The Balance-of-payments Constraint: From Balanced Trade to Sustainable Debt

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CEPA Working Paper 2001.06

January 2002
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Abstract:

This paper extends the balance-of-payments constraint on growth known as Thirlwall's law to incorporate unbalanced trade and debt accumulation. Assuming that small open economies face a liquidity constraint, the text shows the growth and real-exchange-rate policy rules consistent with a stable ratio of net exports to income. Given such rules, the text shows how the trade balance of a small open economy is residually determined by the ratio of foreign debt to income allowed by international conditions.

Key Words: balance of payments constraint, trade, debt, and growth.

JEL classification: F300 and F410

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* Center for Economic Policy Analysis, New School for Social Research. Financial support from CAPES, Brasília, Brazil, is acknowledged. I wish to thank Duncan Foley, John McCombie, Juan Carlos Moreno-Brid, Lance Taylor, Anthony Thirlwall, and two anonymous referees for their comments and suggestions. The opinions and errors remain my own.
**Introduction**

The balance-of-payments (BP) constraint on growth is usually associated with Thirlwall’s (1979) model, which imposes balanced trade as a necessary long-run constraint on open economies. According to Thirlwall's law, international capital flows and interest payments balance out during long intervals of time so that, given a stable real exchange rate, the long-run growth rate of a small open economy is limited by the growth rate of its exports divided by the income elasticity of its imports. Independently on whether such an empirical law holds for some sufficiently long intervals of time, in practice open economies may take several years to show balanced trade and, in the meanwhile, capital flows and interest payments are an important part of the BP constraint.

To deal with capital flows, Thirlwall and Hussain (1982) extended Thirlwall’s (1979) original model to allow trade deficits and showed how the growth rate of a small open economy may also be constrained by capital inflows in addition to trade factors. However, a particular characteristic of their extension is that “although it allowed for nonzero capital inflows, it imposed no restriction whatsoever on their trajectory except for the balance-of-payments accounting principle, which forces debit and credit items to cancel out.” (Moreno-Brid 1998, p 283) In other words, Thirlwall and Hussain obtained a dynamic accounting identity that shows how capital inflows may tighten or relax the BP constraint on growth.

To impose a limit on capital inflows, Moreno-Brid (1998) redefined the BP constraint in terms of a stable ratio of trade deficits to income on the assumption that this is a sufficient condition for a non-explosive accumulation of foreign debt.\(^1\) Notwithstanding its contribution to a better understanding of the BP constraint, Moreno-Brid’s (1998) extension has two important limitations of its own. First, its BP constrained growth rate is not necessarily stable and, second, its BP constraint does not separate interest payments from imports of goods and non-factor services in the analysis of debt accumulation.

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\(^1\) It should be noted that McCombie and Thirlwall (1997) were the first to analyze the implications of redefining the BP constraint in terms of a stable ratio of trade deficits to income.
With the above points in mind, this paper extends Thirlwall’s (1998) model to allow for a “sustainable” accumulation of foreign debt taking into consideration both the potential instability of such a constraint and the impact of interest payments on debt accumulation. The analysis is purely theoretical but heavily inspired by the recent Brazilian experience.\(^2\)

The objective is to model a case where fluctuations in foreign lending are a major determinant of macroeconomic policy and growth, and where the trade balance adjusts residually to the maximum ratio of foreign debt to income allowed by international financial conditions. The underlying principle is that international financial markets are incomplete, so that a small open economy may be liquidity constrained.

The text is organized in four sections in addition to this introduction. Section two outlines Thirlwall’s (1979) the balanced-trade version of the BP constraint and analyzes its implications for growth and real exchange rates. Section three presents the unbalanced-trade version of the BP constraint of Moreno-Brid (1998) and analyzes under which conditions such version is consistent with a stable growth rate. Section four presents the sustainable-debt version of the BP constraint and analyzes its implications for trade, growth, and real exchange rates. Section five concludes the analysis with a summary of the main points of the paper.

**Balanced trade**

Assume that the world economy consists of a large “foreign” country and a small “home” country. To simplify the exposition, assume further that both countries are one-sector economies and that there is imperfect substitution between the foreign and home goods. Finally, assume that the foreign currency is also the international currency, so that the foreign country can create money to finance its BP deficits. Since the home country cannot do the same, it may face a BP constraint when financial markets are incomplete.\(^3\)

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\(^2\) For an overview of the Brazilian trade and finance patterns in 1974-97, see Terra (1999). For an analysis of the 1990s, see Averbug and Giambiagi (1999) and Sainz and Calcagno (1999).

\(^3\) The basic idea is that the foreign good can impose a supply constraint on the home country. In this way the BP constraint is analogous to a capital or labor constraint in Harrod’s (1939) closed model, which is exactly the essence of the “dual-gap” model of Chennery and Bruno (1962).
Following the post Keynesian approach of Thirlwall (1979), assume that the home and foreign goods are produced with constant labor productivity and priced through a stable markup rule over unit labor costs, meaning that the home and foreign supply curves are horizontal in the absence of changes in nominal wages. In this context, Thirlwall (1979) represented the home exports and imports from the demand side, that is:

\[
(1) \quad Q_m = A \left( \frac{P_h}{EP_f} \right)^\alpha Q_h^\beta
\]

and

\[
(2) \quad Q_x = B \left( \frac{EP_f}{P_h} \right)^\gamma Q_f^\delta,
\]

where \( Q_m \) and \( Q_x \) are the real imports and exports of the home country, respectively, \( E \) the home price of the foreign currency (the nominal exchange rate), \( P_f \) the price of the foreign good in foreign currency, \( P_h \) the price of the home good in home currency, \( Q_f \) the real foreign income, and \( Q_h \) the real home income. The nonnegative parameters \( \alpha, \beta, \gamma, \delta \) are the price and income elasticity of home imports and exports, respectively, whereas the nonnegative parameters \( A \) and \( B \) control for other effects than price and income.5

If the BP constraint implies balanced trade as assumed by Thirlwall (1979), then

\[
P_h Q_x = EP_f Q_m \quad \text{and, therefore,}
\]

\[
(3) \quad p_h + q_h = e + p_f + q_m,
\]

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4 Note that constant labor productivity does not preclude active pricing by home and foreign firms and, in fact, changes in unit labor costs due to the pressure of trade surpluses or deficits on employment are usually part of the adjustment mechanism of growth to its BP constraint. For an analysis of markup pricing, see Taylor (1991).

5 If we define \( \eta \geq 0 \) as the price elasticity of the home demand for the home good, then the elasticity of substitution in the home country is \( \alpha + \eta \). By analogy, the same reasoning applies to the elasticity of substitution in the foreign country. The parameters \( A \) and \( B \) are included to represent fixed effects and, in the especial case where \( \alpha = \gamma = 0 \) and \( \beta = \delta = 1 \), analyzed by Chennery and Bruno (1962), they are exactly the import requirements of the home and foreign countries, respectively.
where the lower-case variables represent the exponential growth rates of the upper-case variables in (1) and (2).

From (1), (2), and (3) we obtain Thirlwall's (1979) balanced-trade home growth rate, that is

\[
q_h = \left( \frac{\delta}{\beta} \right) q_f - \left( \frac{1 - \alpha - \beta}{\beta} \right) r,
\]

where \( q_f \) is the exponential growth rate foreign income and, to simplify notation, \( r \) is the exponential growth rate of the real exchange rate \( R = E P_f / P_h \).

Given the foreign growth rate and the trade parameters, (4) implies an adjustment of \( q_h \), \( r \), or both. According to Thirlwall's law the adjustment comes fully through the home growth rate because, in the long run, the real exchange rate does not change (\( r=0 \)) or does not matter (\( \alpha + \gamma = 1 \)).

Notwithstanding the debate over its empirical validity, the theoretical implication of Thirlwall's law is clear, namely: to rule out the mainstream alternative of a full adjustment via the real exchange rate. More specifically, according to neoclassical growth theory both the home and foreign growth rates are determined from the supply side. If (4) is a relevant long-run constraint, the adjustment has to come completely through relative prices. Building upon Harrod's (1933) trade multiplier, Thirlwall's (1979) post Keynesian alternative is to close (4) completely from the demand side.

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6 The adjustment mechanism is assumed but not demonstrated by Thirlwall (1979). Building upon the verbal arguments of McCombie and Thirlwall (1994), Pugno (1998) constructed a model where the employment rate “predates” the inverse of the home-foreign real exchange rate and, in this process, it makes the home growth rate converge to a level that balances trade. The causal chain is the following: (i) a trade surplus leads to an increase in the home employment rate; (ii) the higher employment rate leads to an increase in the home labor costs; and (iii) the increase in labor costs erodes international competitiveness of the home country, reducing its trade surplus. A symmetric mechanism applies to trade deficits and the steady state is reached only when trade is balanced.

7 A survey of the empirical literature on Thirlwall’s law is beyond the scope of this paper. For a summary of the main points, see McCombie and Thirlwall (1994, ch.5). For more recent arguments, see McCombie (1997) and Alexander and King (1998).

8 A third theoretical alternative can be found in Krugman (1989), where \( r=0 \) and \( \delta / \beta \) converges to the ratio of supply-determined growth rates.
The intermediary alternative is an adjustment of quantities and relative prices where, say, the home country uses its macroeconomic policy to control not only growth, but also the real exchange rate. Since this “closure” implies active demand-management, it is perfectly consistent with Thirlwall's (1979) demand-led approach.

2 - Unbalanced trade

If there is unbalanced trade between the home and foreign countries, then

\[ P_s Q_s - E P_f Q_m = NX \]

where naturally \( NX \neq 0 \) represents home net exports in home currency. From this accounting identity Thirwall and Hussain (1982) derived an extended version of the BP constraint where the ratio of capital inflows to the sum of capital inflows and exports enters as an exogenous variable. Although correct from an accounting perspective, such an extension does not elaborate on the implications of limited capital inflows.

Moreno-Brid’s (1998) alternative was to propose a stable ratio of net exports to income as the relevant BP constraint on small open economies on the assumption that this is a sufficient condition for a stable ratio of foreign debt to income. As we shall see in the next section, this is actually a necessary condition for a stable debt-income ratio if one separates interest payments from imports of goods and non-factor services. For the moment, let us proceed under Moreno-Brid’s (1998) assumption that trade deficits or surpluses are not explosive.

Why should the home country stabilize its ratio of net exports to income? The answer involves a “proof by contradiction” since, if the home country has explosive trade surpluses in relation to its income, it will eventually produce all world output without consuming any of it. By analogy, if the home country has explosive trade deficits in relation to its income, it will eventually consume all world output without producing any of it. The history of capitalist economies indicates that these are mathematical

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9 For a real-world example, see the analysis of Brazil’s exchange-rate policy in 1964-97 by Bonomo and Terra (1999).

10 As pointed by Moreno-Brid, this “accounting restriction is insufficient to guarantee that the evolution of foreign capital inflows – whether in real or in nominal terms – generates a pattern of foreign indebtedness that is sustainable in the long run.” (Moreno-Brid 1998, p.284).
possibilities without any economic sense since even the most frugal of the countries would eventually want to use part of its international wealth to consume. In the same vein, even the least frugal of the countries would eventually have to adjust its current expenditures to the demands of foreign creditors. In fact, a stable ratio of net exports to income is nothing more than the “non-Ponzi” condition one finds in mainstream and non mainstream models of international finance to rule out infinite borrowing.

Normalizing $NX$ by the nominal home income, we have $x - m = nx$, where naturally $x = Q_x / Q_h$, $m = RQ_m / Q_h$, and $nx = NX / P_hQ_h$. If the BP constraint implies a stable ratio of net exports to income, then $dx/dt = dm/dt$ and the crucial question is what are the implications of this condition for growth and real exchange rates? To answer this note that

\[ \frac{dm}{dt} = m(r + q_m - q_h) \]  

(5) and

\[ \frac{dx}{dt} = x(q_x - q_h) \]  

(6) by definition. So, after substituting these equations in $dx/dt = dm/dt$ and using (1) and (2) to solve the resulting expression for $q_h$, we obtain the unbalanced-trade home growth rate proposed by Moreno-Brid (1998), that is

\[ q_h = \left( \frac{z\delta}{\beta - 1 + z} \right) q_f - \left( \frac{1 - \alpha - z\gamma}{\beta - 1 + z} \right) r, \]  

(7) where $z = x/m$ is the export-import ratio of the home country.

Equation (7) is a more general definition of the BP constraint to account for unbalanced trade and, not surprisingly, it encompasses (4) as an especial case when trade is initially balanced. Despite this connection, there exists a crucial difference between the “balanced” and “unbalanced” versions of the BP constraint, namely: unlike in (4),
causality runs in both directions in (7) because the home export-import ratio is itself a function of the home growth rate. More formally, since

$$(8) \quad \frac{dz}{dt} = z[\delta q_f - \beta q_h - (1 - \alpha - \gamma) r],$$

when (7) holds, we necessarily have

$$(9) \quad \frac{dz}{dt} = z \left[ (\beta - 1)(1 - z) \delta \frac{q_h}{\beta - 1 + z} - \frac{(1 - \alpha - \gamma + \beta \gamma)(z - 1)}{\beta - 1 + z} r \right]$$

and, therefore, $z$ is not necessarily stable unless trade is initially balanced. Moreover, even if one follow Moreno-Brid (1998) and assume that $r=0$, $z$ is still not necessarily stable unless trade is initially balanced or the income elasticity of home imports equals one.\(^{11}\) Moreno-Brid’s (1998) model is thus one possible case of the unbalanced-trade BP constraint on growth.

To check all cases, let us follow Moreno-Brid’s (1979) approach and assume that $r=0$. The simplest way to represent the dynamics of the BP constraint is to define the growth adjustment of the home country as

$$(10) \quad \frac{dq_h}{dt} = \chi \left[ \left( \frac{z \delta}{\beta - 1 + z} \right) q_f - q_h \right],$$

where $\chi > 0$ measures how fast the home growth rate converges to the BP constraint given by (7). From (8)

$$(11) \quad \frac{dz}{dt} = z(\delta q_f - \beta q_h),$$

which together with (10) form a 2x2 nonlinear dynamical system for the home growth rate and export-import ratio. The joint and nontrivial stationary solution of this system is

\(^{11}\) Neither McCombie and Thirlwall (1997) nor Moreno-Brid (1998) considered this potential source of instability in their unbalanced-trade version of the BP constraint.
\((q_h^*, z^*) = (\delta q_h / \beta, 1)\) and, to analyze local stability about this point, let \(\tilde{q}_h = q_h - q_h^*\) and \(\tilde{z}^* = z - z^*\). In matrix notation the linear version of (10) and (11) is

\[
\begin{pmatrix}
\frac{d\tilde{q}_h}{dt} \\
\frac{d\tilde{z}}{dt}
\end{pmatrix} = 
\begin{bmatrix}
-\chi & \chi \delta (\beta - 1) \beta^{-2} \\
-\beta & 0
\end{bmatrix}
\begin{pmatrix}
\tilde{q}_h \\
\tilde{z}
\end{pmatrix}.
\]

Since \(\chi > 0\) by assumption, it is straightforward that (12) is stable if and only if \(\beta > 1\). In economic terms, given a constant real exchange rate, the home country tends to its BP-constrained growth rate with balanced trade when the income elasticity of its imports is greater than one, as shown in figure 1. If the income elasticity is smaller than one, the equilibrium point is a saddle point and, therefore, (12) is stable only under the strong assumption that exogenous shocks do not drive the state variables out of their stable path. The phase diagram of this case is shown in figure 2. If the income elasticity equals one, the “equilibrium lines” of the home growth rate and export-import ratio coincide and, therefore, the home country tends to the BP-constrained growth rate with balanced or unbalanced trade. In short, (12) has multiple equilibrium points, as shown in figure 3.

[FIGURES 1, 2, AND 3 ABOUT HERE]

From the three cases above we can conclude that, if (7) holds and \(r = 0\) as assumed by Moreno-Brid (1998), then the only way to have stable and unbalanced trade in the long run is to impose the auxiliary assumption that the income elasticity of home imports equals one. Since small open economies usually have income-elastic imports, the case analyzed by Thirlwall (1979) is more likely to occur.

If one wants to preserve stable and unbalanced trade without restricting the income-elasticity of home imports, the natural solution is to include \(r\) in the problem and redefine the unbalanced-trade constraint as stable export-income and import-income ratios.\(^{12}\)

More formally, let \(dm/dt = 0\) and use (1) to solve (5) for the home growth rate. The result is the loci of points \((r, q_h)\) for which the home import ratio is stable, that is

\[\text{[12] By assuming that} \ x \text{ and} \ m \text{ are stable we obtain a stable} \ nx \text{ but rule out the extreme case where} \ nx \text{ is stable with} \ x \text{ and} \ m \text{ tending to zero or infinite.}\]
(13) \[ q_h = [(1 - \alpha)/(1 - \beta)]r. \]

By analogy, the loci of points \((r, q_h)\) for which the home export ratio is stable is given by

(14) \[ q_h = \gamma r + \delta q_f. \]

Solving these equations for \(q_h\) and \(r\),

(15) \[ q_h = \left[ \frac{(1 - \alpha)\delta}{1 - \alpha - \gamma + \beta\gamma} \right] q_f, \]

and

(16) \[ r = \left[ \frac{(1 - \beta)\delta}{1 - \alpha - \gamma + \beta\gamma} \right] q_f. \]

So, assuming for the moment that the home country can control growth and relative prices, (15) and (16) give us the policy rules consistent with stable import and export ratios.\(^{13}\)

Focusing the analysis on the cases where both \(\alpha\) and \(\beta\) are different than one,\(^{14}\) (15) and (16) gives us three qualitatively distinct cases, namely:

(i) when \(\alpha > 1\) and \(\beta < 1\) or \(\alpha < 1\) and \(\beta > 1\): income expansion in the foreign country implies income expansion in the home country and appreciation of the home good, as shown in figure 4;

(ii) when \(1 + \gamma - \alpha \gamma - \beta\gamma > 0\) and either \(\alpha > 1\) and \(\beta > 1\) or \(\alpha < 1\) and \(\beta < 1\): income expansion in the foreign country implies income expansion in the home country and depreciation of the home good, as shown in figure 5; and

\(^{13}\) In terms of Tingerben's (1955) analysis of economic policy, the home country has two instruments (income and the real exchange rate) to achieve two targets (stable export and import ratios).

\(^{14}\) Given a positive growth rate in the foreign country, (15) and (16) give us exactly Thirlwall’s law when the income elasticity of imports equals one. In contrast, when the price elasticity of imports equals one, we have the economically unusual result of zero home growth with an appreciation of the home good.
(iii) income expansion in the foreign country implies income contraction in the home country and appreciation of the home good (when $1 - \alpha + \beta \gamma < 0$ and either $\alpha < 1$ and $\beta > 1$ or $\alpha > 1$ and $\beta < 1$), as shown in figure 6.

By analogy the implications of income contraction in the foreign country can also be grouped in the same three qualitatively distinct cases.

Since for almost any arbitrary division of the world economy one observes positive growth in the “foreign” and “home” blocks during, say, 10-years intervals of time, case (iii) tends to be a rare real-world phenomenon. Moreover, since small open economies usually have income-elastic imports ($\beta > 1$), the distinction between case (i) and (ii) tends to lie on the price elasticity of home imports.\(^{15}\)

Now the crucial question: can the home country really control growth and real exchange rates? Theoretically, this can only happen if there is a stable “technological-institutional” structure connecting income, prices, and exchange rates in the home country where macroeconomic policy and foreign conditions enter as exogenous variables. In the jargon of Keynesian economics, if there exist a stable “Phillips Curve” connecting growth and inflation in which macroeconomic policy enters as an exogenous variable, then it may be possible for the home government to achieve (15) and (16) with the aid of some exchange-rate parity condition. For instance, assume that the home growth rate is a function of monetary policy, fiscal policy, and the appreciation or depreciation of the home good, that is

$$ q_h = \phi_0 + \phi_1 (i_h - p_h) + \phi_2 g + \phi_3 r $$

where $i_h$ is the nominal interest rate in the home country, $g$ an index that measures the demand impact of fiscal policy,\(^{16}\) and $\phi_j \neq 0$ for $j = 0, 1, 2,$ and $3$. If there is also a risk-adjusted parity between the home and foreign interest rates,

\(^{15}\) Case (i) tends to happen when home imports are price inelastic ($\alpha < 1$) and case (ii) when they are price elastic.

\(^{16}\) Say, the ratio of budget deficits to income. Note that the intercept coefficient in (19) can incorporate a “natural” real rate of interest and an “equilibrium” or “optimal” value of $g$ without loss of generality.
where $i_f$ and $\sigma$ are the nominal foreign interest rate and risk premium paid by home borrowers in the foreign financial market, respectively.

Since $e = r - p_f + p_h$, (18) can be used to obtain the home real interest rate consistent with the target growth rate of the real exchange rate. Then, substituting this result in (17), we obtain the fiscal-policy variable necessary to achieve the target income growth. Altogether, the risk-adjusted parity between nominal interest rates and the real-exchange-rate target determine monetary policy, and then monetary policy and the income-growth target determine fiscal policy.

The stable institutional and technological structure implicit in (17) is obviously a very strong assumption for the long run but, on the other hand, it is a reasonable approximation of the reality of small open economies in the short run. Price rigidities, asymmetric information, and fundamental uncertainty usually lead to a short-run relationship of the kind depicted in (17). The increasing integration of world financial markets tends to subordinate monetary policy to foreign conditions, as modeled in (18). The result is a short-run structure that allows the home government to control income and relative prices.

On the empirical side, the experience of some Latin American countries since the end of the Bretton Woods system indicates that stop-and-go policies may indeed be able to control income and relative prices during short intervals of time, at the cost of periodic currency crises.\textsuperscript{17} In fact, the opportunities brought by cheap foreign credit and the inability to issue foreign currency during times of crisis are usually more convincing than the Lucas critique in the determination of macro policy in small open economies.

Overall, (15) and (16) should be interpreted as short-run targets for income and real-exchange-rate growth rates when the home country is constrained to have a non-explosive trade pattern. Since these targets are consistent with balanced and unbalanced

\textsuperscript{17} The exchange-rate based stabilization plans in Mexico, Argentina, and Brazil during the 1990s are a good example of such a BP-determined demand management.
trade, they do not tell us at what level the home export and import ratios are stable. This is exactly where the concept of a sustainable accumulation of foreign debt closes the analysis.

**Sustainable debt**

So far we analyzed the impact of unbalanced trade on the BP constraint without mentioning interest payments and the dynamics of foreign debt. However, since the home country does not issue foreign currency, it can only have persistent trade deficits by receiving a continuous inflow of foreign capital. The counterpart of unbalanced trade is a change in the stock of foreign debt and, therefore, we have to check under which conditions the unbalanced-trade constraint given by (15) and (16) is consistent with a non-explosive accumulation of foreign debt.

Following the approach of Moreno-Brid (1998), let a stable ratio of foreign debt to income be the definition of a sustainable accumulation of foreign debt. Assuming that the home country is a net debtor and that capital flows involve only interest-bearing bonds,

\[(19) \quad P_h Q_h - E P f Q m - (i_f + \sigma) ED + EF = 0,\]

where \(D\) is the net foreign debt of the home country, and \(F\) the net inflow of foreign capital into the home country, both in foreign currency. Normalizing (19) by home income

\[(20) \quad x - m - (i_f + \sigma)d + f = 0,\]

where \(d = ED / P_h Q_h\) and \(f = EF / P_h Q_h\).

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18 By doing so we are merging the foreign debt of the government and private sectors of the home country and ignoring the “twin-deficits” argument that budget deficits are necessarily the driving force of current-account deficits. As the recent experience of the US indicates, the business sector may also be the destabilizing agent.

19 A non-zero balance of payments can be incorporated into the analysis without loss of generality, provided that we redefine sustainable debt accumulation as a stable ratio of foreign reserves to foreign debt. For a real-world example, see Barbosa-Filho (2001).
Given a constant ratio of capital inflows to income, (20) implies not only that \( x \) and \( m \) should be stable like in the previous section, but also that the debt ratio \( d \) should be stable. In other words, given the availability of foreign finance \( (f) \), the BP constraint now implies stable trade \( (x \) and \( m) \) and debt ratios \( (d) \).

Using the fact that the net inflow of foreign capital equals the change in foreign debt \( (F=dD/dt) \),

\[
\frac{dd}{dt} = m - x + (i_f + \sigma + e - p_h - q_h)d.
\]

So, given \( x \) and \( m \), it is straightforward that \( d \) is stable as long as the home growth rate exceeds the real cost of foreign debt in home currency. To insert the trade parameters into the analysis, note that from (15) and (16) we can rewrite (21) as

\[
\frac{dd}{dt} = m - x + \left[ i_f + \sigma - p_f - \left( \frac{\beta - \alpha}{1 - \alpha - \gamma + \beta \gamma} \right) \delta q_f \right] d
\]

and, therefore, the stability condition for \( d \) is

\[
\left( \frac{\beta - \alpha}{1 - \alpha - \gamma + \beta \gamma} \right) \delta > \frac{i_f + \sigma - p_f}{q_f}.
\]

The economic intuition is that, given its trade parameters, the international financial stability of the home country depends on its risk premium and the growth and real interest rates in the foreign country.\(^20\)

From the steady-state solution of (22) we also have

\[
x - m = \left[ i_f + \sigma - p_f - \left( \frac{\beta - \alpha}{1 - \alpha - \gamma + \beta \gamma} \right) \delta q_f \right] d,
\]

which completes the home control problem by setting a target for the home net-export ratio in terms of trade parameters and foreign conditions.
Altogether, the BP constraint implies managing $q_h$ and $r$ according (15) and (16) to keep $x$ and $m$ stable at the level given by (24). The economic intuition is that the trade parameters, the risk premium, and the debt ratio allowed by foreign financial conditions determine the net-export ratio of the home country, which in its turn determine its income and real-exchange-rate growth rates.

Recalling that we assumed earlier that the home country is a net debtor ($d > 0$), the target net-export ratio in (24) is negative when (23) holds and positive otherwise. Thus, when the debt ratio shows stable dynamics, the BP constraint is consistent with trade deficits and vice versa. From (24) we can also see that, independently of the value of the trade parameters, an increase in the risk premium or the foreign real interest rate always increases the “trade burden” of sustainable debt ($x - m$).

The trade parameters are important to determine the impact of foreign growth on the target net-export ratio. Considering the three cases analyzed in the previous section, an increase in the foreign growth rate always reduces the target net-export ratio in case (i). In case (ii) this happens only if $a < \beta$ and, in case (iii), only if $a > \beta$.

Recalling that case (iii) is rare phenomenon and that small open economies usually have income-elastic imports, we can conclude that an increase in foreign growth tends to reduce the target net-export ratio when the price-elasticity of home imports is smaller than its income elasticity ($\alpha < \beta$) and vice versa.

**Conclusion**

Thirlwall's (1979) original specification of the BP constraint can be extended to include unbalanced trade, interest payments, and a sustainable accumulation of foreign debt,

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20 Note that if home imports are price inelastic and income elastic, the left-hand side of (23) is positive.
21 Now the home country has three instruments ($q_h$, $r$, and $x - m$) to achieve three targets (stable $x$, stable $m$, and stable $d$).
22 Again, the exchange-rate based stabilization plans in Mexico, Argentina, and Brazil during the 1980s and 1990s indicate that this usually involves two analytically distinct phases. First, once-for-all changes of the real exchange rate (a “maxi” depreciation or appreciation of the home currency) to put $x - m$ at the level allowed by foreign conditions. Second, a continuous demand management to keep $x$ and $m$ stable at such level. The second part usually breaks down in the medium run and the result is a currency crisis that starts the process all over again.
provided that we expand its definition to include the real exchange rate and the trade balance.

In relation to the previous theoretical literature on the BP constraint, four points should be mentioned. First, similar to the models of Thirlwall and Hussain (1982), McCombie and Thirlwall (1997), and Moreno-Brid (1998), the model of this paper allows persistent trade deficits or surpluses, encompassing Thirlwall's law as a special short-run case or the long-run case. Second, unlike the model of Thirlwall and Hussain (1982) and similar to the models of McCombie and Thirlwall (1997) and Moreno-Brid (1998), the model of this paper imposes non-explosive trade deficits or surpluses on the country in question. Third, the model of this paper gives us theoretical hypotheses about growth and real exchange rates and, differently from the models of McCombie and Thirlwall (1997) and Moreno-Brid (1998), it does not result in a potentially unstable BP-constrained growth rate. Fourth, unlike the models of Thirlwall and Hussain (1982), McCombie and Thirlwall (1997), and Moreno-Brid (1998), the model of this paper separates interest payments from the imports of goods and non-factor services.

Altogether, the model of this paper shows the connection between trade parameters, foreign growth, foreign interest rates, and trade ratios in the determination of a sustainable accumulation of foreign debt. Its main disadvantage in relation to the existing literature lies on the heroic assumption that the country in question can control business fluctuations and relative prices. Notwithstanding the fact that many developing economies have been trying to do exactly this since the end of the Bretton-Woods system, continuous and discontinuous changes in technology, preferences, and institutions restricts the analysis of this paper to the short run.

During long intervals of time the trade parameters and foreign conditions change and, therefore, so do the targets for $q_h$, $r$, and $x-m$. Hence, if the trade parameters do not change, in the long run one of the countries may become infinitely large, one of the goods may become infinitely cheap, or both. Since one does not see this in the world economy, in the long run the BP-constraint equations become accounting identities of any country that does not display explosive trade and debt patterns.
Whether or not the BP constraint proposed in this paper is an adequate description of the short-run operation of small open economies is a point to be investigated empirically. On a first approximation, the recurrent currency crises in developing countries and the autonomous or IMF-imposed adjustments to them indicate that such a constraint usually comes through quantities, relative prices, and debt ratios. Since a BP-oriented demand management involves many targets and variables, the results of this paper offer one possible way to organize the analysis in terms of trade parameters, foreign conditions, and a sustainable debt-income ratio.

References:


FIGURE 3

\[
\frac{\delta}{\beta}q_f
\]

\[q_h\]

\[z\]
FIGURE 4

Stable import ratio

Stable export ratio

$q_h$

$r$
FIGURE 5

Stable import ratio

Stable export ratio
FIGURE 6

Stable export ratio

Stable import ratio

$q_h$

$r$