revenue,
u	:	extraction rate from the non-renewable resource,

The stock of public capital g is allocated among three uses:

ν_1	:	standard infrastructure,
ν_2	:	climate change adaptation,
ν_3	:	climate change mitigation,

$$\nu_1, \nu_2, \nu_3 \geq 0, \quad \nu_1 + \nu_2 + \nu_3 = 1.$$

Dynamic Model of Adaptation and Mitigation

Production function $Y = A(A_K K + A_u u)^{\alpha} \cdot (\nu_1 g)^{\beta}$

Dynamical system

$$
\begin{aligned}
\dot{K} &= Y - C - e_P - (\delta_K + n)K - u\,\psi R^{-\tau}, & K(0) &= K_0,\\
\dot{g} &= \alpha_1 e_P + i_F - (\delta_g + n)g, & g(0) &= g_0,\\
\dot{b} &= (\bar{r} - n)b - (1 - \alpha_1 - \alpha_2 - \alpha_3) \cdot e_P, & b(0) &= b_0\\
\dot{R} &= -u, & R(0) &= R_0\\
\dot{M} &= \gamma\, u - \mu(M - \kappa \widetilde{M}) - \theta(\nu_3 \cdot g)^{\phi}, & M(0) &= M_0.
\end{aligned}
$$

State variable : $X = (K, g, b, R, M) \in \mathbf{R}^5$

Control variable : $U = (C, e_p, u) \in \mathbf{R}^3$

Control System : $\dot{X} = f(X, U), \quad X(0) = X_0$

Planning Horizon : $[0, T]$, terminal time $T > 0$

Welfare Functional and Optimal Control Problem

Optimal Control Problem

Maximize the welfare functional

$$W(T, X, U) = \int_0^T e^{-(\rho-n)\cdot t} \cdot \frac{\left(C(\alpha_2 e_P)^\eta \left(M-\widetilde{M}\right)^{-\epsilon} (\nu_2 g)^\omega\right)^{1-\sigma} - 1}{1-\sigma} \, dt$$

such that for all $t \in [0, T]$:

$$\dot{X}(t) = f(X(t), U(t)), \; X(0) = X_0 \,,$$

$$0 \leq u(t) \leq u_{max} \,,$$

$$K(t) \geq 0, \quad R(t) \geq 0 \,.$$

Further constraints:

terminal constraint : $K(T) = K_T \geq 0$

state constraint : $M(t) \leq M_{max} \quad \forall \, t \in [0, T] \,.$

Model Parameters

Parameter	Value	Definition
ρ	0.03	Pure discount rate
n	0.015	Population Growth Rate
η	0.1	Elasticity of transfers and public spending in utility
ϵ	1.1	Elasticity of CO_2-eq concentration in (dis)utility
ω	0.05	Elasticity of public capital used for adaptation in utility
σ	1.1	Intertemporal elasticity of instantaneous utility
A	$\in [1, 10]$	Total factor productivity
A_K	1	Efficiency index of private capital
A_u	$\in [50, 500]$	Efficiency index of the non-renewable resource
α	0.5	Output elasticity of privately-owned inputs, $A_k k + A_u u$
β	0.5	Output elasticity of public infrastructure, $\nu_1 g$
ψ	1	Scaling factor in marginal cost of resource extraction
τ	2	Exponential factor in marginal cost of resource extraction
δ_K	0.075	Depreciation rate of private capital
δ_g	0.05	Depreciation rate of public capital
i_F	0.05	Official development assistance earmarked for public infrasctureture
α_1	0.1	Proportion of tax revenue allocated to new public capital
α_2	0.7	Proportion of tax revenue allocated to transfers and public consumption
α_3	0.1	Proportion of tax revenue allocated to administrative costs
$\bar{r}$	0.07	World interest rate (paid on public debt)
$\widetilde{M}$	1	Pre-industrial atmospheric concentration of greenhouse gases
γ	0.9	Fraction of greenhouse gas emissions not absorbed by the ocean
μ	0.01	Decay rate of greenhouse gases in atmosphere
κ	2	Atmospheric concentration stabilization ratio (relative to $\widetilde{M}$)
θ	0.01	Effectiveness of mitigation measures
ϕ	$\in [0.2,\ 1]$	exponent in mitigation term $(\nu_3\ g)^{\phi}$

- Dynamic Model for Mitigation and Adaptation to Climate Change
- With Parameter Uncertainty
- Numerical Solutions for Mitigation Efficiency $\phi = 1$
  - Comparing solutions for fixed and optimal values of ν_1, ν_2, ν_3
  - Homotopic Solutions for Efficiency Index A_u
  - Comparing Solutions for $A_u = 100, 200, 500$
- Numerical Solutions for Mitigation Efficiency $0.2 \leq \phi \leq 1$
  - Homotopic Solutions for Exponent $0.2 \leq \phi \leq 1$
  - Comparing Solutions for $A_u = 100, 200, 500$
- Homotopic Solutions for discount rate $0.02 \leq \rho \leq 0.1$

Initial conditions, constraints, choice of ν_1, ν_2, ν_3

Initial conditions

$$K(0) = 1.5,\ g(0) = 0.5,\ b(0) = 0.8,\ R(0) = 1.5,\ M(0) = 1.5.$$

Control constraint : $0 \le u(t) \le 0.1 \quad \forall\, t \in [0, T]$.

Terminal constraint : $K(T) = K_T = 3$.

Strategy 1 : Choose fixed values $\nu_1 = 0.6, \nu_2 = 0.2, \nu_3 = 0.2$.

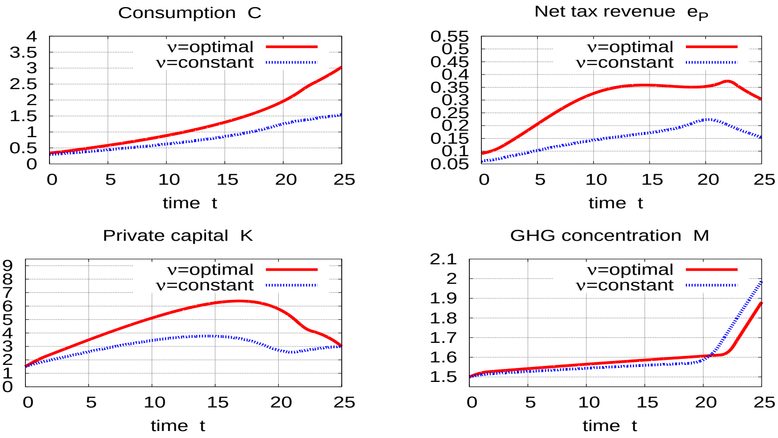
Strategy 2 : Consider ν_1, ν_2, ν_3 as additional optimization variables satisfying the constraints $\nu_1 + \nu_2 + \nu_3 = 1$.

Strategy 3 : Consider $\nu_1 = \nu_1(t), \nu_2 = \nu_2(t), \nu_3 = \nu_3(t),\ t \in [0, T]$, as control functions satisfying the constraints $\nu_1(t) + \nu_2(t) + \nu_3(t) = 1 \quad \forall\, t$.

Strategy 3 improves only slightly on Strategy 2 and will be discarded in the numerical results.

Comparison : fixed and optimal values of ν_1, ν_2, ν_3

Exponent $\phi = 1$ and efficiency index $A_u = 50$:



optimal values $\nu_1 = 0.9534, \quad \nu_2 = 0.04662, \quad \nu_3 = 0$: $W(T) = 5.1086$

fixed values $\nu_1 = 0.6, \quad \nu_2 = 0.2, \quad \nu_3 = 0.2$: $W(T) = -2.1006$

Comparison: fixed and optimal values of ν_1, ν_2, ν_3

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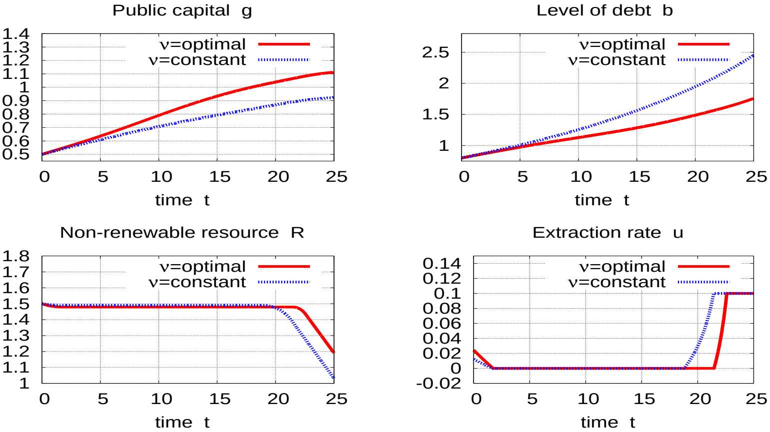

optimal values $\nu_1 = 0.9534, \quad \nu_2 = 0.04662, \quad \nu_3 = 0$: $\quad W(T) = 5.1086$

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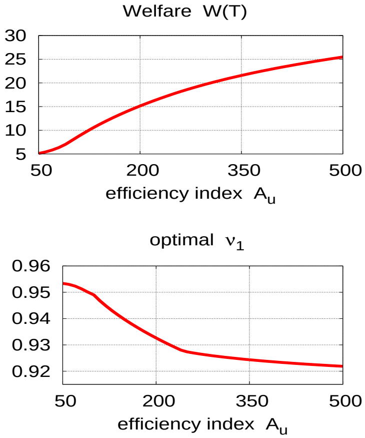
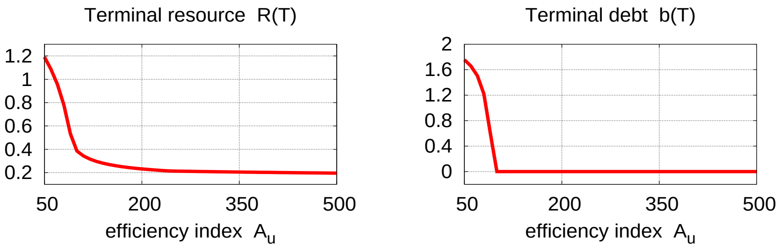
Terminal values for homotopy $A_u \in [50, 500]$



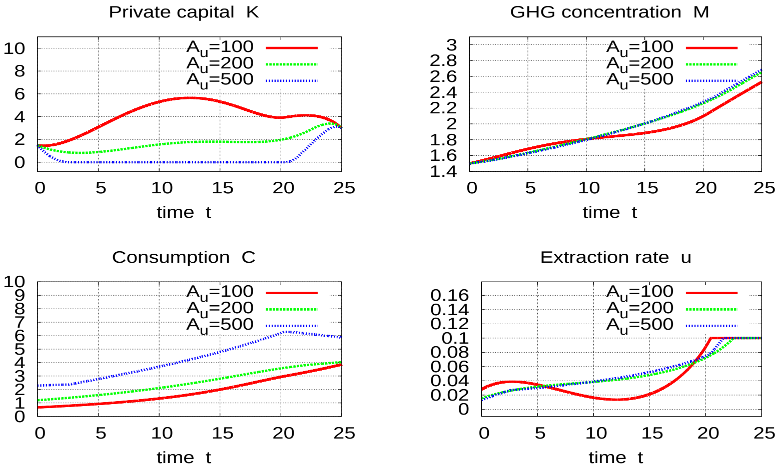
Terminal values $W(T)$ and $M(T)$ and optimal parameters ν_1, ν_2.

Terminal values for homotopy $A_u \in [50, 500]$



Terminal values $R(T)$ and $b(T)$ for $A_u \in [50, 500]$

$\phi = 1$: Solutions for $A_u = 100, A_u = 200, A_u = 500$



$\phi = 1$: comparing solutions for $A_u = 100$, $A_u = 200$, $A_u = 500$.

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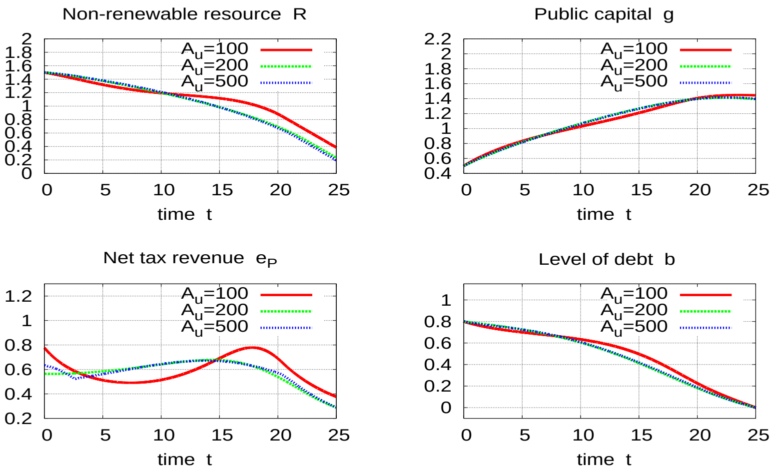


$\phi = 1$: comparing solutions for $A_u = 100$, $A_u = 200$, $A_u = 500$.

Mitigation Exponent $0.2 \leq \phi \leq 1$

Consider the mitigation exponent $0.2 \leq \phi \leq 1$ in

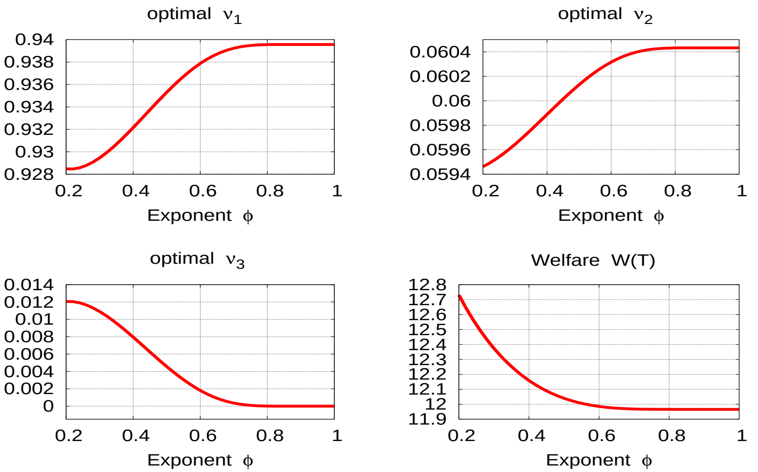
$$\dot{M} = \gamma u - \mu(M - \kappa \widetilde{M}) - \theta(\nu_3 \cdot g)^{\phi}.$$

For $\phi = 1$ we always obtain $\nu_3 = 0$. However, for

$$\phi \leq \phi_0 \approx 0.88$$

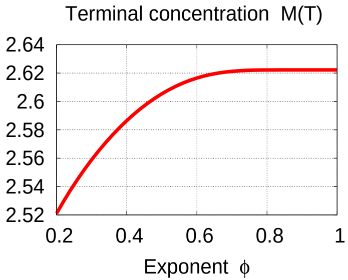
we obtain $\nu_3 > 0$. These findings can be confirmed by computing optimal solutions via a homotopy with respect to $\phi \in [0.2, 1]$.

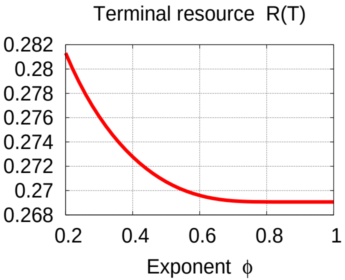
Homotopy for $\phi \in [0.2, 1]$ with $A_u = 150$



$\phi \in [0.2, 1]$: optimal values ν_1, ν_2, ν_3 and welfare $W(T)$.

Homotopy for $\phi \in [0.2, 1]$ with $A_u = 150$

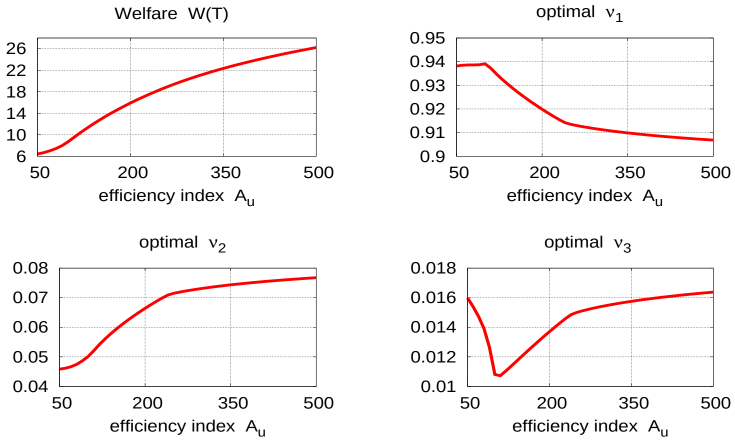




$\phi \in [0.2, 1]$: terminal values $M(T)$ and $R(T)$.

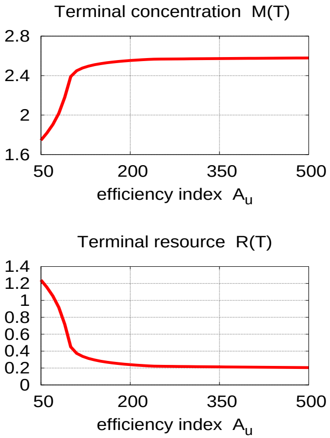
$\phi = 0.2$: terminal values for homotopy $A_u \in [50, 500]$

$\phi = 0.2$: homotopy parameter $50 \leq A_u \leq 500$



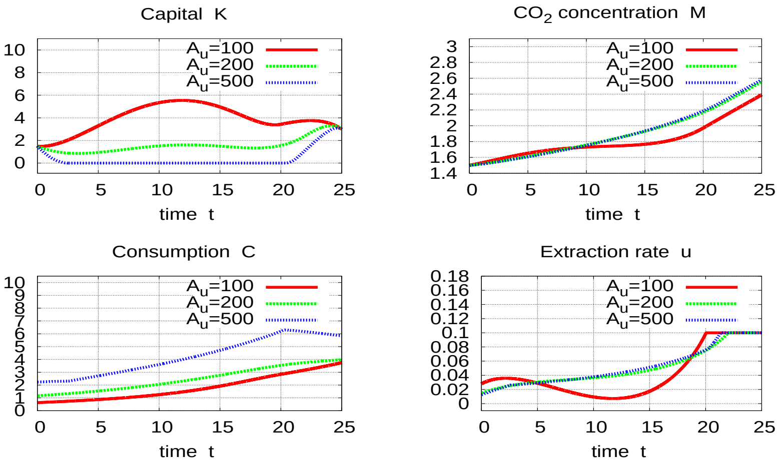
Welfare $W(T)$ and optimal parameters ν_1, ν_2, ν_3.

$\phi = 0.2$: terminal values for homotopy $A_u \in [50, 500]$



Terminal values $M(T)$, $R(T)$, $b(T)$, $g(T)$ depending on A_u

$\phi = 0.2$: Solutions for $A_u = 100, A_u = 200, A_u = 500$



$\phi = 0.2$: comparing solutions for $A_u = 100$, $A_u = 200$, $A_u = 500$.

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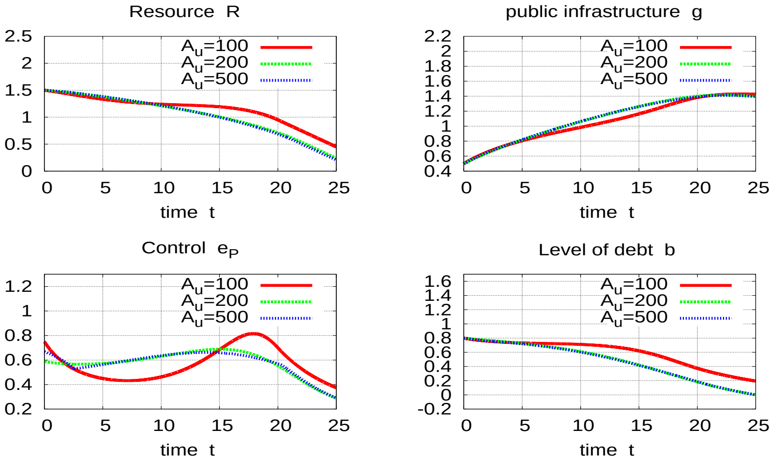
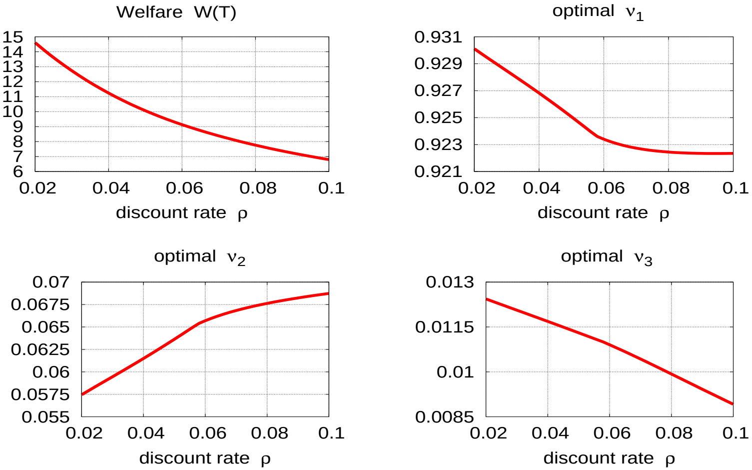


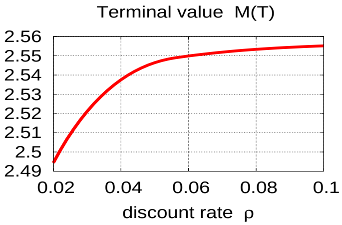
$\phi = 0.2$: comparing solutions for $A_u = 100$, $A_u = 200$, $A_u = 500$.

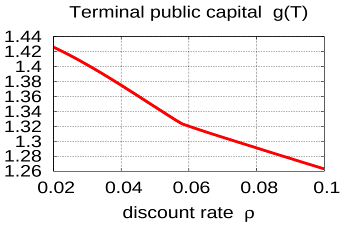
$\phi = 0.2$: homotopy for dicount rate $0.02 \le \rho \le 0.1$

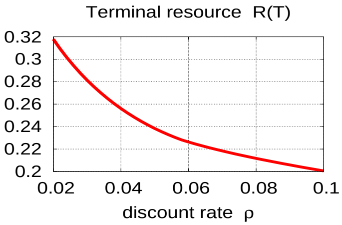


Welfare $W(T)$ and optimal parameters ν_1, ν_2, ν_3.

$\phi = 0.2$: terminal values for homotopy $\rho \in [0.02, 0.1]$







Terminal values $M(T)$, $R(T)$, $g(T)$

Simplified Model–Financing of climate policies

solved with NMPC, see Gevorkyan et al. (2016)
Phase 1: Business As Usual (BAU)

$$Max_C \int_{t=0}^{N} e^{-\rho t} ln(C) dt$$

$$\dot{K} = D \cdot Y - C - (\delta + n)K$$

$$\dot{M} = \beta E - \mu M$$

$$E = (\frac{aK}{A_0})^{\gamma}$$

$$D(\cdot) = (a_1 \cdot M^2 + 1)^{-\psi}$$

Simplified Model–Financing of climate policies

Phase 2: Carbon tax, and green bonds

$$Max_C \int_{t=0}^{N} e^{-\rho t} ln(C) dt$$

$$\dot{K} = D \cdot Y - C - \chi . Y - (\delta + n) K$$

$$\dot{M} = \beta E - \mu M$$

$$\dot{b} = r \cdot b + A$$

$$E = \left(\frac{aK}{5(A + \chi . Y) + A_0}\right)^{\gamma}$$

$$\chi = b_1 \frac{2}{\pi} atan(b_2 M^2 - 0.01)$$

Simplified Model–Financing of climate policies

Phase 3: Debt repayment after elimination of carbon emission

$$\dot{K} = Y(1-\tau) - C - \chi . Y - (\delta + n)K$$

$$\dot{b} = r \cdot b - \tau Y$$

Conclusion

- Dynamic Model for Mitigation and Adaptation to Climate Change
- Optimal control model with 5 state variables and 3 and 6 controls
- Numerical Method Finite Time Horizon, AMPL
- Explorations of Parameter Uncertainty
- Next Step: Calibration for North-South Countries
- Supported by German Science Foundation

Thank you for your attention