A Keynesian Based Econometric Framework for Studying Monetary Policy Rules

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Abstract

In the framework of a Keynesian based monetary macromodel we study the implications of alternative monetary policy rules. Our monetary macromodel exhibits the following features: asset market clearing, disequilibrium in the product and labor markets, sluggish price and quantity adjustments, two Phillips curves for the wage and price dynamics and expectations formulation which represents a combination of adaptive and forward looking behavior. Two alternative monetary policy rules for controlling inflation are considered: the monetary authority 1) targeting monetary aggregates or 2) targeting the interest rate. For those two policy rules the model's dynamic features are explored given certain parameter constellations. Then the key parameters of the model variants are estimated through GMM and single equation estimations employing U.S. time series data 1960.1-1995.1. Stochastic simulations are performed and contrasted with U.S. macroeconomic data in terms of standard deviation of macro variables as well as their cross-correlation to output. The model can be viewed as an alternative to equilibrium macromodels in fitting macroeconomic data. With respect to our two monetary regimes it seems that in terms of volatility the model variant with the second policy rule gives a better fit whereas for cross-correlation with output the variant with the first policy rule performs better.

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1 Introduction

Recently, in macroeconomics the quantitative study of monetary policy rules has been undertaken in a variety of frameworks. Such frameworks are, for example, the macroeconometric approach by Fair (1984), the VAR by Bernanke and Blinder (1993), Sims (1996) and Fuhrer and Moore (1995) and the optimizing approach by Rotemberg (1997) and Woodford (1997) and Christiano, Eichenbaum and Evans (1997). In this literature two alternative monetary policy rules have been considered, namely the monetary authority 1) targeting monetary aggregates or 2) targeting the interest rate. The latter rule originates in Taylor (1993) and has also been called the Taylor rule.\footnote{A rule of this type, however, can already be found in Fair (1984).}

Historically, as has been shown, most central banks of OECD countries switched during the 1980s from the policy of controlling monetary aggregates to targeting short-term interest rates (see Bernanke and Mishkin 1992).\footnote{Although the German Central Bank claims that it targets the money supply there are several papers which show that in fact the Bundesbank uses also interest rate targeting, see Clarida and Gertler, (1995), Bernanke and Clarida, Gali and Gertler (1997).} In particular the second type of monetary policy rule, the Taylor rule, has recently been given much attention in macroeconometric frameworks. This paper employs a small scale Keynesian integrated macromodel to evaluate the above two monetary regimes of central banks.

The Keynesian monetary growth model presented and estimated here exhibits along the lines of Flaschel, Semmler and Franke (1997) asset market clearing, disequilibrium in product and labor market, sluggish price and quantity adjustments, two Phillips-curve for the wage and price dynamics and expectations formulation which represents a combination of adaptive and forward looking. Moreover, as in Flaschel and Chiarella (1998), the paper provides an integration of prototype Keynesian macromodels of real growth, of inflationary dynamics and inventory adjustment. As to the historical tradition, on the demand side it is Keynesian, it makes use of Kaldor's distribution theory, uses the asset market structure as in Sargent's (1987, part I) Keynesian model, employs Malinvaud's (1980) investment theory and the Metzler inventory adjustment process\footnote{See Franke and Lux (1993) and Franke (1992).} and an expectations mechanism which is forward and backward looking.\footnote{See also Groth (1988).}

More specifically, we consider a closed three sector economy (households, firms and government), where there exist five distinct markets; for labor, goods, money, bonds and equity (which are perfect substitutes of bonds).\footnote{We restrict ourselves to this standard, basic framework to stay, at least initially, very close to traditional foundations of Keynesian dynamics, see Sargent (1987, Ch.I-V) in particular.} In order to briefly summarize
our model we use the following table. We employ the index $d$ to denote 'quantities demanded' and no index in the case of 'quantities supplied'. The symbols in the following table should be clear as to their economic meaning (a detailed list of the notation employed will be provided in an appendix to this paper).

<table>
<thead>
<tr>
<th></th>
<th>Labor market</th>
<th>Goods market</th>
<th>Money market</th>
<th>Bonds market</th>
<th>Equities market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td>$L$</td>
<td>$C$</td>
<td>$M^d$</td>
<td>$B^d$</td>
<td>$E^d$</td>
</tr>
<tr>
<td>Firms</td>
<td>$L^d$</td>
<td>$Y, I + \delta K$</td>
<td>$-$</td>
<td>$-$</td>
<td>$E + \Delta E$</td>
</tr>
<tr>
<td>Government</td>
<td>$-$</td>
<td>$G$</td>
<td>$M + \Delta M$</td>
<td>$B + \Delta B$</td>
<td>$-$</td>
</tr>
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</table>

This is the basic structure of the closed economy model considered in this paper.

In accounting for behavior of firms we use the fact that firms seldom operate at their desired capacity and with their desired inventories, but deviate in general from those norms due to unexpected changes in aggregate goods demand. A distinguishing feature of Keynesian models, in particular in contrast to equilibrium macromodels,\(^6\) is that the consideration of under- or over-utilized capital besides an under- or over-utilized labor force is important. Yet, even in Keynesian tradition it has often been neglected, see Malinvaud (1980) for an exception in the context of a stationary economy.

Moreover, we attempt to include growth in $\alpha$, although rudimentary, but consistent way—a feature that is generally not often explored in the literature on Keynesian macrodynamics.\(^7\) The fact that growth is lacking in most monetary disequilibrium macromodels is an important discontinuity in the development of the literature on such macroeconomics. Furthermore, our small scale model is complete in the sense that we consider all the major markets and define the financing conditions and budget restrictions of households, firms and the government. The model gives rise to seven interdependent laws of motion or — via a suitable assumption on tax collection — to a six-dimensional integrated dynamic system.

It should also be noted that all behavioral and technical relationships in the following model have been chosen to be linear as much as possible. It is not difficult to introduce into the model some well-known nonlinearities that have been used in the literature on real, monetary and inventory dynamics of Keynesian type. We use

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\(^6\)See, for example, Flaschel, Franke and Semmler (1997, chs.1-4).

\(^7\)On the other hand, Keynesian models of growth are rarely mentioned in surveys on the literature on monetary growth, as for example, in the recent article on 'Money, inflation and growth' by Orphanides and Solow (1990).
only unavoidable nonlinearities in the model. Such nonlinearities naturally arise from the growth rate formulation of certain laws of motion, certain unavoidable ratios and the multiplicative interaction of variables. Already on the basis of these most basic types of nonlinearities it can be shown that interesting dynamic properties will arise – without any ‘bending of curves’ known to be needed to tame the explosive dynamical behavior of the partial submodels.

The model’s dynamic features for the two policy regimes are explored for certain parameter constellations. For the purpose of doing so we transform the continuous time model of the Chiarella and Flaschel (1998) model into a discrete time model. The general dynamic behavior of our system cannot be studied analytically with currently available techniques – apart from being able to make some basic statements about it. For the model with money supply rule it is shown that for a wide range of parameter constellations interesting dynamics, for example, persistent cycles, may arise. On the other hand, the Taylor rule appears to add further stabilizing forces to this type of model.

In order to match the model with the U.S. macroeconomic time series data we estimate key parameters through GMM and single equation estimations using data from 1960.1-1995.1. In the estimation of the parameters for the wage-price dynamics and for the inventory dynamics as well as investment and consumption functions expectations variables appear which are not observables. We can, however, transform the equations to be estimated by replacing the unobservable by observable variables.

We employ then stochastic simulations and contrast the model with the historical macrodata with respect to the standard deviation of the macro variables as well as their cross-correlation to output. With respect of our two monetary regimes it seems that in terms of volatility the model variant with the second policy rule, the Taylor rule, gives a better fit whereas for cross-correlation with output the variant with the first policy rule performs better. Our macroeconometric framework should be viewed only as first step toward describing the historical data and policy rules. Yet, it appears useful, since its results can compete with currently widely used equilibrium macromodels of RBC type.

The remainder of the paper is organized as follows. Section 2 introduces the small scale monetary macromodel. Section 3 studies the steady state and the dynamics of the integrated model for a wide range of parameters. In section 4 we describe our econometric estimations and report results from the stochastic simulation. Section 5 concludes the paper and the appendices describe some extensions of the model and our econometric procedures to match the model with the facts.

2 A Model of a Monetary Macroeconomy

Turning the continuous time Keynesian model with Metzler inventory adjustment, as developed by Chiarella and Flaschel (1998, Ch.7), into a discrete time version makes the time structure of the model more transparent. We provide a structural form
of the model that is theoretically coherent in its use of budget constraints, dating
of activities and expectations and can be investigated from the empirical point of
view. We formulate the model in terms of modules where each of them allows for any
number of modifications.

We start with some notations. A complete list of notations is given in the appendix
1. Our Keynesian disequilibrium model uses the following income flows' and assets'
definitions:

1. Definitions (remunerations and wealth):

\[ \omega_t = \frac{w_t}{p_t}, \quad \rho_t^e = \frac{(Y_t^e - \delta K_{t-1} - \omega_t L_t^e)}{K_{t-1}}, \]  \hspace{1cm} (1)

\[ W_t = \frac{(M_{t-1} + B_{t-1} + p_{et} E_{t-1})}{p_t}, \quad p_b = 1. \]  \hspace{1cm} (2)

\[ \dot{z}_t = \frac{(z_t - z_{t-1})}{z_{t-1}} \]  \hspace{1cm} (3)

This set of equations represent real wages, \( \omega_t \), the expected rate of return of capital
(RRC), \( \rho_t^e \), based on sales expectations at \( t-1 \) for the present point in time \( t \) and the
definition of the current stock of real wealth \( W_t \). Note that stocks that exist at time \( t \)
are indexed by \( t-1 \), while their actual reallocation and revaluation happens in \( t \) and
is thus indexed by \( t \). Note also that secondary market components of our financial
markets must be integrated with the primary one (new issue and new demand for
such assets), since there is no separation between primary and secondary markets
(new issues and resale).

Current real wealth held by households in \( t \) is here composed of money \( M_{t-1} \), fixed
price bonds \( B_{t-1} \) (\( p_b = 1 \)) and equities \( E_{t-1} \) as in Sargent (1987)\(^8\) and is determined
on the basis of the current market prices for equities, \( p_{et} \), and output \( p \). Real wealth
given at \( t \) is based on assets bought at \( t-1 \), but valued at current prices for equities
and goods. Furthermore, current output is produced with the capital stock given at
\( t-1 \) and with labor that is paid in \( t \). Note finally that the definition of growth rates \( \dot{z} \)
is indexed forward in order to ease the presentation of the intensive form of the
model later on.

Describing income distribution and saving along the line of Kaldor (1966)\(^9\) we
propose the behavior of households, represented by workers and asset holders, to be
determined by the following set of equations. All behavioral equations are at first
chosen as linear as possible. Only intrinsic 'natural' nonlinearities are allowed for.
Later, extrinsic nonlinearities may be added.

2. Households (workers and asset-holders):

\[ C_t = \omega_t L_t^d + (1 - s_e)[\rho_t^e K_{t-1} + \tau_t B_{t-1}/p_t - T_t], \]  \hspace{1cm} (4)

\[ S_{pt} = s_e[\rho_t^e K_{t-1} + \tau_t B_{t-1}/p_t - T_t] \]  \hspace{1cm} (5)

\[ \dot{L}_{t+1} = n = \text{const}. \]  \hspace{1cm} (6)

\(^8\) Assuming consols (\( p_b = 1/\tau_t \)) in the place of the fixprice bonds assumed by Sargent (1987) does
not alter the 6D-dynamics of the private sector to be considered below.

\(^9\) We do, however, not consider corporate saving as Kaldor does.
\[ W_t + S_{pt} = \left( M^d_t + B^d_t + p_{et} E^d_t \right) / p_t, \] see the asset market module below. \( (7) \)

Aggregate consumption of households, \( C_t \), is based on classical saving habits with savings out of wages set equal to zero for simplicity. We assume that real taxes \( T_t \) are paid out of (expected) profits and interest income and in a lump sum fashion (see module 4). Extending our fiscal policy instruments by allowing for a tax on wage income as well does not make much qualitative difference to the model. It is easy to add wage taxation at a constant rate, \( \tau_w \), to the above module and to the module describing the government sector. This would alter the calculation of the steady state rate of return of capital as shown in Proposition 4 of Appendix 3.

Equation (5) provides the definition of real private savings, \( S_{pt} \), here only of wealth owners, which is, in eq. (7), allocated to the actual changes in the stock of money, in bonds and in equity. The supply of labor, \( L_t \), is inelastic at each moment in time with a rate of growth, \( \dot{L}_{t+1} \), given by \( n_t \), the natural rate of growth. Note here again that we use end of period indices to characterize growth rates of quantities and prices. Finally, eq. (7) states how real wealth and real savings act as budget restriction for the sum of the stock demand for real money balances and real bond and equity holdings of asset owners at time \( t \) (Walras' law of stocks and flows).

The production sector and the behavior of firms are described by the following set of equations:

3. **Firms (production, investment and inventory):**

\[ Y^p_t = y^p K_{t-1}, \quad y^p = \text{const.}, \quad U_t = Y_t / Y^p_t = y_t / y^p \quad (y_t = \frac{Y_t}{K_{t-1}}), \] \( (8) \)

\[ L^d_t = Y_t / \pi, \quad \pi = \text{const.}, \quad V_t = L^d_t / L_t = Y_t / (\pi L_t), \] \( (9) \)

\[ I_t = \dot{I}_t (\rho^* - y_t - \tau_t) K_{t-1} + \dot{I}_2 (U_t - \bar{U}) K_{t-1} + n K_{t-1}, \] \( (10) \)

\[ S^f_t = Y^f_t - Y^e_t = I_t, \] \( (11) \)

\[ \Delta Y^e_t = Y^e_t - \delta K_{t-1} - C_t - I_t - G_t = Y^e_t - Y^d_t, \] \( (12) \)

\[ \frac{p_{et} \Delta E_t}{p_t} = I_t + \Delta Y^e_t = I_t + (N_t - N_{t-1} - I_t), \] \( (13) \)

\[ I^e_t = I_t + N_t - N_{t-1} = I_t + \Delta Y^e_t + I_t = p_{et} \Delta E_t / p_t + I_t, \] \( (14) \)

\[ \dot{K}_t = (K_t - K_{t-1}) / K_{t-1} = I_t / K_{t-1}. \] \( (15) \)

According to equations (8)-(15), firms produce output, \( Y_t \), in the technologically simplest way possible, via a fixed proportions technology characterized by the potential output-capital ratio \( y^p = Y^p_t / K_{t-1} \) and a fixed ratio \( \pi \) between actual output \( Y_t \) and labor \( L^d_t \) needed to produce the output. This simple concept of technology allows for a straightforward definition of the rate of utilization of capital, \( U_t \), and labor, \( V_t \).

Note that current investment \( I_t \) will not have a capacity effect in the current point in time.

\(^{10}\) Chiarella and Flaschel (1998, Ch.5) show how such an approach can be extended to the case of smooth factor substitution without substantial change of the model.
time $t$ which is restricted by the capital to $K_{t-1}$, and that labor is paid ex post, at $t$
from the proceeds obtained from current output, $Y_t$.

In equ. (10) investment per unit of capital, $I_t/K_{t-1}$, is driven by two forces, the
excess return of current expected RRC, $\rho_{t-1}^e$, over the real rate of interest, $\tau_t - \pi_t$, and
the deviation of actual capacity utilization $U_t$ from the normal or non-accelerating
-inflation rate of capacity utilization $\bar{U}$. Note that the expected inflation rate $\pi_t$
refers to the medium-run (see the wage-price module below) and thus not to the
time interval $[t-1,t]$. Thus, the two rates of return that are here being compared
may imply different time horizons. In appendix 3 this is made more precise. There
is also an unexplained trend term in the investment equation which is set equal to
the natural rate of growth for reasons of simplicity. An endogenous treatment of the
trend term may be desirable.\textsuperscript{11}

Savings of firms, equ. (11), is equal to the excess of output over expected sales
(caused by planned inventory changes). We assume in this model that expected sales
are the basis of firms’ dividend payments (after deduction of capital depreciation,
$\delta K_{t-1}$, and real wage payments, $\omega_t L^d_t$).\textsuperscript{12}

Equ. (12) defines the excess of expected demand over actual demand. In the
present version of the model this excess demand has to be financed by firms by
issuing new equity. It follows, as expressed in equ. (13), that the total amount of
new equity issued by firms must equal the intended fixed capital investment and
unexpected inventory changes, $\Delta Y_t^e = N_t - N_{t-1} - \Delta I_t$; compare our formulation of
the inventory adjustment mechanism in module 6. Equ.(14) then describes the actual
investment, $I_t^e$, from various perspectives, serving here to add some details about the
accounting framework of our firms. Finally, equ. (15) states that (fixed business)
investment plans of firms are always realized in this Keynesian (demand oriented)
context, by way of corresponding inventory changes. We want to note that it is easy
to add Harrod neutral technical change to the considered model, see the appendix 3
(modules 1. and 3.).

We now turn to a brief description of fiscal and monetary policies:

4. Government (fiscal and monetary authorities):

$$t^n = \frac{T_t - \tau_t B_{t-1}/p_t}{K_{t-1}} = \text{const.}, \quad (16)$$

$$G_t = gK_{t-1}, \quad g = \text{const.}, \quad (17)$$

$$S_{gt} = T_t - \tau_t B_{t-1}/p_t - G_t, \quad (18)$$

$$\dot{M}_t = \mu = \Delta M_t/M_t = \mu_o + \beta_m (\mu_o - n - \pi_t), \quad \mu_o = \text{const.}, \quad (19)$$

$$\Delta B_t = p_t G_t + \tau_t B_{t-1} - p_t T_t - \Delta M_t. \quad (20)$$

\textsuperscript{11}See, for example, Chiarella and Flaschel (1998, ch.8).

\textsuperscript{12}Further instruments for the financing decision of firms are considered in Franke and Semmler’s
(1997) model of the financial market. For simplicity we here, however, stick to the framework used
in Sargent (1987, part I) and assume that firms do not borrow and retain no expected earnings.
Since we want to focus on monetary policy we stylize fiscal policy in as simple a way as possible. In the present model the role of bond financing of government expenditures appears explicitly, but does not yet feed back into the dynamics of the private sector; see the intensive form of the model in section 4. In equ. (16), lump sum real taxes, on income from financial assets only, are, therefore, assumed to be collected in such a way that their ratio (net of interest) $t^n$ to the capital stock remains constant. Using this treatment of taxes, see also Sargent (1987, part I), allows to treat fiscal policies as parameters in the intensive form of the model with no feedback of bond accumulation, via income or wealth effects, on the rest of the model. Hence, in equ. (17), also government expenditures per unit of capital $g$ are assumed to be a constant. The resulting definition of government saving, equ. (18), $S_{gt}$ is obvious.

As regards to the monetary policy we will explore alternative rules. Module 4 above first assumes that monetary authority, for controlling inflation, targets the supply of money, represented in equ. (19). The growth rate of the money supply, $\hat{M}$, is governed by a constant trend term $\mu_o$ in equ. (19) which would imply a steady state rate of inflation of $\mu_o - n$. In addition, money supply also responds to the deviation of the rate of inflation that is expected for the medium-run, see module 7 of the model, from the steady state rate of inflation in an attempt to counteract this deviation by appropriate changes in the growth rate of the money supply ($\beta_m > 0$). This is, of course, still a very simple rule for the growth of money supply which refers to the trend growth in potential output as far as its dynamic part is concerned.

As modern alternative to this money supply oriented policy we will therefore also investigate the Taylor rule according to which the monetary authority aims at setting the nominal rate of interest in response to deviations of the interest rate from its steady state value, the deviations of the actual rate of inflation, $\hat{\pi}_t$, from a target rate of inflation, $\bar{\pi}$, and the deviations of the actual rate of capacity utilization from the target rate of capacity utilization. This alternative rule, often called the central bank reaction function, thus reads:

$$r_{t+1} = r_o + \beta_{r_1} (r_t - r_o) + \beta_{r_2} (\hat{\pi}_{t+1} - \bar{\pi}) + \beta_{r_3} (U_t - \bar{U})$$

Note that the here employed rate of inflation is a forward rate of inflation in line with our assumptions in module 7 of the model where myopic perfect foresight is present in the mutual interdependence of the wage and price setting process. Note also that $\beta_{r_3} (U_t - \bar{U})$ can be rewritten as $\beta_{r_3} (U_t / \bar{U} - 1)$ with only a slight change in the adjustment parameter. Note finally that the above Taylor rule assumes that money demand is always realized at the nominal rate of interest set by the monetary authority.

In view of the fiscal rule for government and either of the monetary rules for the central bank the issue of new bonds by the government (net of open market operations by the central bank) is then determined residually via equ. (20). This states that the resulting money and bond financing must exactly cover the deficit in government
expenditure financing. This holds also for the Taylor rule.\footnote{The rate of change of the money supply $\mu$ implied by the above use of the Taylor rule reads (in terms of continuous time for simplicity):}

We now describe the asset market equilibrium conditions of the model:

5. Equilibrium conditions (asset-markets):

\begin{align*}
W_t + S_{pt} &= (M^d_t + B^d_t + p_{ct}E^d_t)/p_t, \quad (21) \\
M_t &= M_{t-1} + \Delta M_t = M^d_t = h_1p_tY_t + h_2p_tK_{t-1}(r_o - r_{t+1}) \quad (22) \\
p_{ct}E_t &= \hat{\rho}_{t+1}^p\Delta K_{t+1}/(r_{t+1} - \hat{\rho}_{t+1}), \quad (23) \\
[B_t = B_{t-1} + \Delta B_t = B^d_t, E_t = E_{t-1} + \Delta E_t = E^d_t].
\end{align*}

The source of the stock demands for financial assets is again shown in (21) as the aggregate value of the existing stock at current market prices plus savings of asset owning households. Money demand is specified as a simple linear function of nominal income, $p_t Y_t$, and interest $r_t$ ($r_{t}$ the steady state rate of interest) in the usual way. Note here that the employed interest rate $r_{t+1}$ is determined (or set) in $t$, with interest paid in $t+1$ which gives the index to this rate.\footnote{This convention conforms with the definition of $\rho_{t+1}^p$ that we use in the determination of share prices below.} Note also that money market equilibrium (22) does not feed back into the rest of the model in the case of the Taylor monetary policy rule, in which case money supply is always adjusted in order to meet money demand at the nominal rate of interest set by the central bank.

The form (22) of the money demand function is chosen in the above way in order to allow for a simple formula for the nominal rate of interest in the intensive form of the model, i.e., it is chosen for mathematical convenience to some extent and yet not exclusively for economic reasons. Nonlinear money demand functions with real wealth in the place of the capital stock would be more appropriate and thus should replace this simple function in further extensions of this model if it is desirable to assume that stocks held should appear in the money demand function.\footnote{The above simple money demand function can be obtained as a Taylor approximation of a general money demand function if it is assumed that money demand is homogeneous of degree 1 in income and wealth and if the variable $K$ is used as a proxy for real wealth.} Note that in appendix 3 of our paper, see there the modules 4. and 5., we make use of the Cagan type of money demand function (which is linear in logarithms). This choice of money demand function simplifies the calculation of the growth rate of money demand considerably.

Asset markets are assumed to clear at all times. Equ. (22) describes this assumption for the money market providing the equation for the current market rate of interest to be used for the payments of interest in the next point in time in the case of
the money supply rule (19). Bonds and equity are assumed to be perfect substitutes (on the basis of expected or actual rates of return in \(t + 1\) which are formed in \(t\) based on information known at \(t\)) when the trade on the stock market is performed, see equation (23). This amounts to assuming, in the light of the assumed Walras's law of stocks and flows that the clearing of the money market implies that the bond and equity market are then cleared as well, with wealth holders accepting any allocation of their wealth with respect to bonds and equity.

The disequilibrium in the goods market is an important driving force for the dynamics of our economy. This is described by the following equations:

6. Disequilibrium in the goods market (adjustment mechanism):

\[
\begin{align*}
S_t &= S_{p_t} + S_{g_t} + S_{f_t} = p_t \Delta E_t / p_t + I_t = I_t + N_t - \bar{N}_{t-1} = I_t^\alpha, \quad (24) \\
Y_t^d &= C_t + I_t + \delta K_{t-1} + G_t, \quad (25) \\
N_t^d &= \beta_n Y_t^a, \quad I_t = nN_t^d + \beta_n(N_t^d - \bar{N}_{t-1}), \quad (26) \\
Y_t &= Y_t^e + I_t, \quad (27) \\
Y_{t+1}^e &= Y_t^e + nY_t^a + \beta_e(Y_t^d - Y_t^e), \quad (28) \\
N_t &= N_{t-1} + Y_t - Y_t^d = S_t - I_t. \quad (29)
\end{align*}
\]

It is easy to check, by means of the presented budget equations and savings relationships, that the consistency of new money and new bonds flow supply and demand implies the consistency of the flow supply and demand for equity. Equ. (24) of this disequilibrium block of the model describes simple identities that can be related with the ex post identity of total savings \(S_t\) and total investment \(I_t^\alpha\) for a closed economy. It is here added for accounting purposes solely. Equ. (25) then defines aggregate demand, \(Y_t^d\), which is assumed to be never constrained in the present model.

In equ. (26) desired inventories \(N_t^d\) are assumed to be a constant fraction of expected sales, \(Y_t^a\), and intended inventory investment, \(I_t\), is determined on this basis via the adjustment speed \(\beta_n\) multiplied by the current gap between intended and actual inventories, \((N_t^d - N_t)\). The latter is augmented by a growth term that integrates in the simplest way the fact that this inventory adjustment rule is operating in a growing economy. Output of firms, \(Y_t\), in eq. (27) is the sum of expected sales and planned inventory adjustments. Sales expectations are here formed in a purely adaptive way, again augmented by a simple term that accounts for growth; see equation (28). Finally, in eq. (29), actual inventory changes are given by the discrepancy between actual output, \(Y_t\), and actual sales, \(Y_t^d\), equal to the difference between total savings, \(S_t\), and fixed business investment, \(I_t\).

We now turn to the last module of our model which is the wage-price module. It decomposes the Phillips curve into two dynamic equations augmented by a law of motion for expectations formation.

7. Wage-Price-Sector (adjustment equations):
\[ \hat{w}_{t+1} = \beta_w (V_t - \bar{V}) + \kappa_w \hat{p}_{t+1} + (1 - \kappa_w) \pi_t, \]  
\[ \hat{p}_{t+1} = \beta_p (U_t - \bar{U}) + \kappa_p \hat{w}_{t+1} + (1 - \kappa_p) \pi_t, \]  
\[ \pi_{t+1} = \pi_t + \beta (\alpha \hat{p}_{t+1} + (1 - \alpha) (\hat{p}_t - \pi_t). \]  

Our above description is based on fairly symmetric assumptions on the causes of wage- and price-inflation. Wage inflation for \([t, t + 1]\) according to eq. (30) is driven, on the one hand, by a demand pull component, given by the deviation of the actual rate of employment, \(V_t\), from the NAIRU-rate, \(\bar{V}\). On the other hand, it is driven by a cost push term measured by a weighted average of the short-run future rate of price inflation, \(\hat{p}_{t+1}\) (representing myopic perfect foresight) and expected rate of inflation, \(\pi_t\), which we interpret as a term of the medium run. Similarly, in equ. (31), price inflation is driven by the demand pull term, \((U_t - \bar{U})\), \(\bar{U}\) the NAIRU rate of capacity utilization, and the weighted average of the short-run future rate of wage inflation \(\hat{w}_{t+1}\), representing myopic perfect foresight, and again the medium-run expected rate of inflation \(\pi_t\). The expected rate of inflation is in turn determined by assuming that it follows a weighted average of short-run (perfect) and medium-run oriented (regressive) expectations, leading to an inflationary expectations mechanism as in (32). Depending on the monetary policy rule that is in action we determine the value of \(\hat{p}_t^*\) used in this expectations generating mechanism either by

- \(\mu - \hat{Y}^p\) (in the case of the money supply rule) or
- \(\hat{\pi}\) (in the case of the interest rate rule).

In the first case, the money supply rule (19), one assumes in addition that the monetary authority (and the public) forecast that the rate of inflation will converge in the medium run to the rate of inflation based on the so-called \(p^*\) concept: \(p^* = \bar{y}M/Y^p\), an estimate of future prices based on the quantity theory of money and an estimate of potential output, which in our model is given by: \(Y^p = \gamma^pK\), \((\mu = \bar{M}/M)\). We have assumed above in addition that the growth rate of potential output is further simplified by replacing its actual rate by the trend rate it exhibits which is given by \(n\), the natural rate of growth. This leads to the estimate \(\mu_0 - n\) for the growth rate of \(p^*\) and it influences the expectations on medium run inflation (which are revised in each quarter according to the formula given above) with a weight \(\alpha\).

---

16A determination of the price dynamics by cost-push and demand components can also be found in Fair (1997); although in Fair wages are not driven by a demand component.

17Note that our formulation of the wage-price dynamics in module 7 would, of course, not result in a linear Phillips-curve (regardless of what is measured on the vertical axis; the inflation rate net of the expected inflation rate or the rate of change of the inflation rate); for a discussion of linear and nonlinear Phillips-curves and the implied NAIRU hypothesis, see Laxton, Rose and Tambakis (1997).

18This is a concept that the German Bundesbank, according to its statements, follows. At some point, it has also been viewed by the Fed, according to Hall, Porter and Small (1991), as a concept to target inflation.
In the case of the application of the Taylor rule by the monetary authority we, of course, should use \( \bar{\pi} \) in the place of the growth rate of \( p^* = \bar{\theta} M/Y \) as an estimate for the rate of inflation that holds over the medium run. In the case of the Taylor rule the growth rate of money supply is endogenous.

It is easy to extend this mechanism in a way such that price forecasting rules, as for example used by the German Bundesbank, according to their so-called \( p^* \)-concept, are explicitly introduced into the medium-run component of this expectations mechanism (in the place of the steady state value \( \mu_0 = \pi \)).\(^{19}\)

It is obvious from this description of the model that it is, on the one hand, already a very general description of macroeconomic disequilibrium dynamics. On the other hand, it is still dependent on some simplifications with respect to financial markets and the fiscal policy rules. This can be justified at the present stage by observing that many of its simplifying assumption are indeed typical for macrodynamic models which attempt to provide a complete description of a closed economy, see in particular the model of Keynesian dynamics of Sargent (1987, Part I). Also, those in incomplete specifications may not prevent us from successfully calibrating the model.

On the other hand, one can considerably extended such a conventional model of a three sector - five markets approach to monetary macroeconomics by extending the sector of firms (module 3), the disequilibrium adjustment process of the quantities produced (module 6) and the wage-price sector and the determination of inflationary expectations (module 7). The model here provides a demand determined description of the macroeconomy with a particular emphasis on the behavior of firms and sluggish price as well as quantity adjustments. In appendix 3 we will provide some extensions of the model which add further realism to the model and which also put the treatment of the medium-run and expectations on a better footing. Other extensions of this framework must here remain for future research.\(^{20}\)

Lastly we want to remark that we have assumed in the investment function (10) as expression for the expected rate of inflation a medium-run \( \bar{\pi} \), defined in (32), but we use this rate there in conjunction with the short-run rate of interest and the RRC. This asymmetric treatment of time horizons in investment behavior can be overcome in two ways: Either by using the (perfectly expected) short-run rate of inflation, \( \hat{\pi}_{t+1} \), in place of \( \pi_t \) to define the real rate of interest used for investment or to refer to medium-run concepts of interest and RRC in the \( i_1 \)-term of the investment function as well, e.g. in the following simple way:

\[
\begin{align*}
I_t &= i_1(r^m - (r^m_t - \pi_t))K_{t-1} + i_2(U_t - \bar{U})K_{t-1} \\
\rho^m_{t+1} &= \rho^m_t + \beta(r_{t+1} - \rho^m_t) \\
r^m_{t+1} &= r^m_t + \beta_\pi(r_{t+1} - r^m_t)
\end{align*}
\]

\(^{19}\)Note that a similar concept has, at some point, also been suggested for the Fed, see Hallman, Porter, and Small (1991).

\(^{20}\)See for example, Franke and Semmler (1997) for the formulation of an advanced financial sector in a macromodel. Furthermore, see Chiarella and Flaschel (1998), ch.8.
Here it may also be appropriate to relabel the variable \( \pi_t \), by \( \pi_t^{\text{m}} \), to clearly show where we use concepts that refer to a medium run horizon. Note that such an extension of the model introduces further lags into the model that reflect the adjustment of expectations with respect to a medium-run horizon; the details of this reformulation of the model are presented in appendix 3 of this paper, see the modules 3. and 7. of the extended model presented there.

3 Alternative Monetary Policy Rules and the Dynamics of the Private Sector

In this section we first study the money supply rule (19) of the monetary authority where the nominal rate of interest is only indirectly set through the decision on the change in money supply by the central bank. After that we explore the dynamics of the macromodel in case the monetary authority follows the Taylor rule. For both versions the dynamics of the model has to be written and studied in intensive form.

For the derivation of the intensive form of the wage-price sector, module 7, we use formulations and procedures from Rose (1990). Indeed, the formal structure of our wage and price adjustment equations is the similar to Rose (1990) although our interpretation of it differs from the one by Rose. These equations can be viewed as a considerable generalization of many other formulations of wage-price inflation, for example, of models which only employ cost push forces in the market for goods.

The wage and price equations of module 7 can be reformulated as two linear equations in the unknowns \( \hat{\omega}_{t+1} - \pi_t, \hat{p}_{t+1} - \pi_t \) which are easily solved giving rise to the following expressions for the two unknowns:

\[
\begin{align*}
\hat{\omega}_{t+1} - \pi_t &= \kappa [\beta_w (V_t - \bar{V}) + \kappa_w \beta_p (U_t - \bar{U})], \\
\hat{p}_{t+1} - \pi_t &= \kappa [\kappa_p \beta_w (V_t - \bar{V}) + \beta_p (U_t - \bar{U})].
\end{align*}
\]

These equations in turn imply for the dynamics of the real wage \( \omega_t = w_t/p_t \) the law of motion:

\[
\hat{\omega}_{t+1} = \hat{\omega}_{t+1} - \hat{p}_{t+1} = \kappa [(1 - \kappa_p) \beta_w (V_t - \bar{V}) - (1 - \kappa_w) \beta_p (U_t - \bar{U})].
\]

This statement, however, is only true when one neglects the second order term \( \omega_{t+1} p_{t+1} \) in the formula that relates the nominal rates of wage and price inflation with the growth rate of the real wage. Such second order terms are repeatedly neglected in all following calculations of the intensive form of the model. The above law (35) provides the first dynamical equation of this intensive form. Note also, that the formula for \( \hat{p}_{t+1} - \pi_t \) is repeatedly used in the following laws of motion of the intensive form of the model.

\[21\] See also the early work of Rose (1967) and Solow and Stiglitz (1986) for similar presentations which, however, still neglect the influence of inflationary expectations on wage-price determination.
Neglecting the second order terms in the calculation of rates of change we get from the model of the preceding section the following autonomous six-dimensional dynamic system in the variables $\omega_t = w_t/p_t, l_t = L_t/K_{t-1}, m_t = M_t/(p_tK_{t-1}), \pi_t, y^d_t = Y^d_t/K_{t-1}$ and $\nu_t = N_{t-1}/K_{t-1}$,\footnote{We have to assume here $\kappa_w, \kappa_p \neq 1$ and use as abbreviations $\kappa = (1 - \kappa_w \kappa_p)^{-1}, \beta_{\pi} = \beta_{\pi} \kappa, \beta_{\pi_2} = \beta_{\pi} (1 - \alpha)$. Note that there are two further differential equations in this model (for $B_t/K_{t-1}, E_t/K_{t-1}$) which however do not feed back into the dynamics (36) – (41).} which describe the laws of motion of the private sector.

\begin{align*}
\omega_{t+1} &= \omega_t + \omega_t \kappa [(1 - \kappa_p) \beta_p (V_t - \bar{V}) + (\kappa_w - 1) \beta_p (U_t - \bar{U})], \quad (36) \\
l_{t+1} &= l_t + l_t (-\delta (\rho^t - \tau_t + \pi_t) - i_2 (U_t - \bar{U})), \quad (37) \\
m_{t+1} &= m_t + m_t (\mu - \pi_t - n - \kappa [\beta_p (U_t - \bar{U}) + \kappa_p \beta_w (V_t - \bar{V})] - (i_1 (\rho^t - \tau_t + \pi_t) + i_2 (U_t - \bar{U}))), \quad (38) \\
\pi_{t+1} &= \pi_t + \beta_{\pi_1} [\beta_p (U_t - \bar{U}) + \kappa_p \beta_w (V_t - \bar{V})] + \beta_{\pi_2} (\mu - n - \pi_t), \quad (39) \\
y^d_{t+1} &= y^d_t + \beta_y (y^d_t - y^p_t) - (i_1 (\rho^t - \tau_t + \pi_t) + i_2 (U_t - \bar{U})) y^p_t, \quad (40) \\
\nu_{t+1} &= \nu_t + \nu_t - \delta (\rho^t - \tau_t + \pi_t) + i_2 (U_t - \bar{U}) + (n + \delta + g). \quad (41)
\end{align*}

For output per capital $y_t = Y_t/K_{t-1}$ and aggregate demand per capital $y^d_t = Y^d_t/K_{t-1}$ we have the following expressions:

\begin{align*}
y_t &= (1 + n \beta_{\pi_2}) y^d_t + \beta_n (\beta_{\pi_2} y^p_t - \nu_t), \quad (42) \\
y^d_t &= \omega_t y_t / \tau + (1 - s_c) (\rho_t - t^n) + i_1 (\rho_t - \tau_t + \pi_t) + i_2 (U_t - \bar{U}) + (n + \delta + g) \\
&= y^d_t + (i_1 - s_c) \rho_t - i_1 (\tau_t - \pi_t) + i_2 U_t + \text{const.} \quad (43)
\end{align*}

Here we make use of the abbreviations:

\begin{align*}
V_t &= l_t^d/l_t, \quad U_t = y_t/y^p, \quad l_t^d = L_t^d/K_{t-1} = y_t/x \quad (y_t, l_t^d \text{ not const.}), \quad (44) \\
\rho_t &= y_t - \delta - \omega_t l_t^d = y_t - \delta - \omega_t y_t/x, \quad (45) \\
r_{t+1} &= r_0 + (h_1 y_t - m_t)/h_2, \quad (46) \\
t^n &= \text{const.} \quad (47) \\
\mu &= \mu_0 + \beta_m (\mu_0 - n - \pi_t) \quad (48)
\end{align*}

By neglecting second order terms the above intensive form of the model is identical to the Euler approximation (with step length 1) frequently used in the numerical simulation of continuous time systems in intensive form. This presentation is therefore justified both for continuous time modeling as in Chiarella and Flaschel (1998) as well as for discrete time modeling used in this paper.

The reformulation of the dynamic system (36) – (41) for step length $h$ is obtained by using in particular $x_{t+h} = x_t + x_t h$ in the place of $x_{t+1} = x_t + x_t$ for the growth rate expressions in the laws of motion for the real wage, $\omega_t$, labor intensity, $l_t$, and
real balances, \( m_t \), or more generally by multiplying all expressions on the right hand side of the dynamics (36)-(41) (after the first +) by the step length \( h \).

We also stress again that the model's structural equations have been chosen as simple (i.e. as linear) as possible in order to clearly separate its basic dynamic implications from the additional effects arising from nonlinear behavioral relationships. In this way we can show that there are always some 'natural' nonlinearities involved in the construction of such a model of the Keynes- Metzler disequilibrium macromodel. These nonlinearities arise from two sources:

- laws of motion that depend on growth rate formulations (either on the left or on the right side of the above difference equations)
- expressions that depend on products, or ratios, of state variables as they derive from, \( \omega L^d \), the sum of wages, and \( L^d/L \), the rate of employment in various places of the model

These two features of the dynamics of the private sector are responsible for the occurrence of the non degenerate Hopf-bifurcation observed in Chiarella and Flascher (1998, Ch.7).

Proposition 1
There is a unique steady-state solution or point of rest of the dynamics (36)-(41) fulfilling \( w_0, l_0, m_0 \neq 0 \) which is given by the following expressions:\(^{23}\)

\[
\begin{align*}
y_0 &= \bar{U} y^\rho, & l_0 &= l_0^\rho/ V = y_0/(x V), & y_0^\sigma = y_0^d = \frac{y_0}{1 + n_\rho n^\sigma}, \\
m_0 &= h_1 y_0, & \tau_0 &= \mu_0 - n, & \nu_0 &= \beta_n y_0^\sigma, \\
\rho_0^\sigma &= t^n + g - t^n + n, & \omega_0 &= \frac{y_0 - \delta - \rho_0^\sigma}{l_0^\rho}.
\end{align*}
\]  

We assume that the parameters of the model are chosen such that the steady state values for \( \omega, l, m, \rho^\sigma, r \) are all positive.

Proposition 2
The orbits of the dynamics (36)-(41) approximate those of the continuous time model of Chiarella and Flascher (1998, Ch. 7) under certain regularity conditions on their system of differential equations, if the step length \( h \) in formulating this system is chosen sufficiently small.

Proof: See Hairer et al. (1987, I.7).

The findings in Chiarella and Flascher (1998, Ch.7) on local asymptotic stability and Hopf bifurcation thus can be transferred to the above discrete time dynamics

\(^{23}\)Chiarella/Flascher (1998, Ch. 7) for a proof.
(36)-(41) provided that the step length $h$ in this Euler-approximation of the continuous time model is chosen sufficiently small.\textsuperscript{24} The system will therefore often give rise to convergent behavior back to the steady state (if the system has been shocked) if prices and expectations adjust sluggishly (and output adjustment is sufficiently fast) while it will lose this stability property in a cyclical fashion (by way of Hopf bifurcations) if the parameters that characterize wage, price and inflationary expectations adjustment are increased to a sufficient degree.

We want to note that there are two further laws of motion implied by the assumed structure of our Keynes-Metzler macromodel, one for bonds per value unit of capital, $b = \frac{B}{PK}$ and one for equities per capital, $e = \frac{E}{K}$ (implied by the Government budget constraint and the budget constraints of firms, respectively). These laws of motion read, in the case of a money supply policy rule and expressed in continuous time for simplicity:\textsuperscript{25}

\begin{align*}
\dot{b} &= g - r^n - [\mu_o + \beta_m(\mu_o - n)]m - (\dot{p} + \dot{K})b \\
\dot{e} &= \frac{r - \dot{p}}{\rho^e} \left[ I \frac{I}{K} + (y^e - y^d) \right] - \frac{I}{K}
\end{align*}

In the steady state these laws of motion imply:

\begin{align*}
b_o &= g - r^n - \mu_o m \\
e_o &= \frac{(r - \dot{p})}{\rho^e} = \left( \frac{p_k E}{pK} \right)_o = 1
\end{align*}

We stress that the case of the Taylor rule, where the interest rate is targeted, is much more difficult to treat, since the change in money supply is then endogenously determined by money demand (by the interest rate fixed by the monetary authorities). This means that this change and thus also the interest rate policy rule must be inserted into the Government budget constraint in order to determine correctly how much of government expenditure is financed by new money and how much (the remainder) by the issue of new government debt. We consequently get that the Taylor interest rate rule reappears in other places of the model and induces thereby a complicated feedback of interest rate policy on output and inflation and this again impacts the interest rate set by the monetary authority.\textsuperscript{26}

\textsuperscript{24}In the present type of model a step length of $h = 0.01$ is generally sufficient to remove any observable discrepancy between the Euler method of simulating systems of differential equations and much more advanced ones. Note also that reducing $h$ is equivalent to reducing the reactions to the disequilibria of the model, i.e., reducing the parameters that are there involved.

\textsuperscript{25}The evolution of Tobin's $q = \frac{p_k E}{pK}$ is by contrast simply given by: $q = \frac{\rho^e}{r - \dot{p}}$.

\textsuperscript{26}Note that in the current variant of our model there is no feedback from bonds to other macrovariables of the economy. Introducing such a feedback would again open a complicated channel through which the Taylor rule would affect macrovariables.
Employing the money supply rule, Figure A provides a brief numerical impression for situations of stable steady states or (at least locally) explosive ones and also shows the creation of a limit cycle for a certain set of parameter values.  

Figure A:  
Hopf Bifurcation-curves, stable limit cycles and stable corridors:  
Real, monetary and inventory cycles (2D-projections of the 6D dynamics).

\(^{27}\)We use \(bx\) in the place of \(\beta_x\) in this figure.
Figure A shows on its left hand side the Hopf–bifurcation loci for the three parameter sets \((\beta_p, \beta_w), (\beta_p, \beta_{n1})\) and \((\beta_{p*}, \beta_n)\), i.e. the loci where a supercritical (birth of limit cycles), subcritical (death of stable corridors) or degenerate Hopf–bifurcation occurs. The vertical lines separate sub– from super–critical Hopf–bifurcations.

Let us consider the \((\beta_w, \beta_p)\)–space as an example. For any given \(\beta_p\), increasing \(\beta_w\) from 0 to 1 means that the system will reach a point where it will lose its stability in a cyclical fashion (at \(\beta_w^H\)). At a supercritical Hopf bifurcation this will happen via the birth of an attracting limit cycle which 'surrounds' the now unstable steady state \((\beta_w > \beta_w^H)\). At a subcritical Hopf–bifurcation an unstable limit cycle (which exists for \(\beta_w < \beta_w^H, \beta_w\) sufficiently close to \(\beta_w^H\)) will disappear as \(\beta_w\) approaches \(\beta_w^H\), where the corridor of local asymptotic stability that existed before has vanished. At a degenerate bifurcation, the same loss of stability need not be accompanied by either the 'birth' of a stable limit cycle (above \(\beta_w^H\)) or the 'death' of an unstable limit cycle (below \(\beta_w^H\)), but purely implosive behavior may here simply be changed into a purely explosive one. These various types of Hopf–bifurcations are treated and depicted in detail, for example, in Wiggins (1990, Ch.3). The \((\beta_p, \beta_w)\)–diagram in figure A thus basically shows that except at very small parameter values of \(\beta_p\) the birth of a stable limit cycle occurs as \(\beta_w\) crosses the depicted Hopf locus.

The same happens in the next bifurcation diagram for \(\beta_{n1}\), the adjustment speed of inflationary expectations in the place of \(\beta_w\), the adjustment speed of wages. This figure in addition shows that a choice of the parameter \(\beta_p\), the adjustment speed of prices, sufficiently small will make the 6D–dynamics locally unstable. The two plots considered would suggest that flexible wages and inflationary expectations and very sluggish prices work against local asymptotic stability.

The last Hopf–bifurcation diagram is for the two adjustment speeds of the Metzlerian inventory mechanism of our 6D–dynamical system, i.e., \(\beta_{n*}\) and \(\beta_n\), the speed of adjustment of sales expectations and of planned inventory adjustments towards desired inventory stocks. It shows that there exists now a band of stable steady states, limited by a region of unstable steady states for low values of \(\beta_{n*}\) and \(\beta_n\) as well as for high values of these parameters. Moreover, loss of stability via increased \(\beta_n\) is always 'subcritical', while loss of stability via a decreased \(\beta_n\) may be sub– or supercritical (as shown in the diagram).

Hopf–bifurcations have generally been considered for 2D– and 3D–systems in the economics literature. Surely, a 6D–system like ours is much more demanding with respect to an analysis of the complete set of the Routh–Hurwitz stability conditions and at present seems out of reach for this system. Yet, even for 2D–systems (and even much more for 3D–systems) it is generally a horrendous algebraic task to investigate analytically whether the Hopf–bifurcation is sub– or supercritical and a proof of this is therefore generally missing in the economics literature (an exception to this rule is Lux (1993)). Numerical methods therefore have to be used in all such cases in order to decide on an important aspect of the Hopf–theorem, the existence of stable or unstable limit cycles.

18
On the right-hand side of the above figure we in addition show an example of a stable limit cycle, generated via a supercritical Hopf-bifurcation. Note that this stable limit cycle is generated by the natural nonlinearities of our Keynes-Metzler monetary macro model solely and not by assumptions on nonlinearities in the behavioral relationships of this model. The parameters for these phase plots of the 6D-dynamics are the following:

\[
\begin{align*}
\beta_p &= 1, \beta_w = .21, \beta_{\pi_1} = .22, \beta_{\pi_2} = .5, \beta_n = .2, \beta_{\nu^*} = .75, \\
\kappa_w &= \kappa_p = .5, n = \mu_o = .05, i_1 = .25, i_2 = .5, s_c = .8, \\
h_1 &= .1, h_2 = .2, \beta_{n^*} = .3, g = \tau^r = .3, \delta = .1, y^p = 1, x = 2 \quad [r_0 = \rho_0 = 0.36875].
\end{align*}
\]

This parameter set gives a point just above the Hopf locus in \((\beta_p, \beta_w)\)-space and \((\beta_p, \beta_{\pi_1})\)-space and just below the lower Hopf-curve in \((\beta_{\nu^*}, \beta_n)\)-space (point A in the above figures). On this basis, the above figures to the right show how the limit cycle is approached when the steady state of the model is disturbed via a small \(l\)-shock. We here show the \((\omega, l)^-, (m, \pi)^-, (y^*, \nu)^-\)-projection of this limit cycle.

Let us consider now the alternative case of a Taylor type monetary policy rule, where short-run interest rates are directly targeted by the monetary authority. In this case we have to remove the money supply equation \((19)\) (and eq. \((46)\)) and thus the law of motion \((38)\) from the dynamics of the private sector and to employ in its place the following law of motion:\(^{28}\)

\[
r_{t+1} = r_0 + \beta_1(r_t - r_0) + \beta_2(\tilde{h}_{t+1} - \tilde{\pi}) + \beta_3(U_t - \bar{U}).
\]

\((52)\)

In this case we obtain in the place of proposition 1 the following proposition:

**Proposition 3.**

There is a unique steady-state solution or point of rest of the dynamics \((36)-(41)\) fulfilling \(w_0, l_0, m_0 \neq 0\) which is given by the following expressions:\(^{29}\)

\[
\begin{align*}
y_0 &= \bar{U} y^p, \quad l_0 = l_0^d / \bar{V} = y_0 / x / \bar{V}, \quad y_0^d = y_0 / (1 + n \beta_{n^*}), \\
\rho_0 &= \tau^r + g - \tau^r + n, \quad \pi_0 = \bar{\pi}, \quad r_0 = \rho_0^* + \bar{\pi}, \\
\nu_0 &= \beta_{n^*} y_0^d, \quad \omega_0 = y_0^d - \delta - \rho_0^* / l_0^d, \\
m_0 &= h_1 y_0.
\end{align*}
\]

\((53)-(55)\)

Note here again that \((51)\) replaces the two interacting equations

\[
\begin{align*}
m_{t+1} &= m_t + m_t \mu - \pi_t - n - \kappa[\beta_p(U_t - \bar{U}) + \kappa_p \beta_w(V_t - \bar{V})] \\
&\quad - (i_1(\rho_t^r - \pi_t) + i_2(U_t - \bar{U})), \\
r_{t+1} &= r_0 + (h_1 y_t - m_t) / h_2,
\end{align*}
\]

\(^{28}\)\(\hat{\pi}\) in the place of \(\beta_{\pi_1}\) and \(\beta_{\pi_2} \bar{U}(\bar{U} - 1)\) in the calibration of the model however.

\(^{29}\)See Chiarella and Flaschel (1998, Ch.7) for a proof.
of the model by the interest rate targeting rule. It thus considerably simplifies the structure of the dynamics. We expect from this simplification that it will increase the stability of the steady state of the model, but can at present only show this for submodels of the full dynamics here considered. In order to get more insight into this situation one should at first transform the present dynamics back into a continuous time version. This can be done as follows as far as the new law of motion of the nominal rate of interest is concerned:

$$\dot{r} = (\beta_{r_1} - 1)(r - r_o) + \beta_{r_2}(\hat{p} - \bar{p}) + \beta_{r_3}(U - \bar{U}).$$

(56)

This equation shows that there will be a negative effect of the nominal rate of interest onto itself if $\beta_{r_1} < 1$ holds which may also extend to the other two terms in this policy rule if nominal interest increases decrease economic activity and thus decrease the rate of capacity utilization and the rate of price inflation.

Finally, we want to note that it is at present not clear in what form proposition 2 will hold in the case of the Taylor rule, i.e., in the case where the monetary authority pursues an interest rate policy and then accommodates the money demand that is generated through the interest rate they have targeted.

4 Parameter Estimation and Calibration

We now turn to the empirical assessment of our Keynesian monetary macromodel. However, we shall first remark that the above model was designed purely for the convenience of theoretical and dynamic analysis. We have pointed out that from the empirical point of view there are still some strong assumption such as the fiscal policy rule, for example, our tax spending and deficit rules. Moreover, some feedback effects are not studied yet. On the other hand, some variables, such as those related to expectations, are not observable, although they are important in determining the dynamics of the model. Further, all the parameters are structural parameters in the sense that they have specific economic meaning and hence boundaries. All these problems cause difficulties for us to apply the standard econometric method for our empirical assessment. We thus adopt the calibration method as has been employed for RBC models, yet attempt to estimate the parameters instead of picking them from different sources. We did not intend to estimate all the parameters of the dynamic model simultaneously. In our procedure some parameters are obtained from the simultaneous estimation of subsystems using GMM, other parameters are obtained from single equations.

Our main objective here is, to compare, for some crucial variables, the time series moments generated from the model's stochastic simulation to those of the actual economy. The stochastic innovation of our system is assumed to be generated from the following three shocks: the money supply shock (only applied to the standard version), the government spending shock and the labor supply shock. We assume that these three shocks, all normalized by the capital stock, follow an AR(1) process:
\[ m_{t+1} = m_0 + m_1 m_t + \epsilon_{t+1}, \]  
(57)

\[ g_{t+1} = g_0 + g_1 g_t + \nu_{t+1}, \]  
(58)

\[ l_{t+1} = l_0 + l_1 l_t + \eta_{t+1}. \]  
(59)

where \( m_t, g_t \) and \( l_t \) are respectively the normalized money, government expenditure and labor supply. The parameters from the above shocks are estimated from the data.\(^{30}\)

Unlike the typical study of RBC model, where the parameters (which are anyway not so many) are selected from different sources, we do not have enough information to appropriately define our parameters. We, therefore, use actual macro economic time series data to estimate the parameters along the above line of reasoning. Table 1 provides the parameters that we have used for our stochastic simulation. In appendix 2, we provide a detailed description of how these parameters are obtained.

| Set 1 | price-wage dynamics | \( \kappa_p = 0.0416, \ k_w = 0.9998, \ \beta_p = 5.5e - 7, \)  
\[ \beta_w = 6.5e - 10, \ \beta_r = 0.0027, \ \alpha = 0.5533. \] |
| Set 2 | sales expectations | \( \beta_n = 1.5321, \ \beta_{n^d} = 0.1589, \ \beta_{y^*} = 0.2372. \) |
| Set 3 | consumption function | \( s_c = 0.1842. \) |
| Set 4 | investment function | \( i_1 = 0.0463, \ i_2 = 0.0286 \) |
| Set 5 | money demand function (applied to standard version) | \( h_1 = 0.1849, \ h_2 = 0.6056 \) |
| Set 6 | interest reaction function (applied to extended version) | \( \beta_{r_1} = 0.9446, \ \beta_{r_2} = 0.3118, \ \beta_{r_3} = 0.0465 \) |
| Set 7 | steady state | \( r_0 = 0.0840, \ \rho_0 = 0.0745, \ g = 0.0657. \) |
| Set 8 | shock equations | \( m_0 = 0.0003, \ m_1 = 0.9996, \ \sigma_\epsilon = 0.0007, \)  
\( l_0 = 0.0389, \ l_1 = 0.9928, \ \sigma_\eta = 0.0294, \)  
\( g_0 = 0.0006, \ g_1 = 0.9907, \ \sigma_\nu = 0.0014. \) |
| Set 9 | other parameters | \( \gamma^p = 0.5045, \ \alpha = 0.0437, \ \delta = 0.0478 \)  
\( n = 0.0047, \ \mu_\epsilon = 0.0142, \ t^n = 0.0695. \) |

It is interesting to remark that \( \beta_p \) and \( \beta_w \) are very small.\(^{31}\) The set of parameters for the wage price dynamics, module 7, are estimated by the method of GMM.

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\(^{30}\)All data are taken from Citibase (1997).

\(^{31}\)We want to note here that the low estimated coefficient \( \beta_w \), for example, may have been impacted by a possible collinearity between \( U_t \) and \( V_t \), see appendix 2.
estimation (see appendix 2). Given our results we can expected that the standard
demand-supply forces in determining prices and wages do not appear to be empirically
significant, at least according to U.S. time series data. The parameter estimates
appear to support a markup theory of pricing.

Employing our parameter estimates the actual and predicted macroeconomic time
series are reported in Figures 1-7. The Figures are collected in appendix 4. Note that
in this exercise the fitted line is obtained not by simulation of the system equations but
by a partial prediction of the respective macroeconomic variables using the estimated
parameters.

Figure 1-7 about here

From the Figures 1-7 one can observe that most macroeconomic variables are well
predicted by this partial exercise. The fit is, however, less successful for the growth
rate of the money wage and investment. It is even less successful for the interest
rate. In the latter case the standard monetary rule has been applied, namely the
assumption that the monetary authority targets monetary aggregates. Thus, the
interest rate is not well predicted using the money demand function resulting from
the standard monetary policy rule of module 7. We also want to note that when
the model variants with the estimated parameters as reported in table 1 for the two
policy rules are simulated deterministically the models are asymptotically stable.

Given the parameters as reported in table 1, we also perform 1000 times stochastic
simulations of the mode variants. Each simulated series is detrended by the HP-filter.
The percentage deviation from the trend can then be calculated. Table 2 reports
some second moments of those percentage deviations for some crucial variables in
comparison to the sample economy. Moreover, we report the cross-correlation of the
variables with output.
Table 2: The percent deviation from the trend: comparison between the sample economy and the simulated economy.

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Cross-correlation with output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sample economy</td>
<td>simulated economy (standard version)</td>
</tr>
<tr>
<td>Y</td>
<td>0.0176</td>
<td>0.0136</td>
</tr>
<tr>
<td>C</td>
<td>0.0139</td>
<td>0.0118</td>
</tr>
<tr>
<td>I</td>
<td>0.0543</td>
<td>0.1502</td>
</tr>
<tr>
<td>K</td>
<td>0.0028</td>
<td>0.0016</td>
</tr>
<tr>
<td>L⁴</td>
<td>0.0101</td>
<td>0.0136</td>
</tr>
<tr>
<td>ρ̂</td>
<td>0.0083</td>
<td>0.0006</td>
</tr>
<tr>
<td>ɵ̂</td>
<td>0.0088</td>
<td>0.0006</td>
</tr>
<tr>
<td>r</td>
<td>0.1623</td>
<td>0.0947</td>
</tr>
</tbody>
</table>

For the convenience of our analysis, we may classify these variables into three groups in terms of their volatility implied by the sample economy. The first group includes investment and interest rate. These are the variables with highest volatility. The second group includes output, consumption and labor. They are in the middle. The third group includes price, wage and the capital stock. These are the stable variables and have very low volatility. In terms of this classification, we find that the simulated economies for both monetary policy rules can match the sample economy quite well, although within the same group, some discrepancies might occur.

In the sample economy, labor is less volatile than output and consumption. In the simulated economies of both monetary policy rules, labor shows the same volatility as output and hence is more volatile than consumption. This is certainly due to our special labor demand function, which is linear in output. Also in the sample economy, the capital stock is less volatile than price and wage, but it is more volatile in the simulated economy. Indeed, the price and wage seem to be excessively stable in the simulated economies. This indicates that there might have been other forces that influence the price and wage determination. These forces may stem, for example, from the external sector which is neglected here.³²

In terms of the cross-correlation, we find that consumption, investment and labor are significantly correlated with output in both the sample and simulated economies. Prices seem to move counter-cyclically in the sample economy. This appears to be particularly due to the abnormal price increase and output fall during the oil-shock in the 1970s, see also Fair (1997). This counter-cyclical relation is not matched in the

³²Note that frequently the price of imports is employed in the price determination, see, for example, Fair (1997), which may give better fit of the sample and simulated economies.
sample economies of both versions. For the capital stock, wage and interest rate,\textsuperscript{33} we do not find the significant correlation with output both in the sample and simulated economies.

For the comparison of our two versions, in terms of volatility, we find that the version that employs the Taylor rule does match the sample economy better than the version that uses the money supply rule. In the standard version, the interest rate is less volatility than investment, which is in contrast to the sample economy. However, in the version with the Taylor rule, this order is reversed and thus correctly matches the facts. However, in terms of cross-correlation comparison, the variant with the Taylor rule does not seem to perform better. We thus can say, from the empirical point of view the model variant with the Taylor rule matches better some aspect of the data whereas the variant with the money supply rule fits better other aspects of the data.

5 Conclusions

In the paper we have chosen a Keynesian disequilibrium framework for studying monetary policy. Disequilibrium is allowed in the product and labor markets whereas the financial markets are always cleared. There are sluggish price and quantity adjustments and expectations are a combination of adaptive and forward looking ones. The main objective of the paper is to study, on the basis of such a model, the effects of recently discussed alternative monetary policy rules. These policy rules are (1) the money supply rule or (2) the interest rate targeted by the monetary authority. We demonstrate the implication of those policy rules for a monetary macromodel of Keynesian type, study the different dynamic properties that they may give rise to and calibrate the model employing US macroeconomic time series data from 1960.1-1995.1.

Based on the estimation of the parameters, obtained partly from subsystems using GMM estimation and partly from single equations, we study, using stochastic simulations, the volatility of the variables and their cross-correlation to output for the sample economy as well as for our two simulated economies which employ either the money supply or the Taylor rule. As we could show with respect to volatility of the macroeconomic variables the model with the Taylor rule seems to describe the data better whereas with respect to cross-correlation to output, the model with money supply rule seems to better match the properties of the historical data. We must

\textsuperscript{33}Note that we have calculated here only the simultaneous relation of macroeconomic variables with output; lagged relationships may be important as well, see Fuhrer and Moor (1995) who, for example, compute the lagged relation between the 3 months TB and output where lags in output, starting with two quarters, show a negative correlation between those two variables. Thus, our above results do not contradict the usual negative relation of TB and output. Note that also the above relation between inflation and output may be changed by using lags. With lags there might also be a more significant co-variation of output and real wage.
however, add that many features of our theoretical model could still be improved. The match of an extended version of our model with macroeconomic time series data is subject to future research.

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7 Appendices

7.1 Appendix 1: Notations

The model of this paper is based on the following basically standard macroeconomic notation:

A. Statically or dynamically endogenous variables:

\[ Y_t \] Output
\( Y_t^d \) 
Aggregate demand \( C_t + I_t + \delta K_{t-1} + G_t \)

\( Y_t^e \)
Expected aggregate demand (from \( t - 1 \) to \( t \))

\( N_t \)
Stock of inventories

\( N_t^d \)
Desired stock of inventories

\( I_t \)
Desired inventory investment

\( L_t \)
Level of employment

\( C_t \)
Consumption

\( I_t^p \)
Fixed business investment

\( I_t^p \)
Planned total investment \( I_t + I_t^d \)

\( I_t^a \)
Actual total investment = \( I_t + N_t^d - N_{t-1} \) total investment

\( r_t \)
Nominal rate of interest (from \( t - 1 \) to \( t \), price of bonds \( p_t = 1 \))

\( p_t \)
Price of Equities

\( S_{pt} \)
Private savings

\( S_{ft} \)
Savings of firms (= \( Y_{ft} \)), the income of firms

\( S_{gt} \)
Government savings

\( S_t = S_{pt} + S_{ft} + S_{gt} \)
Total savings

\( T_t \)
Real taxes

\( G_t \)
Government expenditure

\( \rho_t^e \)
Rate of profit (Expected rate of profit)

\( \rho_t^m \)
Medium-run Average of Rate of profit

\( r_t^m \)
Medium-run Average of Rate of Interest

\( V_t = L_t^d / L_t \)
Rate of employment

\( Y_t^p \)
Potential output

\( \Delta Y_t^e = Y_t^e - Y_t^d \)
Sales expectations error

\( U_t = Y_t / Y_t^p \)
Rate of capacity utilization

\( K_{t-1} \)
Capital stock used for production in \( t \)

\( w_t \)
Nominal wages.

\( p_t \)
Price level

\( \pi_t \)
Expected rate of inflation (medium-run) for \( [t - 1, t] \)

\( p_t^* \)
The \( p^* \) concept of the FED

\( M_t \)
Money supply (index d: demand)

\( L_t \)
Normal labor supply

\( B_t \)
Bonds (index d: demand)

\( E_t \)
Equities (index d: demand)

\( W_t \)
Real Wealth

\( \omega_t \)
Real wage \( (u_t = \omega_t / \pi \text{ the wage share}) \)

\( \nu_t = N_t / K_{t-1} \)
Inventory-capital ratio

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B. Parameters

\( \bar{V} = 1 \)  
1. NAIRU-type normal utilization rate concept (of labor)

\( \bar{U} = 1 \)  
2. NAIRU-type normal utilization rate concept (of capital)

\( \delta \)
Depreciation rate

\( \mu \)
Growth rate of the money supply

\( n \)
Natural growth rate

\( i_{1,2} > 0 \)
Investment parameters

\( k_{1,2} > 0 \)
Money demand parameters

\( \beta_w \geq 0 \)
Wage adjustment parameter

\( \beta_p \geq 0 \)
Price adjustment parameter

\( \beta_{\pi} \geq 0 \)
Inflationary expectations adjustment parameter
\( \alpha \in [0, 1] \) Weights for forward and backward looking expectations
\( \beta_{i} \geq 0 \) Parameters of Interest Rate Policy (the Taylor Rule, \( i = 1, 2, 3 \))
\( \pi \) Target Inflation Rate of the FED
\( \beta_{u} > 0 \) Desired Inventory output ratio
\( \beta_{p} > 0 \) Inventory adjustment parameter
\( \kappa_{w, p} \in [0, 1], \kappa_{w, \kappa p} \neq 1 \) Weights for short- and medium-run inflation
\( \kappa = (1 - \kappa_{w, \kappa p})^{-1} \)
\( y^{p} > 0 \) Potential output-capital ratio (\( \neq y \), the actual ratio)
\( x > 0 \) Output-labor ratio
\( t(t^{n} = t - rb) \) Taxes (net of interest) per capital
\( \tau_{w} \) Taxation Rate (Wage Income)
\( s_{c} \in [0, 1] \) Savings−ratio (out of profits and interest)
\( s_{w} \in [0, 1] \) Savings−ratio (out of wages, = 0 here)

C. Mathematical notation

\( \hat{x}_{t} = \frac{x_{t} - x_{t-1}}{x_{t-1}} \) Growth rate of \( x \) for the interval \([t-1, t]\)
\( \Delta \) Difference Operator
\( r_{o}, etc. \) Steady state values
\( y_{t} = Y_{t}/K_{t}, etc. \) Real variables in intensive form
\( m_{t} = M_{t}/(p_{t}K_{t-1}) \) Nominal variables in intensive form

7.2 Appendix 2: Parameter Selection and Estimation Method

7.2.1 The Selection and Estimation of the Parameters in Set (5)-(9) of table 1.

The parameters in Set 9 (as reported in Table 1) are all selected by matching the first moments of the data. The parameters in Set 8 are estimated by applying OLS regression for equations (56)-(58). The parameters in Set 7 are selected according to the steady state relations as expressed in proposition 1, after the parameters in Set 9 have been selected. Specifically, \( \rho_{o}^{*} \) is calculated from (49) for the given \( r_{o}, \mu_{o} \) and \( n \), and then \( g \) is derived from (50).

The parameters in Set (6) are estimated by applying OLS regression to the following equation:

\[ \check{r}_{t+1} = \beta_{r_{1}} (r_{t} - r_{o}) + \beta_{r_{2}} (\bar{\pi}_{t} - \bar{\pi}) + \beta_{r_{3}} (U_{t}/\bar{U} - 1) \quad (A1) \]

where \( \check{r}_{t+1} \equiv r_{t+1} - r_{o} \); \( r_{o} \) is given in Set 9 and \( \bar{\pi} = \mu_{o} - n \). To estimate the parameters in Set (5), we first rewrite (22) as

\[ \check{r}_{t+1} = \left( \frac{h_{1}}{h_{2}} \right) \left( \frac{Y_{t}}{K_{t-1}} \right) - \left( \frac{1}{h_{2}} \right) \left( \frac{M_{t}}{p_{t}K_{t-1}} \right) \quad (A2) \]

We then regress (A1), which gives us two parameters, say \( b_{1} \) and \( b_{2} \). Then, \( h_{1} \) and \( h_{2} \) can be uniquely derived by setting \( b_{1} = h_{1}/h_{2} \) and \( b_{2} = -1/h_{2} \).\(^{34}\)

\(^{34}\)Note that this indicates that we try to select the parameters in such a way that the variation of interest rate (given \( r_{o} \)), rather than the money demand, can be matched as closely as possible.
7.2.2 The GMM Estimation of the Parameter Set Related to Price-Wage Dynamics

The rest sets of parameters is more complicated to be estimated, for their estimations needs, either directly or indirectly, the expectation variables that are not observable. We thus have to reduce the relevant systems into those only including the observable variables.

The parameter set with respect to price-wage dynamics appear in equation (30)-(32) in the model, and are further reduced into (33) and (34). Note, that $\pi_t$ is the unobservable expectation variable and thus we shall first substitute it out from our estimation equations. For this, we use equations (32)-(34). For notional convenience, we re-write them as follows:

$$\hat{\omega}_{t+1} = w_1(V_t - 1) + w_2(U_t - 1) + \pi_t \quad (A3)$$

$$\hat{p}_{t+1} = p_1(V_t - 1) + p_2(U_t - 1) + \pi_t \quad (A4)$$

$$\pi_{t+1} = \pi_1\pi_t + \pi_2\hat{p}_{t+1} + \pi_3 \quad (A5)$$

where $w_1 = \kappa\beta_w$, $w_2 = \kappa\kappa_w\beta_p$, $p_1 = \kappa\kappa_p\beta_w$, $p_2 = \kappa\beta_p$, $\pi_1 = (1 - \beta_t)$, $\pi_2 = \beta_t\alpha$ and $w_1 = \kappa\beta_w$.

Using (A4) (or A3, which, anyway, gives the same result) to express $\pi_t$ in (A5):

$$\pi_{t+1} = \pi_4\hat{p}_{t+1} + \pi_5(V_t - 1) + \pi_6(U_t - 1) + \pi_3 \quad (A6)$$

where $\pi_4 = \pi_1 + \pi_2$, $\pi_5 = -\pi_1p_1$ and $\pi_6 = -\pi_1p_2$. Next we use (A6) to express $\pi_t$ in (A3) and (A4). This gives us the following two equations that can be used for our estimation:

$$\hat{\omega}_{t+1} = w_1(V_t - 1) + w_2(U_t - 1) + \pi_4\hat{p}_t + \pi_5(V_{t-1} - 1) + \pi_6(U_{t-1} - 1) + \pi_3 \quad (A7)$$

$$\hat{p}_{t+1} = p_1(V_t - 1) + p_2(U_t - 1) + \pi_4\hat{p}_t + \pi_5(V_{t-1} - 1) + \pi_6(U_{t-1} - 1) + \pi_3 \quad (A8)$$

Note that all these $\pi_t$, $w_t$ and $p_t$ are complicated functions of our original six parameters. The method used here is the GMM estimation, which can handle the complicated nonlinear system. Since the objective function of our GMM estimation is nonlinear, local optimum might occur. We thus use a global optimization method, called simulated annealing, that searches parameter space, in our case in six dimensions, iteratively so that the distance function as defined in the GMM estimation is minimized.
7.2.3 The Estimation of the Parameter Set Related to Sales Expectations

This set of parameters appear in equations (26) - (29). It can be demonstrated that equation (26) - (29) can be reduced to the following single equation:

\[ Y_t = (1 + n - \beta_{v^*})Y_{t-1} + (n - \beta_{v^*})\beta_n N_{t-1} + (1 + (n\beta_n)\beta_{v^*})\beta_{v^*} Y_{t-1}^d \]  \hspace{1cm} (A9)

The usual OLS regression on (A9) generates three parameters, say \( a_1, a_2, \) and \( a_3, \) with the implication given by

\[ a_1 = 1 + n - \beta_{v^*} \]  \hspace{1cm} (A10)

\[ a_2 = (n - \beta_{v^*})\beta_n \]  \hspace{1cm} (A11)

\[ a_3 = (1 + (n + \beta_n)\beta_{v^*})\beta_{v^*} \]  \hspace{1cm} (A12)

Therefore, \( \beta_n, \beta_{v^*} \) and \( \beta_{v^*} \) can be uniquely determined from these three equations. Specifically,

\[ \beta_{v^*} = 1 + n - a_1 \]  \hspace{1cm} (A13)

\[ \beta_n = a_2/(n - \beta_{v^*}) \]  \hspace{1cm} (A14)

\[ \beta_{v^*}^d = \frac{a_3/\beta_{v^*} - 1}{n + \beta_n} \]  \hspace{1cm} (A15)

7.2.4 The Estimation of Consumption and Investment Functions

Once the above two sets of parameters are obtained, we then can use them to generate the time series of the expectations variables, \( \psi_t^e \) and \( \pi_t, \) that are necessary for estimating the consumption and investment functions. According to (1), \( \psi_t^e \) depends on \( Y_t^e \) where \( Y_t^e \) is generated from (28) given the parameters \( n, \beta_{v^*}, \) and \( Y_0^e \) the initial condition of \( Y_t^e. \) We assume that \( Y_0^e \) equals \( Y_0^d. \) The time series of \( \pi_t \) is generated simply from (A6) without using any initial condition. Once these two time series are obtained, we apply OLS regression to obtain the parameters \( s_0, i_1 \) and \( i_2. \) We remark that the dependent variable of the regression equation for consumption function is \( C_t = \omega L_t^d; \) and for the investment function it is \( I_t = (n + \delta)K_{t-1}. \)
7.3 Appendix 3: Some Extensions of the Macromodel

7.3.1 Extensions of the Modules

The equations of this extended form of the Keynes-Metzler I/S–LM–growth model are:

1. Definitions (remunerations, wealth and growth):
   \[\omega_t = \frac{w_t}{p_t}, \quad \rho_t^e = \frac{(Y_t^e - \delta K_{t-1} - \omega_t L_t^d)}{K_{t-1}}\]
   \[\rho_t = \frac{(Y_t^d - \delta K_{t-1} - \omega_t L_t^d)}{K_{t-1}}\] (60)
   \[W_t = \frac{(M_{t-1} + B_{t-1} + p_{et} E_{t-1})}{p_t}, \quad p_b = 1.\] (61)
   \[\dot{z}_t = \frac{(z_t - z_{t-1})}{z_{t-1}} \quad \text{Growth rate of a variable } z_t\] (62)
   \[n = n_1 + n_2 \quad \text{Natural rate including Harrod-neutral technical change}\] (63)

This set of definitions is the same as before with the exception of the formulation of the natural rate of growth which now includes Harrod-neutral technical change (at a constant rate) to be defined later on and the definition of the actual rate of profit which is based on actual sales in the place of expected ones. The first extension does not change the model’s intensive form significantly since it only means that state variables must now use labor measured in efficiency units whenever necessary (which is just a renormalization of the original model).

Household behavior is described next by the following set of equations:

2. Households (workers and asset-holders):
   \[C_t = (1 - \tau_w)\omega_t L_t^d + (1 - s_c)[\rho_t^e K_{t-1} + \tau_t B_{t-1} / p_t - T_t^c]\] (64)
   \[S_{pt} = s_c[\rho_t^e K_{t-1} + \tau_t B_{t-1} / p_t - T_t^c]\] (65)
   \[\dot{L}_{t+1} = n = \text{const.}\] (66)
   \[W_t + S_{pt} = (M_t^d + B_t^d + p_{et} E_t^d) / p_t, \quad \text{see the asset market module below.}\] (67)

This module is subject to one change only, the addition of wage taxation – besides profit taxation – to the consumption functions of the considered types of households.

The behavior of the production sector of the economy is described by the following set of equations:

3. Firms (production-units and investors):
   \[Y_t^p = y^p K_{t-1}, \quad y^p = \text{const.}\] (68)
   \[U_t = Y_t / Y_t^p = y_t / y^p \quad (y_t = \frac{Y_t}{K_{t-1}})\] (69)
   \[L_t^d = Y_t / x_t, \quad x_t = x_0 \exp(n_2 t), x_0 = \text{const.}\] (70)
   \[V_t = L_t^d / L_t = Y_t / (x_t L_t)\] (71)
   \[I_t = I_1 (\rho_t^p - (r_t^m - n_t^m)) K_{t-1} + I_2 (U_t / U - 1) K_{t-1} + n K_{t-1},\] (72)
   \[\rho_t^m = \beta_t^p \rho_{t+1} + \beta_t^p \rho_t + \beta_t^m \rho_{t-1}, \quad \beta_t^p + \beta_t^m + \beta_t^{m-1} = 1\] (73)
   \[r_t^{m+1} = r_t^{m+1} + \beta_t^m r_t + \beta_t^{m-1} r_{t-1}, \quad \beta_t^m + \beta_t^m + \beta_t^{m-1} = 1\] (74)
   \[S_{ft} = Y_{ft} = Y_t - Y_t^e = T_t,\] (75)
   \[\Delta Y_t^e = Y_t^e - \delta K_{t-1} - C_t - I_t - G_t = Y_t^e - Y_t^d,\] (76)
\[
\frac{\Delta E_t}{p_t} = I_t + \Delta Y_t^e = I_t + (N_t - N_{t-1} - I_t), \tag{77}
\]
\[
I_t^a = I_t + N_t - N_{t-1} - I_t = I_t + \Delta Y_t^e + I_t = p_t \Delta E_t/p_t + I_t, \tag{78}
\]
\[
\dot{K}_t = \frac{(K_t - K_{t-1})}{K_{t-1}} = I_t/K_{t-1}. \tag{79}
\]

In the sector of firms we have assumed that labor productivity \( x \) is now increasing at a constant rate \( n_2 \). According to eqs \((69),(71)\), firms produce commodities in amount \( Y_t \) in the technologically simplest way possible, via a fixed proportion technology characterized by the potential output/capital ratio \( Y_t^p/K_{t-1} \) and a fixed ratio \( x_0 \) between actual output \( Y_t \) and labor \( L_t^{da} = L_t^a \exp(n_2 t) \) – measured in efficiency units – needed to produce this output. This simple concept of technology allows for a straightforward definition of the rate of utilization \( U_t, V_t \) of capital as well as labor. Furthermore, in view of the remark at the end of section 2 we have now assumed a uniform time horizon for the profitability expressions in the investment function by assuming that they all relate to the medium run. Medium run or average estimates of expected profitability and the nominal rate of interest are described in a very basic way as moving averages of corresponding rates of return by equations \((73)\), \((74)\) while the law of motion of the expected medium run rate of price inflation is provided in the wage-price module below.

We now turn to a brief description of the government sector:

4. **Government (fiscal and monetary authority):**

\[
t^{-e} = \frac{T_c - r_t B_{t-1}}{K_{t-1}} = \text{const.}, \tag{80}
\]
\[
g = \text{const.}, \tag{81}
\]
\[
\frac{G_t}{K_{t-1}} = \dot{g} = \text{const.}, \tag{82}
\]
\[
S_{gt} = T_t - r_t B_{t-1}/p_t - G_t, \quad T_t = \tau_o \omega L_t^d + T_c^e, \tag{83}
\]
\[
r_t+1 = r_o + \beta_1 (r_t - r_o) + \beta_2 (\pi_t^{wu} - \bar{\pi}) + \beta_3 (U_t/U - 1) \tag{84}
\]
\[
M_t = h_1 p_t Y_t \exp(h_2 (r_o - r_{t+1}))) \tag{85}
\]
\[
\mu_t = (M_t - M_{t-1})/M_{t-1} \tag{86}
\]
\[
B_t = B_{t-1} + p_t G_t + r_t B_{t-1} - p_t T_t - (M_t - M_{t-1}) \tag{87}
\]

We have now wage taxation in the model (with a constant tax rate) and continue to assume that profit and interest taxes net of interest payments of the government preserve a fixed ratio to the capital stock. This assumption is made for mathematical convenience and basically says that the feedback from this magnitude to the rest of the model is not considered as important for the moment. Furthermore money supply (and thus its rate of growth) is now assumed as endogenous and determined by money demand function (see the asset market block) in the face of the interest rate \( r_{t+1} \) set by the monetary authority in the new interest rate feedback rule we now use in this module of the model. Finally, the endogenous supply of additional money (exercised by open market operations in the present context) must be inserted into the GBC in order to get the change in bonds held by the public and caused by the government deficit. We see that money supply must grow in a growing economy and that this growth serves to finance some of the government expenditure as shown by budget restriction of this sector, the GBC.

We now describe the equilibrium conditions of the model:

5. **Equilibrium conditions (asset-markets):**
\[ W_t + S_{pt} = \frac{(M^d_t + B_t^d + p_{et}E_t^d)}{p_t}, \]  
\[ M_t = M^d_t = h_1 p_t Y_t \exp(h_2(r_o - r_{t+1})) \]  
\[ p_{et}E_t = \rho_{t+1}p_{t+1}K_t/(r_{t+1} - \dot{p}_{t+1}), \text{ if } B_t = B^d_t, \quad E_t = E^d_t. \]

Money demand is now formulated in a way that is popular in the empirical investigation of the money demand function and it is fully served by the central bank on the basis of the interest rate \( r_{t+1} \) it sets by its interest feedback rule. With this exception, there is nothing new to comment upon in this module of the model. Asset markets are assumed to clear at all times as in the previous version of the model (on the basis of the perfect substitute assumption made.

The disequilibrium situation in the goods market is an important component in the sluggish responses of the economy to disequilibria in goods and labor markets and it is not changed in the present extension of the model of section 2 – up to the sales expectations mechanism that is now based on a moving average of current and past actual sales.

6. Disequilibrium situation (goods-market adjustments):

\[ S_t = S_{pt} + S_{gt} + S_{ft} = p_{et}\Delta E_t/p_t + I_{t} = I_{t} + N_{t} - N_{t-1} = I_{t}, \]  
\[ Y^d_t = C_t + I_t + \delta K_{t-1} + G_t, \]  
\[ N^d_t = \beta_{n}Y^e_t, \quad I_{t} = nN^d_{t} - \beta_{n}(N^d_{t} - N_{t-1}), \]  
\[ Y_t = Y^e_t + I_t, \]  
\[ Y_{t+1} = (1 + n)Y^e_{t} + \beta_{n}(Y^d_{t} - Y^e_{t}), \]  
\[ N_{t} = N_{t-1} + Y_{t} - Y^d_{t} = S_{t} - I_{t}. \]

We now come again to the last module of our model which is the wage-price sector or the supply side of the model.

7. Wage-Price-Sector (adjustment equations):

\[ \dot{w}_{t+1} = \beta_{w_1}(V_t/V - 1) + \beta_{w_2}(V_t - V_{t-1}) \]  
\[ + \\kappa_w(\dot{p}_{t+1} + n_2) + (1 - \kappa_w)(\pi_t^m + n_2), \]  
\[ \dot{p}_{t+1} = \beta_p(U_t/U - 1) + \kappa_p(\dot{w}_{t+1} - n_2) + (1 - \kappa_p)\pi_t^m, \]  
\[ \pi_{t+1} = \alpha(\beta_{1}\dot{p}_{t+1} + \beta_{2}\dot{p}_{t} + \beta_{3}\dot{p}_{t-1}) + (1 - \alpha)\pi_t. \]

The first adjustment equation (for nominal wages) now includes a derivative term \( V_t - V \) in line with Phillips' (1958), Kuh's (1967) and others' observations, see also the justification of this term in Chiarella and Flashel (1997, Ch. 8) by its relationship to the over- or under-time work of workers inside the firm. Note also that the growth rate of labor productivity \( n_2 \) enters the wage bargain in a positive fashion and that it takes pressure from the formation of the price rate of inflation as shown in (97). Note also that the rate of inflation expected for the medium run is now a weighted average of a moving average of observed inflation rates and the target rate set by the central bank. In sum we therefore now have in the model a fairly symmetric treatment of medium run or average concepts for nominal interest, profitability, sales expectations and price inflation, which we need in the investment function, for the output decisions of firms and in the wage price spiral of the model.
7.3.2 The intensive form of the extended model

In the formulation of the wage-price module we have added a derivative term (related to so-called Phillips loops) to the dynamics of money wages and have augmented wage and price inflation by the effects of a constant growth rate of labor productivity which exercises extra pressure on the increase of money wages and which removes some pressure from price inflation. The laws of motion for wages and prices can then again be reformulated as two linear equations now in the unknowns \( \dot{\omega}_{t+1} - (\beta^m_t + n_2) \), \( \dot{\pi}_t^{m} - \pi_t^{m} \), which are easily solved and give rise to the following expressions for the two unknowns:

\[
\dot{\omega}_{t+1} - (\beta^m_t + n_2) = \kappa_\beta (V_t / \bar{V} - 1) + \beta_w (V_t - V_{t-1}) + \kappa_\beta \beta_p (U_t / \bar{U} - 1),
\]

\[
\dot{\pi}_t^{m} - \pi_t^{m} = \kappa_\pi (\beta_w (V_t / \bar{V} - 1) + \beta_w (V_t - V_{t-1})) + \beta_p (U_t / \bar{U} - 1).
\]

These equations in turn imply for the dynamics of the wage share \( u_t = w_t L_t^p / (p_t Y_t) = u_t / (p_t x_t) \) the law of motion:\textsuperscript{35}

\[
\dot{u}_{t+1} = \dot{\omega}_{t+1} - n_2 - \dot{\pi}_t^{m},
\]

\[
= \kappa [(1 - \kappa_\beta) (\beta_w (V_t / \bar{V} - 1) + \beta_w (V_t - V_{t-1})) - (1 - \kappa_\pi) \beta_p (U_t / \bar{U} - 1)].
\]

This statement is again only true when one neglects the second order term \( \omega_{t+1} \rho_{t+1} \) in the formula that relates the nominal rates of wage and price inflation with the growth law for the real wage. Such second order terms are neglected in all following calculations of the intensive form of the model where the above law provides the first dynamical equation of this intensive form. Note also, that the formula for \( \dot{\pi}_t^{m} - \pi_t^{m} \) is repeatedly used in the following laws of motion of the intensive form of the model.

Neglecting the second order terms in the calculation of rates of change we get from the model of the preceding section the following autonomous dynamical system\textsuperscript{36} in the state variables \( u_t = u_t / (p_t x_t), l_t = x_t L_t / K_{t-1}, r_t, \pi_t^{m}, y_t^e = Y_t^e / K_{t-1}, \nu_t = N_{t-1} / K_{t-1}, \rho_t^m \) and \( r_t^{m} \):\textsuperscript{37}

\[
u_{t+1} = u_t + u_t \kappa_\pi [(1 - \kappa_\beta) (\beta_w (V_t / \bar{V} - 1) + \beta_w (V_t - V_{t-1})) + (\kappa_\beta - 1) \beta_p (U_t / \bar{U} - 1)],
\]

\[
l_{t+1} = l_t + l_t (-1 (\beta^m_t - r_t^{m} - \pi_t^{m})) - \kappa_\pi (U_t / \bar{U} - 1),
\]

\[
r_{t+1} = r_t + \beta_w (r_t - r_{t-1}) + \beta_w (\pi_t^{m} - \bar{\pi}) + \beta_p (U_t - U_t / \bar{U} - 1),
\]

\[
\pi_{t+1} = \alpha (\beta^m_t r_{t+1} + \beta^m_t \rho_{t+1} + \beta^{m}_{t+1} \beta_{t+1}) + (1 - \alpha) \bar{\pi},
\]

\[
\rho_{t+1}^m = \beta_{t+1}^2 \rho_{t+1}^m + \beta_{t+1}^m \rho_{t+1}^m + \beta_{t+1} \beta_{t+1}^m \beta_{t+1}^m + \beta_{t+1}^m + \beta_{t+1}^m = 1
\]

\[
r_{t+1}^{m} = \beta_{t+1}^m r_{t+1} + \beta_{t+1}^m r_{t+1} + \beta_{t+1}^{m} r_{t+1} - \beta_{t+1}^m \beta_{t+1}^m + \beta_{t+1}^m + \beta_{t+1}^m = 1
\]

\[
y_{t+1}^e = y_t^e + \beta_{t+1} (y_t^e - y_t^e) - (1 (\beta^m_t - r_{t+1}^{m} + \pi_{t+1}^{m}) + \beta_p (U_t - U_t / \bar{U})) y_t^e,
\]

\[
\nu_{t+1} = \nu_t + y_t^e - \nu_t + (l_{t+1} - n_2) \nu_t,
\]

\textsuperscript{35}Note here that the real wage \( \omega_t = u_t / p_t \) is no longer constant in the steady state of the model as it there grows in line with the growth rate of the labor productivity \( x_t \).

\textsuperscript{36}If \( \kappa_\beta, \ell_{t+1} = \ell_{t+1} / \ell_t \).

\textsuperscript{37}We have to assume here \( \kappa_\beta, \ell_{t+1} \neq 1 \) and use as abbreviation \( \kappa = (1 - \kappa_\beta, \ell_{t+1})^{-1} \). Note that there are three further difference equations in this model (for \( M_t / K_{t-1}, B_t / K_{t-1}, E_t / K_{t-1} \)) which however do not feed back into the above system of dynamical equations.
For output per capital $y_t = Y_t/K_{t-1}$, aggregate demand per capital $y^d_t = Y^d_t/K_{t-1}$ and price inflation we have the following expressions:

\begin{align}
  \nu_t &= (1 + n) \beta_{n_t} y^d_t + \beta_n (\beta_{n_t} y^s_t - \nu_t) \\
  y^d_t &= (1 - \tau_w) u_t y_t / x_0 + (1 - s_c) (\rho^s - \tilde{t}^{nc}) \\
  &\quad + t_1 (\rho^m - \tau^m_t + \pi^{m_t}) + t_2 (U_t / \bar{U} - 1) + n + \delta + g \\
  \hat{\rho}_{t+1} &= \tilde{\pi}^m + \kappa [\beta_p (U_t / \bar{U} - 1) + \kappa_p (\beta_{w_t} (V_t / \bar{V} - 1) + \beta_{w_1} (V_t - V_{t-1}))] 
\end{align}

Here we make use of the abbreviations:

\begin{align}
  V_t &= y^d_t / l_t, \quad U_t = y^d_t / y^p, \quad l^d_t = y_t / x_0 \quad (y_t, l^d_t \text{ not const.}), \\
  \rho^s_t &= y^s_t - \delta - u_t t^d_t, \\
  \tilde{t}^{nc} &= \text{const.}
\end{align}

**Proposition 4**

There is a unique steady-state solution or point of rest of the dynamics (102)-(109) with $u_0, l_0, m_0 \neq 0$ which is given by

\begin{align}
  y_0 &= \bar{U} y^p, \quad l^d_0 = y_0 / x_0, \quad U_0 = \bar{U}, \quad y^s_0 = \bar{y}^s_0 = y_0 / (1 + n \beta_{n_t}), \quad l_0 = l^d_0 / \bar{V} \\
  \pi_0 &= \bar{\pi}, \quad \hat{\rho} = \hat{\omega}_0 - n_2 = \bar{\pi}, \quad m_0 = h_1 y_0, \\
  u_0 &= \frac{n + g - (1 - s_c) \tilde{t}^{nc} - s_c (y^s_0 - \delta)}{(s_c - \tau_w) y_0 / x_0}, \quad \rho^m_0 = \rho^s_0 = y^s_0 - \delta - u_0 y_0, \\
  r^m_0 &= r_0 = \rho^s_0 + \bar{\pi}, \quad \nu_0 = \beta_{n_t} y^s_0.
\end{align}

This steady state is in particular is characterized by

$M_0 = \mu_0 = \bar{\pi} + n, \quad \bar{K}_0 = \bar{Y}_0 = n = n_1 + n_2, \quad \bar{\rho}_0 = \bar{\pi}, \quad \bar{\omega}_0 = \mu_0 - n_1, \quad \bar{\omega}_0 = n_2, \quad \bar{L}_0 = \bar{L}_0 = n_1.$

We assume that the parameters of the model are chosen such that the steady state values for $u, l, m, \rho^s, r$ are all positive.

**7.3.3 Important subcases of the extended model**

The minimal 4D dynamics:

In this minimal version of the model we do no distinguish short- and medium-run rates of return, set inflationary expectations equal to the steady state value of price inflation and reduce the
Metzlerian inventory adjustment process to the dynamic multiplier version of Keynesian quantity adjustment. We still have two laws of motion that describe sluggish wage / price and quantity adjustments and one law of motion for factor and output growth and of course the interest rate policy rule of the central bank:

\[ u_{t+1} = u_t + u_t \kappa((1 - \kappa_p)(\beta w_1(V_t/\bar{V} - 1) + \beta w_2(V_t - V_{t-1})) + (\kappa_w - 1)\beta_p(U_t/\bar{U} - 1)), \]
\[ l_{t+1} = l_t + l_t(-i_t(\rho_t - (r_t - \pi))) - i_t(U_t/\bar{U} - 1)), \]
\[ r_{t+1} = r_o + \beta r_1(r_t - r_o) + \beta r_2(U_t/\bar{U} - 1)), \]
\[ y_{t+1} = y_t + \beta y(y_t^d - y_t) + \tilde{l}_{t+1}y_t, \]

(120)

(121)

(122)

(123)

For aggregate demand per capital \( y_t^d = Y_t^d/K_{t-1} \) and price inflation we have the following expressions:

\[ y_t^d = (1 - \tau_w)u_t y_t / x_o + (1 - \sigma_c)(\rho_t - t^{nc}) + i_t(\rho_t - \pi) + i_t(U_t/\bar{U} - 1) + n + \delta + g \]
\[ \dot{p}_{t+1} = \pi + \kappa[\beta_p(U_t/\bar{U} - 1) + \kappa_p(\beta w_1(V_t/\bar{V} - 1) + \beta w_2(V_t - V_{t-1}))] \]

(124)

(125)

Here we make use of the abbreviations:

\[ V_t = l_t^d / l_t, \ U_t = y_t / y_P, \ l_t^d = y_t / x_o \ (y_t, l_t^d \ \text{not const.}), \]
\[ \rho_t = y_t - \delta - u_t l_t^d, \]
\[ t^{nc} = \ \text{const.}. \]

(126)

(127)

(128)

We have the Rose real wage effect in the first law of motion (which can be set equal to zero by assuming a markup based real wage rigidity. We have the growth law which can be suspended from the perspective of medium run analysis. And we have the generally stable dynamic multiplier process as the final law of motion of the economy.

The Keynes-Metzler 5D dynamics

Here, the 1D dynamic multiplier process is again replaced by the 2D Metzlerian quantity adjustment process which to some extend prefers the properties of the dynamic multiplier process:

\[ u_{t+1} = u_t + u_t \kappa((1 - \kappa_p)(\beta w_1(V_t/\bar{V} - 1) + \beta w_2(V_t - V_{t-1})) + (\kappa_w - 1)\beta_p(U_t/\bar{U} - 1)), \]
\[ l_{t+1} = l_t + l_t(-i_t(\rho_t - (r_t - \pi))) - i_t(U_t/\bar{U} - 1)), \]
\[ r_{t+1} = r_o + \beta r_1(r_t - r_o) + \beta r_2(U_t/\bar{U} - 1)), \]
\[ y_{t+1} = y_t^d + \beta y^{d_1}(y_t^d - y_t^d) + \tilde{l}_{t+1}y_t, \]
\[ \nu_{t+1} = \nu_t + y_t - y_t^d + (\tilde{l}_{t+1} - n)\nu_t, \]

(129)

(130)

(131)

(132)

(133)

For output per capital \( y_t = Y_t/K_{t-1} \), aggregate demand per capital \( y_t^d = Y_t^d/K_{t-1} \) and price inflation we have the following expressions:

\[ y_t = (1 + n\beta_{n*})y_t^d + \beta_n(\beta_{n*}y_t^d - \nu_t) \]

(134)
\begin{align*}
y_t^d &= (1 - \tau_u)u_t y_t / x_o + (1 - s_c)(\rho^c_t - \tau^{nc}) \\
&+ i_1(\beta^{\alpha}_u - \pi_t^{\alpha} + \pi_t^z) + i_2(U_t / \tilde{U} - 1) + \alpha + g \\
\hat{p}_{t+1} &= \pi_t^{\alpha} + \kappa [\beta_p (U_t / \tilde{U} - 1) + \alpha (\beta^{\alpha}_u (V_t / \tilde{V} - 1) + \beta^{\alpha}_w (V_t - V_{t-1}))] \\
\end{align*}

Here we make use of the abbreviations:
\begin{align*}
V_t &= l_t^d / l_t, \quad U_t = y_t / y^p, \quad l_t^d = y_t / x_o \quad (y_t, l_t^d \text{ not const.}) , \\
\rho^c_t &= y_t - \delta - u_t^d t_t^d, \\
\tau^{nc} &= \text{const.} \quad (139)
\end{align*}

The Tobin 5D dynamics:
We here allow for adjusting inflationary expectations and thus for locally explosive Mundell/Tobin/Cagan effects, but not yet for Metzlerian quantity adjustments in the place the dynamic multiplier:
\begin{align*}
u_{t+1} &= u_t + u_t [(1 - \kappa_p) (\beta^{\alpha}_w (V_t / \tilde{V} - 1) + \beta^{\alpha}_w (V_t - V_{t-1})) \\
&+ (\kappa_w - 1) \beta_p (U_t / \tilde{U} - 1)), \\
l_{t+1} &= l_t + l_t (\rho - (\pi_t^{\alpha} - \pi_t^z)) - i_2(U_t / \tilde{U} - 1), \\
r_{t+1} &= \rho_o + \beta_1 (\pi_t^z - \pi^z) + \beta_2 \pi_t^z + \beta_3 (U_t / \tilde{U} - 1), \\
\pi_t^{\alpha} &= \alpha (\beta^{\alpha}_p \hat{p}_{t+1} + \beta^{\alpha}_p \hat{p}_{t+1} + \beta^{\alpha}_w \hat{p}_{t+1} + (1 - \alpha) \hat{\pi} , \\
y_{t+1} &= y_t + \beta_p (y_t^d - y_t) + \hat{t}_{t+1} y_t.
\end{align*}

For aggregate demand per capital \( y_t^d = Y_t^d / K_{t-1} \) and price inflation we have the following expressions:
\begin{align*}
y_t^d &= (1 - \tau_u)u_t y_t / x_o + (1 - s_c)(\rho_t - \tau^{nc}) \\
&+ i_1(\rho_t - \pi_t^z + \pi_t^{\alpha}) + i_2(U_t / \tilde{U} - 1) + \alpha + g \\
\hat{p}_{t+1} &= \pi_t^{\alpha} + \kappa [\beta_p (U_t / \tilde{U} - 1) + \alpha (\beta^{\alpha}_u (V_t / \tilde{V} - 1) + \beta^{\alpha}_w (V_t - V_{t-1}))]
\end{align*}

Here we make use of the abbreviations:
\begin{align*}
V_t &= l_t^d / l_t, \quad U_t = y_t / y^p, \quad l_t^d = y_t / x_o \quad (y_t, l_t^d \text{ not const.}) , \\
\rho_t &= y_t - \delta - u_t^d t_t^d, \\
\tau^{nc} &= \text{const.} \quad (149)
\end{align*}

The 6D Keynes-Metzler-Tobin dynamics
This version of the model unifies the Tobin case with the Metzlerian inventory adjustment process:
\begin{align*}
u_{t+1} &= u_t + u_t [(1 - \kappa_p) (\beta^{\alpha}_w (V_t / \tilde{V} - 1) + \beta^{\alpha}_w (V_t - V_{t-1})) \\
&+ (\kappa_w - 1) \beta_p (U_t / \tilde{U} - 1)), \\
l_{t+1} &= l_t + l_t (\rho - (\pi_t^z - \pi_t^{\alpha})) - i_2(U_t / \tilde{U} - 1),
\end{align*}

37
\[ r_{t+1} = r_o + \beta_r (r_t - r_o) + \beta_v (\pi_t - \bar{\pi}) + \beta_u(U_t/\bar{U} - 1), \]  
(152)

\[ \pi_t = \alpha(\beta^c_t \hat{p}_{t+1} + \beta^n_t \hat{c}_t + \beta^u_{t-1} \hat{p}_{t-1} + (1 - \alpha)\bar{\pi}), \]  
(153)

\[ \nu_{t+1} = \nu_t + \beta_* (\nu_t' - \nu_t) + \bar{i}_{t+1} \nu_t', \]  
(154)

\[ \nu_{t+1} = \nu_t + \nu_t - \nu_t' + (\bar{i}_{t+1} - n)\nu_t, \]  
(155)

For output per capital \( y_t = Y_t/K_{t-1} \), aggregate demand per capital \( y_t^d = Y_t^d/K_{t-1} \) and price inflation we have the following expressions:

\[ y_t = (1 + n\beta_{n*})y_t^c + \beta_n (\beta_{n*} y_t^c - \nu_t) \]  
(156)

\[ y_t^d = (1 - \tau_w)u_t y_t/x_o + (1 - s_o)(\rho^c_t - \epsilon^{nc}) \]

\[ + \bar{i}_1 (\rho^c_t - r_t + \pi_t^m) + s_2 (U_t/\bar{U} - 1) + n + \delta + g \]  
(157)

\[ \hat{p}_{t+1} = \bar{\pi} + \kappa_2 (\beta_p (U_t/\bar{U} - 1) + \kappa_p (\beta_w_1 (V_t/\bar{V} - 1) + \beta_w_2 (V_t - V_{t-1})) \]  
(158)

Here we make use of the abbreviations:

\[ V_t = \frac{t_t^d}{t_t}, \quad U_t = y_t/y^p, \quad t_t^d = y_t/x_o \quad (y_t, t_t^d \text{ not const.}) \]  
(159)

\[ \rho^c_t = y_t^c - \delta - u_t^d, \]  
(160)

\[ \epsilon^{nc} = \text{const.} \]  
(161)

This model only differs from the general 8D case by the two laws of motion for the medium-run rate of profit and the medium-run rate of interest.
7.4 Appendix 4: Figures 1 – 7 of section 4

The following figures represent actual and predicted time series data as discussed in section 4 of the paper for the standard model (with the money supply rule) as well as the extended or modified one (with the Taylor interest rate policy rule).

Figure 1. Predicted (solid line) and Actual (dashed line) GDP (matched by selecting the parameters related to sales expectations, applied both to standard and extended model).

Figure 2. Predicted (solid line) and Actual (dashed line) Consumption (matched by selecting the parameters related to consumption function, applied both to standard and extended models).

Figure 3. Predicted (solid line) and Actual (dashed line) Inflation Rate (matched by selecting the parameters related to price-wage dynamics, applied only to standard model).
Figure 4. Predicted (solid line) and Actual (dashed line) Wage Growth Rate (matched by selecting the parameters related to price-wage dynamics, applied both to standard and actual model).

Figure 5. Predicted (solid line) and Actual (dashed line) Investment (matched by selecting the parameters related to the investment function, applied both to the standard and extended models).

Figure 6. Predicted (solid line) and Actual (dashed line) Interest Rate (matched by selecting the parameters related to money demand function, only applied to standard model).
Figure 7. Predicted (solid line) and Actual (dashed line) Interest Rate
(matched by selecting the parameters related to interest reaction function,
only applied to extended model)