Background images and text colors have been adjusted for editing

Lecture 4
Social Interaction

What is social science?

- One way to think about social science is to ask what it means to say behavior is *social*.
- The specifically social aspect of behavior arises when the presence of other people in an interaction has an effect on individual behavior.
- We can look at this issue through the *canonical social interaction model*.

The canonical social interaction model

- Consider the interaction of a “large” number $i = 1, \ldots, N$ of identical individuals , each controlling some action variable x^i. (x^i may be treated as a vector eventually, but for the moment, assume it is a scalar.) For notational convenience write $\mathbf{x} = \{x^1, \ldots, x^N\}$ for the vector of actions of the individuals. We arbitrarily and without loss of generality consider the *n*th individual as the *typical individual*, and let the average level of x across the other individuals be $z[\boldsymbol{x}] = \bar{\boldsymbol{x}} = \langle \boldsymbol{x} \rangle = \frac{1}{N-1} \sum_{i=1}^{N-1} x^i$. (Individuals might also have a "state", y^i to represent heterogeneity of condition.)

- The idea of a "social" interaction is represented by making the perceived payoff of each agent (all identical) depend on her own action and on the average action of the rest of society], $u\left[x^i, z\right]$, since all the individuals are assumed to be identical. Assume that $u[., z]$ has a maximum in x for any value of z. (Heterogeneity would lead to the more complicated version $u\left[x^i, y^i, z\right]$. In these notes, we will often suppress the y argument, and assume all the y^i are equal.)

- This is a “mean-field” theory, because individual behavior depends on social behavior only through the mean of the social behavior. But other summary statistics of the behavior of the rest of society, such as the median or a mode, could be substituted within the logic of the model.

Equilibria

- Individual behavior when the individual takes others' behavior as given is a *best response*.
- The individual's best response, taking *z* as given is the solution $x[z]$ to:

$$x^{br}[z] = \underset{x}{\mathrm{ArgMax}}[u[x, z]] \tag{1}$$

- Because all the individuals are identical in relevant aspects, the equilibrium individual level of the action (in the Nash or Cournot sense), x^e, must be a best response to itself:

$$x^{\mathrm{br}}[x^e] = x^e \tag{2}$$

- If there is a policy contribution to the social action such as a stimulus or subsidy, ξ, the typical individual responds to the social signal $z + \xi$, and the equilibrium condition becomes:

$$x^{\mathrm{br}}[x^e + \xi] = x^e \tag{3}$$

Best response

- The *Mathematica* code to compute the typical individual’s best response is:

```
Clear[br];
br[u_] := Maximize[{u[xbr]}, xbr][[
   2]];
br[u_, xlo_, xhi_] :=
  Maximize[{u[xbr], xlo ≤ xbr ≤ xhi},
    xbr][[2]];
```

- The *Mathematica* code for a quadratic payoff is:

```
Clear[uQuad];
uQuad[α_, β_, γ_: 0][z_][x_] =
  (α + β z) x - 1/2 x^2 + γ z
```

$$-\frac{x^2}{2} + x\,(\alpha + z\,\beta) + z\,\gamma$$

- $(\alpha + \beta z)\,x$ represents the private payoff to the individual of her own action, x, which can be influenced positively or negatively by the social action. The quadratic term represents the cost to the individual of the action, and γz represents a positive or negative externality from the social action to the individual.

- The unconstrained best response for an individual with a quadratic payoff is linear in *z*:

```
br[uQuad[α, β][z]]
```

$\{xbr \to \alpha + z\,\beta\}$

- When the individual action is constrained by upper and lower bounds, the best response is a piecewise linear function:

```
Clear[uQuadbr];
uQuadbr[α_, β_, xlo_: - Infinity,
    xhi_: Infinity][z_] =
 br[uQuad[α, β][z], xlo, xhi] //
  FullSimplify[#,
    Assumptions →
     {Element[α | β | z | xhi, Reals],
      xlo < xhi}] &
```

$$\left\{\text{xbr} \to \begin{cases} \text{xlo} & \text{xlo} \geq \alpha + z\,\beta \\ \alpha + z\,\beta & \text{xlo} < \alpha + z\,\beta \;\&\& \\ & \;\text{xhi} \geq \alpha + z\,\beta \\ \alpha + z\,\beta - & \text{True} \\ \;\text{Abs}[-\text{xhi} + \alpha + z\,\beta] & \end{cases}\right.$$

uQuadbr[α, β][z]

$$\left\{xbr \to \begin{cases} -\infty & -\infty \geq \alpha + z\,\beta \\ \alpha + z\,\beta & -\infty < \alpha + z\,\beta \;\&\&\; \infty \geq \alpha + z\,\beta \\ -\infty & \text{True} \end{cases}\right\}$$

- The best response of the individual does not depend on the externality $\gamma\,z$, which does not vary with the individual’s behavior.

Equilibrium

- Equilibrium depends on the best response function and the social stimulus:

```
Clear[soceq];
soceq[br_, ξ_] :=
  Quiet@
   Solve[{xeq == (xbr /. br[xeq + ξ])},
    {xeq}];
```

- Equilibrium for a society of individuals with quadratic payoff functions is:

```
Assuming[(α | β | ξ) ∈ Reals,
 soceq[uQuadbr[α, β], ξ] // Simplify]
```

$$\left\{\left\{\text{xeq} \to \frac{\alpha + \beta\,\xi}{1 - \beta}\right\}\right\}$$

- When individual behavior is constrained, there can be up to three equilibria, depending on the parameters:

```
soceq[uQuadbr[α, β, xlo, xhi], ξ] //
 FullSimplify[#,
   Assumptions →
    {xlo < xhi, (α | β | ξ | xlo | xhi) ∈
      Reals}] &
```

Socially-coordinated outcomes in the social interaction model

- The socially-coordinated outcome, which would be implemented by a Social Planner trying to maximize average payoff under the constraint that all the participants choose the same level of the action, x^{sc}, will be:

$$x^{sc} = \underset{x}{\text{ArgMax}}\, u[x, x] \quad (4)$$

```
Clear[soccoord];
soccoord[u_] :=
 Maximize[{u[xcoord][xcoord]},
   {xcoord}][[2]]
soccoord[u_, xlo_, xhi_] :=
 Maximize[{u[xcoord][xcoord],
    xlo ≤ xcoord ≤ xhi}, {xcoord}][[2]]
```

- When $\beta < \frac{1}{2}$ the socially coordinated outcome when the payoff is quadratic is interior:

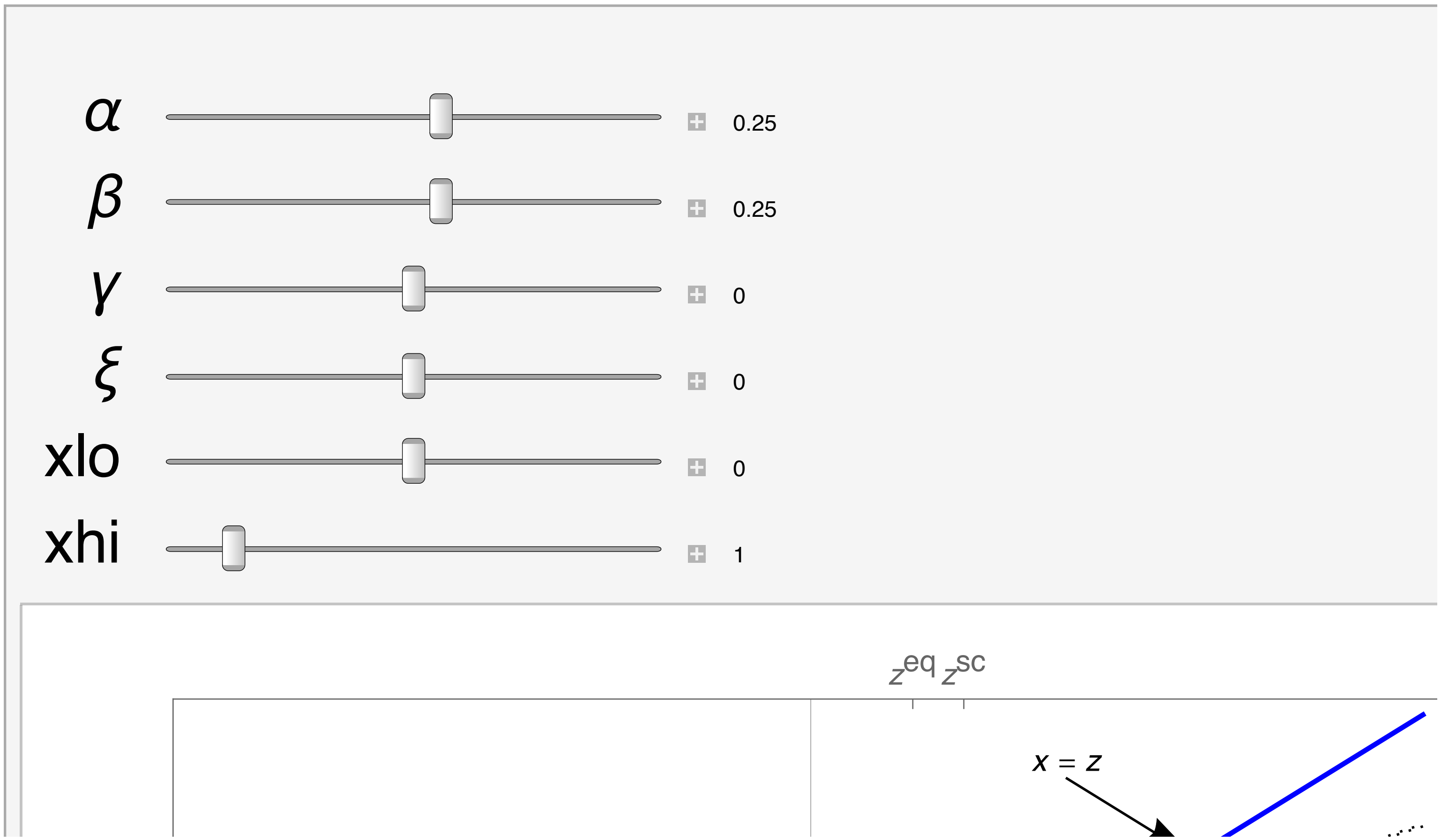
```
soccoord[uQuad[α, β, γ]] //
  Simplify[#, Assumptions → {β < 1/2}] & //
 TraditionalForm
```

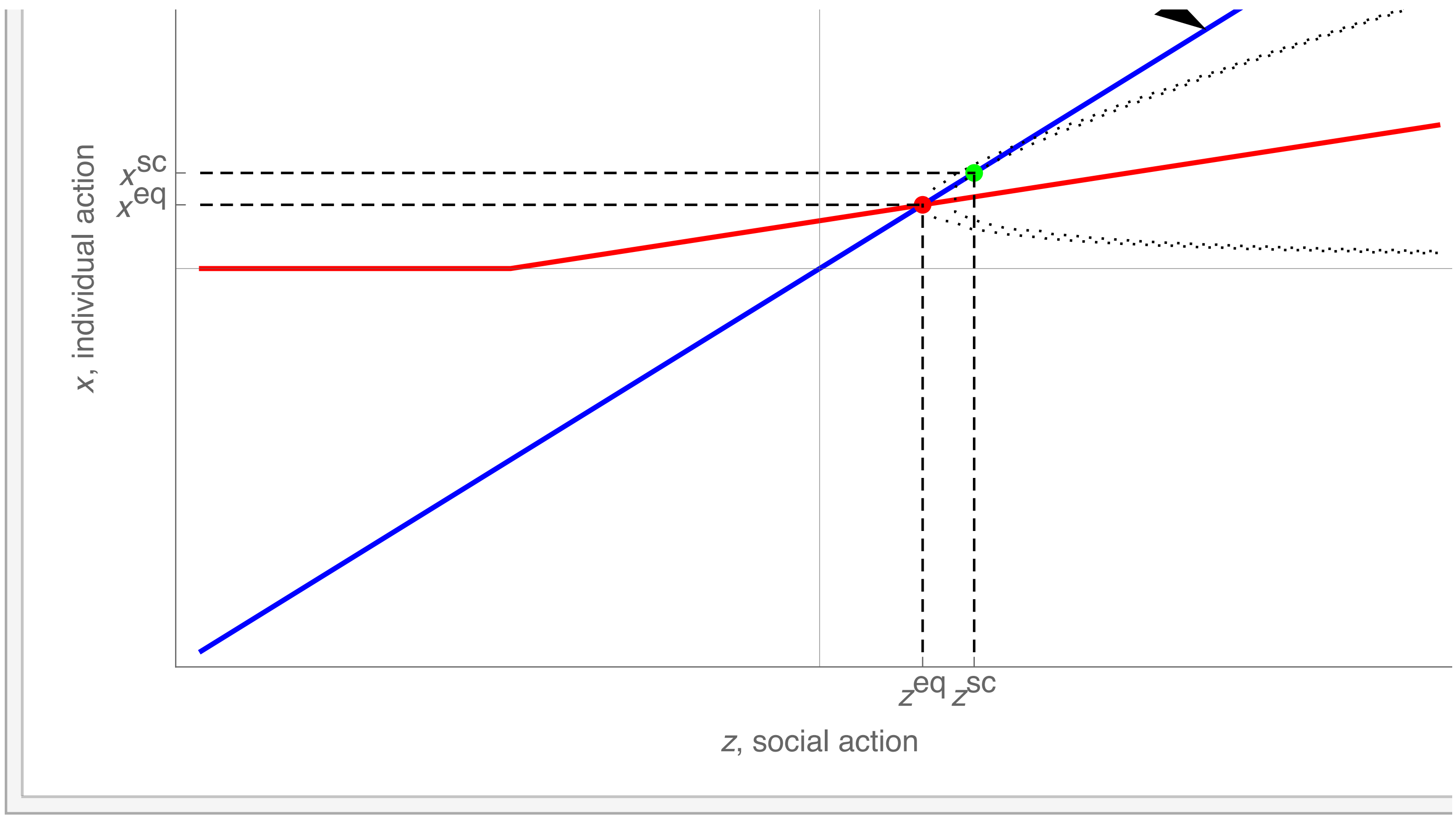
$$\left\{\text{xcoord} \to \frac{\alpha + \gamma}{1 - 2\beta}\right\}$$

- When $\beta > \frac{1}{2}$, the socially coordinated outcome when the payoff is quadratic is at one of the constraints:

```
soccoord[uQuad[α, β, γ], xlo, xhi] //
 Simplify[#, Assumptions →
    {β ≠ 1/2, xlo < xhi}] &
```

Visualizing equilibrium in the social interaction model




x^{sc}
x^{eq}
x, individual action
z^{eq} z^{sc}
z, social action

Social coordination and equilibrium

- The fact that equilibria of the social interaction model are not necessarily socially coordinated is a main feature of social life.
- When $\beta < 0$, social and individual actions are *strategic substitutes*. The equilibrium is a Prisoners' Dilemma where the equilibrium level of the individuals' actions are higher than the social coordination levels. A positive social stimulus can move the equilibrium closer to the social coordination level.
- When $1 > \beta > 0$, social and individual actions are

weak strategic complements. The equilibrium is a Prisoners' Dilemma where the equilibrium level of the individuals' actions are lower than the social coordination levels. A negative social stimulus can move the equilibrium closer to the social coordination level.

- When $\beta = 0$, a "knife-edge" case, the equilibrium level of the action is the same as the socially coordinated level of the action. This is the "invisible hand".

Quadratic Example 1

- The derivatives of the quadratic u with respect to x and z at the social coordination action level are reported in the following table:

	0	1
0	0	γ
1	α	β
2	$-\frac{1}{2}$	0

- The economic interpretation of the coefficients is as follows. $u_{10} = \alpha > 0$ when x is a good, and $u_{10} < 0$ when x is a bad (like effort) to the individual. Because individual utility with respect to x is maximized at both the socially coordinated and the equilibrium action levels, x is neither a bad nor a good. $u_{01} = \gamma > 0$ if there is a positive additive externality to the typical individual from the rest of society, while $u_{01} < 0$ if there is a negative externality. $u_{20} = -\frac{1}{2} < 0$ represents diminishing marginal utility (increasing marginal disutility for a bad) to the individual. $u_{11} = \beta > 0$ represents strategic complementarity between the individual action and the social action, while $u_{11} = \beta < 0$ represents strategic substitutability.

Political Economy--Tragedy of the Commons 1

- To see how the social interaction model works, let’s apply it to the classic problem of the “tragedy of the commons”, from which a good deal of modern marginalist and neoclassical economic thinking flows.
- A farming village in which all the (identical) farmers graze cows is provided with a limited well-watered, fertile grassy area held by the village as a *commons*, on which anyone may graze their cows. Outside this grassy area is a

vast expanse of less fertile land, the *waste*.

- Cows grazed on the waste produce M_W pounds of milk a year each, regardless of how many of them, X_W, there are. The milk output of a cow grazed on the commons, y_C, depends on how many cows graze the commons altogether, X_C, according to the relation $y_C = M_C - X_C$ where $M_C > M_W$.
- The socially coordinated grazing plan chooses maximizes the total milk production, $(M_C - X_C) X_C + M_W X_W$, given the total number of cows, $X = X_C + X_W$, by setting $X_C = \frac{1}{2}(M_C - M_W)$, thereby equalizing the marginal milk production of a cow on the commons, $M_C - 2 X_C$, to the

marginal (and average) milk production of a cow grazed on the waste, M_W.

- If the farmers choose the grazing plan as individuals, each taking the number of cows already on the commons as given, they will equalize the *average* milk production of a cow on the commons, $M_C - X_C$, to the average (and marginal) milk production of a cow the waste, M_W, and graze $X_C = M_C - M_W$ cows on the commons, twice as many as in the socially coordinated grazing plan, incurring a loss in total milk production.

Social interaction and the tragedy of the commons

- In the social interaction model, the action of a typical farmer who, like all the others, has a herd of X cows, is the number of cows she grazes on the common, x, (grazing the remainder of her cows, $X - x$, on the waste). The variable z represents the average number of cows grazed on the commons by the $n - 1$ other farmers.

- Since the total number of cows on the commons is $(n - 1)\,z + x$, when the typical farmer grazes x cows on the commons, her herd's milk production will be:

$$x\,(M_c - ((n-1)\,z + x)) + (X - x)\,M_w = \\ (M_C - M_W - (n-1)\,z)\,x - x^2 + M_W\,X \tag{5}$$

- This payoff is the social interaction model with $\alpha = \frac{1}{2}\,(M_C - M_W)$ and $\beta = -\frac{1}{2}\,(n-1)$.
- The equilibrium grazing level is $\frac{\alpha}{1-\beta} = \frac{M_C - M_W}{n+1}$. The n farmers will graze a total of $\frac{n}{n+1}$ cows on the commons in equilibrium. The typical farmer for any finite n does not quite equalize the average productivity of her cows on the commons and the waste because she partially internalizes the externality: moving one more cow from the waste to the commons would degrade the productivity of her own cows on the common

too much. Only as $n \to \infty$ does the commons equilibrium actually equalize average productivity on the waste and the commons.

- The social coordination grazing level is $\frac{\alpha}{1-2\,\beta} = \frac{M_C - M_W}{2\,n}$, instructing the n farmers to put a total of $\frac{1}{2}\,(M_C - M_W)$ cows on the common.

Avoiding the tragedy of the commons

- The social coordination problem represented by the parable of the commons has been a major theme of economic theory for all segments of the ideological spectrum.
- For advocates of the system of private property, the theoretical and historical solution to the commons dilemma is *enclosure*, the conversion of the commons into private property. The idea here is that if each farmer controls a share of the commons, she will internalize the tradeoffs between grazing cows on the waste and the common. We have seen that this internalization

can be significant for small numbers even without enclosure.

- The theoretical idea of enclosure raises some important complications. In its simplest form it seems to envision breaking up the original commons into *n* equal parcels, each of which is a model of the whole commons, particularly in the sense of reproducing exactly the same tradeoff between productivity and the number of cows grazing the commons land as the scale of grazing gets smaller.
- But there is good reason to think that the crowding experienced by each of 100 cows grazing 10 acres is less than that experienced by

each of 10 cows grazing 1 acre, so that the typical farmer would not face the same productivity tradeoff as the collective farmer.

- In historical fact, of course, enclosure hardly ever actually results in an equal division of the land among those who have access to a commons. What happens is that the process of privatization leading to enclosure results in a highly skewed distribution of the resulting land holdings, with a small number of participants who get very large parcels and a large number of participants who get very little or no land at all. If the titles to land are initially equally distributed, the result of a free market in land transfers tends to be a highly unequal equilibrium distribution of land.

- Examples include the distribution of land claims to the veterans of the American Revolutionary war, which rapidly concentrated in the hands of wealthy speculators, various land reforms which broke up large quasi-feudal estates in Latin American countries which often resulted in a re-consolidation of land ownership in the hands of a few, and the privatization of socialized assets after the collapse of communist regimes in Eastern Europe through voucher schemes, which also led to a rapid concentration of asset ownership through speculation.

Interventions and the commons dilemma

- In terms of the social interaction model, there are two abstract forms of intervention. Society can provide a subsidy (or tax), σ, for the action, changing the private return to $\alpha + \sigma$. Or the society could provide a stimulus, ξ, to add or subtract to the social average level of the action (for example, by putting some number of collectively owned cows on the common). The resulting equilibrium will be equal to the social coordination level if:

$$\frac{\alpha + \sigma + \beta\,\xi}{1 - \beta} = \frac{\alpha + \gamma}{1 - 2\,\beta} \text{ or}$$

$$\sigma = \frac{\alpha\,\beta + \gamma(1 - \beta)}{1 - 2\,\beta} - \beta\,\xi$$

- In the case of the commons, $\gamma = 0$, and it is not clear where the cows would come from to provide a stimulus, so we can take $\xi = 0$, too. The efficient subsidy is:

$$\sigma = \frac{\frac{1}{2}(M_C - M_W)\left(-\frac{1}{2}\right)(n - 1)}{1 + n - 1} = -\frac{1}{4}\,\frac{n - 1}{n}\,(M_C - M_W) \qquad (7)$$

- With this tax, the equilibrium level of cows per farmer will be equal to the socially coordinated level:

$$x^e[\sigma] = \left(\frac{1}{2} - \frac{1}{4}\,\frac{n-1}{n}\right)\frac{M_C - M_W}{1 + \frac{1}{2}(n-1)} = \frac{1}{2\,n}\left(M_C - M_W\right) \quad (8)$$

Rent

- Solving the commons problem with a subsidy or tax requires a large degree of centralization to collect information and implement the system. Could some simple system of privatization resolve the social coordination problem?
- Suppose that the commons is divided into n identical parcels, and each farmer (now a landlord) is given a parcel with the right to exclude cows from it.
- Let us also make the rather heroic assumption that the parcels are big enough to reflect the

same crowding effects as the whole commons.

- Each farmer-landlord has to decide how many cows, x, to allow on her parcel. In considering this question, the landlord knows that the price (in milk) grazers will pay to graze a cow is going to be the difference between the productivity of cows on her parcel and their productivity on the waste. If the landlord allows *x* cows to graze her parcel, the productivity of a cow grazing that parcel, assuming that productivity on the parcel declines proportionately to productivity on the whole commons, will be

$$y_C[x] = M_C - n\,x \tag{9}$$

- The rent per cow the farmer (as landlord) can charge will be:

$$t[x] = y_C[x] - M_W = M_C - n\,x - M_W \qquad (10)$$

- The rent revenue of the typical farmer-landlord will be

$$t[x]\,x = (y_C[x] - M_W)\,x = (M_C - M_W)\,x - n\,x^2 \qquad (11)$$

- This is again the social interaction model, with $\alpha' = \frac{1}{2\,n}(M_C - M_W)$, $\beta' = 0$, and $\gamma' = 0$ (so there is no social interaction). The equilibrium number of cows per parcel and the rent per cow will be:

$$x^e = \frac{\alpha'}{1 - \beta'} = \frac{1}{2\,n}(M_C - M_W) \qquad (12)$$

$$t^e = M_C - M_W - n\, x^e = \frac{1}{2}\,(M_C - M_W)$$

The marginal product of ancestors

- Following the hints of Eleanor Ostrom's work, we might also imagine a society where the commonly held belief is that deeply venerated deceased ancestors disapprove of over-grazing the commons.
- Belief holds that the ancestors punish households that put more than their fair share of cows on the commons with bad luck: for example, their crops are subject to mysterious fires or their children do not find favorable marriage partners.
- Such a society would avoid over-grazing the

commons without introducing rent and private property. The resulting surplus production would be realized equally by all the households in the form of higher milk production from the limited number of cows on the commons.

- This example raises the question of just what an input to production is and what a “marginal product” is. Is it the commons land that has a marginal product, or the social relations that govern its exploitation?

Strong strategic complementarity

- When $\beta > 1$, the interior equilibrium of the social interaction model is unstable in the sense that an increase (decrease) in the social action level leads to a larger than proportional increase (decrease) in the typical individual action level. This is the case of *strong strategic complementarity*.
- With strong strategic complementarity the social interaction model makes economic sense only if we assume limits on the typical agent's action, $x_{lo} \leq x \leq x_{hi}$.
- In this case there is always at least one social

interaction equilibrium at one of the action limits, and there can be three social interaction equilibria, two stable equilibria at each of the action limits and one unstable interior equilibrium.

- When there are three equilibria, the unstable interior equilibrium is the boundary of the *basins of attraction* between two stable equilibria at the constraints.
- In this case we can predict with considerable confidence that the system will be at one of the stable equilibria. (Unstable equilibria are knife-edge cases that we never expect to see.) But which stable equilibrium depends on the initial

level of the action, so the system exhibits *path-dependency*.

Social coordination with strong strategic complementarity

- When there is strong strategic complementarity, $\beta > 1 > \frac{1}{2}$, and the social coordination outcome always lies at one of the limit actions.
- When there is a single stable equilibrium it may or may not coincide with the social coordination outcome.
- When there are two stable equilibria, one of them does coincide with the social coordination outcome, and the other is worse for the typical agent.

- Many of the most interesting problems of social theory can be analyzed as cases of strong strategic complementarity.

Assurance games and network externalities

- The social interaction model with strong strategic complementarity is often called an *Assurance Game*. One of the two stable extreme equilibria is also the social coordination action level, and the other represents a “trap” where social coordination fails.

- Strong strategic complementarity is also sometimes referred to as a *network externality*, where the choice of a network or standard by one agent increases the incentives for other agents to join the same network or adopt the same standard.

- Network externalities are at the heart of many modern technological monopolies where “winner-take-all” dynamics prevail, such as social networking sites, computer operating systems and the like.

Actions as probabilities: the quantal response model

- A particularly useful application of the social interaction model is to situations where the typical agent adopts a mixed strategy. In this case the action is the probability, P, of doing something, rather than the level, and the average action of the rest of the system can be thought of as the proportion of the other agents adopting the given action, $\overline{P}$.
- This interpretation implies action limits of $P_{\text{lo}} = 0$, $P_{\text{hi}} = 1$, since the probability of doing

something cannot be negative or greater than 1.

- This interpretation underlies the *quantal response* models widely used in econometric investigations.
- The interior equilibrium of the social interaction model will be a *polymorphic state*, where some proportion P^e, $0 < P^e < 1$ of the population chooses the action, and the extreme equilibria will be *monomorphic states*, where all the agents uniformly either choose the action or avoid it.

Equilibrium in the quantal response social interaction model

- Suppose now the typical agent has to choose either to take an action $x = 1$, or not take it, $x = 0$. Suppose the payoff depends on whether or not the other participants take the action, $z = 0, 1$, according to:

$$\pi[x, z] = (\alpha + \beta z)\, x \qquad (13)$$

- This gives the payoff matrix for the interactive game:

$$\begin{pmatrix} x\downarrow z\rightarrow & 0 & 1 \\ 0 & 0,0 & 0,\alpha \\ 1 & \alpha,0 & \alpha+\beta,\alpha+\beta \end{pmatrix}$$

- If the probability of others taking the action is $\overline{P}$, the expected payoffs to the individual agent taking the action with probability P is:

$$\pi[P, \overline{P}] = (\alpha + \beta \overline{P}) P \tag{15}$$

- In this case we have the limits $P_{\text{lo}} = 0$, $P_{\text{hi}} = 1$.

- The best response is

$$P[\overline{P}] = \begin{cases} 1 & \alpha + \beta \overline{P} > 0 \\ 0 & \alpha + \beta \overline{P} < 0 \end{cases} \tag{16}$$

- If $0 < -\frac{\alpha}{\beta} < 1$, there is an interior equilibrium at

$P^e = -\frac{\alpha}{\beta}$. When $\beta < 0$, the system exhibits strategic substitutability and the interior equilibrium is stable and the only equilibrium; when $\beta > 0$, the system exhibits strategic complementarity, the interior equilibrium is unstable, and is the boundary of the basin of attraction of two stable equilibria at $P^e = 0, 1$.

- If $-\frac{\alpha}{\beta} < 0$, there is a unique stable equilibrium at $P^e = 0$, and if $-\frac{\alpha}{\beta} > 1$, there is a unique stable equilibrium at $P^e = 1$.

Social coordination in the quantal response social interaction model

- The social coordination outcome in the quantal response social interaction model solves the maximum problem:

$$\underset{0 \le P \le 1}{\text{Max}} (\alpha + \beta P) P = (\alpha + \gamma) P + \beta P^2 \tag{17}$$

- If $\beta < 0$, the interaction exhibits strategic substitutability, and when $0 < \alpha + \gamma < -2\beta$, the social coordination outcome is at the interior critical point, $P^{sc} = -\frac{\alpha+\gamma}{2\beta}$. If $\alpha + \gamma < 0$, $P^{sc} = 0$, and if $\alpha + \gamma > -2\beta$, $P^{sc} = 1$.
- If $\beta > 0$ the interaction exhibits strategic

complementarity, and the social coordination outcome is at the extreme point with the higher payoff. Since $P = \overline{P} = 0$ gives the typical individual a payoff of 0, and $P = \overline{P} = 1$ gives the typical individual a payoff of $\alpha + \gamma + \beta$, $P^{sc} = 0$ when $\alpha + \gamma + \beta < 0$, and $P^{sc} = 1$ when $\alpha + \gamma + \beta < 0$ in the case of strategic complementarity.

Example: the Hawk-Dove game

- Each player chooses to be a Hawk (with probability P) or a Dove (with probability $1 - P$).
- The payoff matrix for interactions of Hawks and Doves is

$$\begin{pmatrix} \text{Row}\downarrow\text{Col}\rightarrow & \text{Hawk} & \text{Dove} \\ \text{Hawk} & -20, -20 & 100, 0 \\ \text{Dove} & 0, 100 & 50, 50 \end{pmatrix} \qquad (18)$$

- There is a prize with total payoff 100. When a Dove meets a Dove they divide the prize; when a Hawk meets a Dove the Hawk gets the whole prize; when a Hawk meets a Hawk they fight and wind up doing damage to each other and destroying the prize.

- If $\overline{P}$ is the proportion of the population who play Hawk (or the probability of any one agent playing Hawk) and therefore the probability that the other participant in an interaction will be a Hawk, the expected payoffs to playing Hawk and Dove are:

$$\begin{aligned} \pi_{\text{Hawk}}[\overline{P}] &= -20\,\overline{P} + 100\,(1-\overline{P}) = 100 - 120\,\overline{P} \\ \pi_{\text{Dove}}[\overline{P}] &= 0\,\overline{P} + 50\,(1-\overline{P}) = 50 - 50\,\overline{P} \end{aligned} \tag{19}$$

- The expected payoff to playing Hawk with probability P is:

$$\pi\left[P, \overline{P}\right] = P\left(100 - 120\,\overline{P}\right) + (1 - P)\left(50 - 50\,\overline{P}\right) = P\left(50 - 70\,\overline{P}\right) + 50 \tag{20}$$

- If $0 \leq \overline{P} < \frac{5}{7}$, $\pi_{\text{Hawk}} > \pi_{\text{Dove}}$, and if $\frac{5}{7} < \overline{P} \leq 1$, $\pi_{\text{Hawk}} < \pi_{\text{Dove}}$. The best response of an risk-neutral agent maximizing expected payoff is

$$\begin{cases} P = 1 & 0 \leq \overline{P} < \frac{5}{7} \\ P = 0 & \frac{5}{7} < \overline{P} \leq 1 \end{cases} \tag{21}$$

- The Hawk-Dove game exhibits *strong strategic substitutability*: the best response function is a downward step function at $\overline{P} = \frac{5}{7}$; $\beta = -70$. There is a unique interior polymorphic equilibrium at $P^e = \frac{5}{7}$; in $\frac{5}{7}$ of the interactions the typical agent acts as a Hawk (or equivalently $\frac{5}{7}$ of the agents always act as Hawks) and in $\frac{2}{7}$ of the interactions the typical agent acts as a Dove.

Social conventions

- With strong strategic complementarity, the social interaction model is a simple explanation of *social conventions*.
- In this interpretation the action of the typical agent is the probability of acting in accord with some social convention, such as working in a particular scientific paradigm, or conforming to some social expectations based on racial differences, or establishing an account with Facebook, or buying a Windows computer.
- The strategic complementarity in these situations

arises because the larger the proportion of the members of society who adopt the convention, the stronger are the incentives for the typical member of the society to adopt it too.

- The best response of the typical member of society in this case is an upward step function that shifts from $P = 0$ to $P = 1$ at a certain *tipping point*.

- The interior polymorphic equilibrium in the case of strong strategic complementarity is unstable and separates the basins of attraction of the two monomorphic extreme equilibria, at one of which the social convention is universally adopted, and at the other of which the social convention is universally rejected.

Example: computer operating system

- Suppose $\overline{P}$ is the proportion of computer users who have a Windows operating system, and the alternative, Linux or Apple, requires somewhat different knowledge of interfaces and occasionally incompatible file transfer methods and standards.
- The payoff matrix for interactions of computer users might look like:

$$\begin{pmatrix} \text{Row}\downarrow\text{Col}\rightarrow & \text{Windows} & \text{Linux} \\ \text{Windows} & 10,10 & 5,5 \\ \text{Linux} & 5,5 & 10,10 \end{pmatrix} \tag{22}$$

- When a Windows user exchanges files with another Windows user, or a Linux user with another Linux user, they both get a payoff of 10. When a Windows and Linux user exchange files, the occasional incompatibilities reduce the payoff to 5.

- If $\overline{P}$ is the proportion of the users who adopt Windows and therefore the probability that the other participant in an interaction will be a Windows user, the expected payoffs to being a Windows or Linux user are:

$$\begin{aligned}\pi_{\text{Windows}}[\overline{P}] &= 10\,\overline{P} + 5\left(1 - \overline{P}\right) = 5 + 5\,\overline{P} \\ \pi_{\text{Linux}}[\overline{P}] &= 5\,\overline{P} + 10\left(1 - \overline{P}\right) = 10 - 5\,\overline{P}\end{aligned} \tag{23}$$

- If $0 \leq \overline{P} < \frac{1}{2}$, $\pi_{\text{Windows}} < \pi_{\text{Linux}}$, and if $\frac{1}{2} < \overline{P} \leq 1$,

$\pi_{\text{Windows}} > \pi_{\text{Linux}}$. The best response of a risk-neutral agent maximizing expected payoff is

$$\begin{cases} P = 0 & 0 \leq \overline{P} < \frac{1}{2} \\ P = 1 & \frac{1}{2} < \overline{P} \leq 1 \end{cases} \tag{24}$$

- The computer user interaction exhibits *strong strategic complementarity*: the best response function is an upward step function at $\overline{P} = \frac{1}{2}$; $\beta = \infty$. There are two extreme stable monomorphic equilibria at $P = 0, 1$; the system will become uniformly Windows or uniformly Linux but which system will dominate depends on path-dependent accidents that break the symmetry of the $\overline{P} = \frac{1}{2}$ starting point.

Explanatory power in the social interaction model

- The various scenarios of the social interaction model represent different types of explanation with different explanatory power.
- With weak social interaction, $\beta < 1$, the unique equilibrium changes with intensity of the social interaction and the individual incentives, α. Theory itself can predict that a polymorphic equilibrium will emerge which is different from the social coordination outcome (unless by a fluke $\beta = 0$).

- This equilibrium outcome is essentially independent of initial conditions and contingent events in the path to equilibrium, and depends only on the individual incentives and the strength of the social interaction.
- But the theory has little to say about the actual configuration of the outcome, except to point the researcher to finding out just what evidence there is on the levels of individual incentives and the intensity of the social interaction effect
- With strong social interaction, $\beta > 1$, the theory makes the strong qualitative prediction that one or the other extreme monomorphic outcome will prevail, regardless of the details of the size of the

individual incentives and the intensity of the social interaction as long as strong strategic complementarity prevails.

- The theory, however, is uninformative as to which extreme outcome will prevail, because of path-dependency, so the actual outcome can be highly sensitive to the specific initial conditions.

The individual and society

- The social interaction model represents the interaction of the *individual* and *society* at an abstract level in dialectical terms.
- What characterizes the individual from this point of view is not her difference from other individuals, since all individuals are identical in relevant aspects.
- What characterizes the individual is the separation of her individual sphere of action from the social sphere of action; what she takes as given and beyond her control as opposed to

what she sees as subject to her will.

- The irony of the social interaction model lies in its representation of the behavioral outcome for the individual as being determined by her interaction with society.
- Despite the fact that all the individuals are identical and share the same interests at some level, social interaction condemns them to equilibrium outcomes none of them would necessarily have chosen.
- It is as futile for the individual to try to change the social outcome by altering her individual behavior as it would be for her to try to alter physical laws of nature.
- Nonetheless, the equilibrium outcome is the

reflection of the typical individual's behavior, the results of social, not natural constraints. To the degree that the individual can transcend her situation and induce others to coordinate their actions with her, she can change the outcome.

- These observations underline the philosophy of the political as the thrust of the social interaction model. Social change flows from coordinated individual action based on a shared understanding of the institutions structuring the definition of the individual in society.

The village of clean-shaven men

- Philosophical issues like *causation* and *explanation* can be problematic in social science.
- The parable of the village of clean-shaven men can throw some light on these questions.
- Most of the time, most of the men in the village appear clean shaven. Each man has a preference to look the same as the majority of the others.
- The direct cause of my being clean-shaven is that I shaved myself this morning. The direct cause of your being clean-shaven is that you

shaved yourself this morning.

- But the reason I shaved myself was partly my (as it turned out, correct) prediction that you would shave yourself, and the reason you shaved yourself was partly your (as it turned out, correct) prediction that I would shave myself.
- Shaving is “behavior” but in this case (and innumerable similar ones) the behavior of individuals cannot be explained without reference to their social context.

Dynamics in the social interaction model

- While it is intuitively plausible to regard an equilibrium of the social interaction model as "stable" when the typical agent best response cuts the 45° line from above and unstable when the intersection is from below, there is a degree of handwaving involved.
- The problem is that in order to study the stability of a dynamical system rigorously, we have to specify its non-equilibrium dynamics explicitly and use the tools of dynamical system analysis to determine stability.
- It would be possible to specify a variety of

dynamic models for any of the scenarios we have discussed, but there are some considerations that recommend against making this the primary path of a research program.

- Dynamic detail is hugely informative when information is available to pin down the exact parameters determining time constants in dynamic processes. But this type of information is often hard to come by in economic contexts where information is scarce and noisy.
- To present the model in an explicitly dynamic form risks misleading a reader into thinking that the specific example is the theory, whereas in fact there is a large range of models that have

similar qualitative properties.

The cusp catastrophe

- A good example of the usefulness of broad qualitative analysis is the possibility of the *cusp catastrophe* in the strong strategic complementarity social interaction models.
- A little experimentation with the interactive figure shows that a stable extreme equilibrium of the social interaction model may be destroyed by a shift in individual incentives, α, or in the external environmental stimulus, ξ.
- In this case the qualitative behavior of the system is to collapse to the remaining stable

equilibrium.

- The exact dynamics by which this occurs may be quite difficult to untangle from data, or to model explicitly.
- In this catastrophe it is likely that the dispersion of individual action levels will temporarily explode. As a result the state space for an explicit model will become very large and the analysis of its dynamics can become impossibly complex.
- In thermodynamics such movements from one equilibrium to another are viewed as *irreversible*. The study of irreversible changes in thermodynamic systems involves very different

concepts and tools from the study of equilibria or reversible movements of the system due to slow “adiabatic” parameter changes.

Phase transition in the social interaction model

- A particularly interesting situation arises in the social interaction model when $\beta = 1$, or is close to 1.
- In this case the best response of the typical individual can coincide with the 45° equilibrium line. Any of a wide range of action levels in this case are equilibria.
- In physical models, this kind of boundary situation is often called a *phase transition*. In the

case of the social interaction model the phase transition occurs at the boundary between weak and strong strategic complementarity.

- At the bifurcation point $\beta = 1$, the system "does not know where to go". In physical models phase transitions are characterized by *scale-free* fluctuations. Because there are no stabilizing forces present, fluctuations can develop at any scale.
- A similar situation can arise in the cusp catastrophe. Near the catastrophe point the unstable and stable equilibria are close together, so the stabilizing forces become weak, and the system may exhibit a high degree of volatility.
- If for some reason the system stabilizes instead

of undergoing the catastrophe, for example if the individual incentive, α, retreats, then the volatility will calm down without the catastrophe occurring.

- It seems likely that macroeconomic time series reflect these highly non-linear phenomena, and that this is an important reason why linear methods fail to identify the important dynamic features of macroeconomic instability.

The social interaction model in higher dimensions

- We have studied the social interaction model when the individual action is confined to one dimension.
- In this case it is relatively straightforward to visualize the equilibrium and quasi-dynamics of the social equilibrium in terms of the best-response curve and the equilibrium condition represented by the 45° line.
- The logic of the social interaction model is not

confined, however, to the one-dimensional case. The action of the typical individual could be a vector with $n > 1$ components.

- In higher dimensions the typical agent’s best response will be a mapping $x : R^n \to R^n$, representing the best response of the agent to the average behavior of the other members of society.
- Equilibria will continue to be appropriately defined fixed points of this mapping. The stability of equilibria will depend on the out-of-equilibrium dynamics of individual behavior.
- As we know, dynamics in higher dimensions can lead to a much richer variety of equilibria than in

one dimension, including limit cycles and strange attractors.