Lecture 6
Social coordination
problems in classical political economy
Classical political economy

- The ideas of entropy-constrained behavior and social interaction can open up new perspectives in our reading of the classical political economists, particularly Smith, and Marx.

- Entropy constrained behavior generates endogenous fluctuations and can offer a mathematical representation of classical notions such as the fluctuations of market prices around natural prices.

- The social interaction model can address the
central problem of the classical political economists, how an economy spontaneously organized through commodity exchange can coordinate production.
The typical producer

- The central figure in Smith’s initial discussion of the division of labor is an *independent producer* who is free, at least over some time horizon, to choose a particular line of production.

- Smith does not elaborate much on the exact nature of the producer. We might think of the producer as an individual in an “early, rude” hunter-gatherer state of society shifting from hunting deer to hunting beaver.

- Initially the typical producer either owns or
creates her own means of production as part of the production process.

Smith argues that the mobility of producers among different lines of production will equalize the “advantages and disadvantages” of each line of production.

The “disadvantages” of a line of production include the effort and unpleasantness of the production tasks themselves, and the effort and time required to learn whatever skills are necessary.

The “advantages” of a line of production in a commodity producing economy where products exchange through money are primarily the
money income the production affords the producer.

- The equalization of advantages and disadvantages can be thought of as a general version of a *labor theory of value* in which the disadvantages of a line of production are summarized as the labor effort required to produce a commodity.
Capitalist firms

- Looking ahead, in the later development of this line of thinking in Marx’s writings, the individual producer will become a capitalist firm. The capitalist firm, as Marx emphasizes, rests on more complex social relations of production, particularly the fact that labor takes the form of wage-labor.

- But many of the features of commodity production with independent producers apply to economies with capitalist firms that are free to move from one line of production to another.

- In fact, many of the features of commodity
production would apply to economies with enterprises that were organized as producer cooperatives, or to worker households that supply particular types of labor-power to the market.

For the moment, however, let us focus our thinking on the abstract case of an economy made of Smith’s independent producers.
The hub-and-spoke model

- To fix ideas on a simple case, suppose we have an economy made up of a (large) number, $N$, of identical independent producers.

- These producers require $K$ different products as inputs to production and as consumption.

- To make things as simple as possible, assume that the producers require the products in strict proportions (the products are, in the terminology of modern economics, *perfect complements*.) We can choose the units in which we measure
the quantities of products so that the ratios in which the products are required are all 1 : 1. The payoff to a producer who winds up with a bundle \( \{x_1, \ldots, x_n\} \) is the \textit{Leontief utility}:

\[
\text{Min}\{x_1, \ldots, x_K\} = \min\{x_1, \ldots, x_K\}
\]

Each of these products is a \textit{good}, or in Smith’s terms, a \textit{use-value}.

All of the independent producers have access to the same technology of production, which consists of two alternatives.

- The producer can \textit{diversify} and produce all of the products herself, in which case she produces the output bundle \( \{1, 1, \ldots, 1\} \).
- The producer can \textit{specialize} in the production
of one good, \( k = 1, \ldots, K \). Specialization requires effort, which the typical individual views as equivalent to \( \sigma \) times the payoff from diversification (1 unit). If she specializes she produces \( A >> (1 + \sigma)K \) (that is, \( A \) is significantly larger than \( (1 + \sigma)K \) units of that good.

- The “hub” of the model is diversified production, which we can think of as the center of a wheel, and the “spokes” are the \( K \) different lines of production.

- This specification takes the quantity of each product required to meet the typical producer’s needs as the unit for measuring
that product. The assumption that productivity is the same for each spoke implicitly assumes that it requires the same amount of producer effort to produce a unit of each product. If this is not the case, we could allow for differential productivities by assigning the same use-value to more than one spoke.
Social coordination in the hub-and-spoke model

The social coordination problem in the hub-and-spoke model arises because if K producers specialize and share their outputs equally, they will each receive the bundle
\[ \left\{ \frac{A}{K}, \frac{A}{K}, \ldots, \frac{A}{K} \right\} = \frac{A}{K} \{1, 1, \ldots, 1\}, \] and a payoff \[ \frac{A}{K} \sigma. \] Given the assumption that \( A \gg (1 + \sigma) K \), or \( \frac{A}{K} \gg (1 + \sigma) \), the division of labor results in a significantly higher payoff from the bundle of products for each producer allowing for the
effort cost of specialization than diversified production.

- Social coordination in this abstract setting is transparent: it would be advantageous to each independent producer to engage in specialized production and share the resulting collective product. The social coordination outcome locates the producers at the spokes.

- Given the indivisibilities of this production model, strictly speaking we need to have $N$ be a multiple of $K$, $N = nK$, so that the producers can divide up into sets of complete teams of $n$ producers to accomplish the division of labor.

- The payoff to the typical producer in the social
coordination outcome will be $\frac{A}{K} - \sigma > 1$, which is higher than the payoff of 1 she can achieve by diversifying.

- There are many equivalent socially coordinated outcomes depending on which spoke each producer is assigned to, but since we regard all the producers as identical, these different outcomes are economically equivalent.

- The social coordination outcome might be achievable in small communities through direct negotiation and enforcement by community norms.

- Diversification in this scenario has some similarities to what conventional economic
analysis calls “leisure”, the withdrawal of labor from the specialized division of labor to household or domestic production activities.
Visualizing the hub-and-spoke model
The social interaction problem in the hub-and-spoke model

- Suppose we apply the social interaction model to the hub-and-spoke system. The typical individual producer can be thought of as deciding whether to specialize as part of a team or to diversify and be self-sufficient. The typical producer chooses a frequency $P$ of choosing specialization, conditional on the proportion of other producers, $\overline{P}$, who are specializing, with a
frequency $1 - P$ of diversifying.

- The division of labor has the effect of dividing the typical producer into $K$ separate individuals, which poses a social coordination problem. If one individual could choose the behavior of all (as in Robinson Crusoe scenarios), she would choose the socially coordinated outcome, but if she has to choose her own action individually, she may find herself at an equilibrium far from the socially coordinated outcome.

- Can the socially coordinated division of labor be sustained as the equilibrium of a social interaction model?

- How this will work depends crucially on the
specific institutions and conventions that structure distribution under specialization.
Spontaneous socialism through sharing

- The simplest scheme of specialization is a two-part \textit{spontaneous socialism}.

- The first part of the spontaneous socialist system instructs the producers to line up in some random order and then each go in turn to find the subset of lines of production (spokes) already occupied by the smallest number of producers, choose one from this subset at random with equal probabilities, and specialize in its production.

- The first producer to choose will find all the
spokes empty, and successive producers will fill up the remaining spokes until there is one producer at each spoke; then a new round of spoke-choice will begin. The assumption that \( N = nK \) assures that every producer will find a seat in this game of musical chairs, and that every spoke will eventually be occupied by the same number of producers.

- The second part instructs each producer to specialize and deliver her output unconditionally in equal proportions to all \( N \) producers, including herself.

- Note that this is not exchange, because each producer shares her product unconditionally with
all the others, while exchange conditions the delivery of part of the product on a quid pro quo consisting of reciprocal delivery of the other producer’s product.

The spontaneous production system aims at the social coordination outcome in the following sense. $n = \frac{N}{K}$ producers will specialize in each line of production, producing $A \frac{N}{K}$ units of each product, so that on average the typical producer will receive $\frac{A}{K}$ units of each product, giving an expected payoff of $\frac{A}{K} - \sigma$, accounting for the effort cost of specialization. If the spontaneous production system could be instituted, it would solve the social coordination problem.
Spontaneous socialism might also be thought of as a kind of spontaneous self-organization, in which every producer acts “rationally” in the interests of the whole society.
Best response and free-riding in spontaneous organization

- The spontaneous production system, however, is vulnerable to free-riding, unless the productivity of specialized labor is very high. Assume that \( N - 1 \) of the producers do follow both parts of the instructions. Would the remaining agent’s best response be to carry out the conform to the socialist behavior?

- Not necessarily, because the agent could do better to diversify, and collect her share of the rest of the agents' specialized product without
paying the effort cost of specialization, if specialized productivity is not too high.

- The typical producer’s payoff to following the spontaneous socialist action is $\frac{A}{K} - \sigma$, allowing for the specialization cost.

- The typical producer’s payoff to diversifying when everyone else specializes and shares (assuming that she would have produced in the first sector if she had specialized) is:

$$\min \left[ \left\{ \frac{A}{K} \frac{n - 1}{n} + 1, \ldots, \frac{A}{K} + 1 \right\} \right] =$$

$$\frac{A}{K} + 1 - \frac{A}{nK} > \frac{A}{K} - \sigma \text{ if and only if}$$

$$\text{(2)}$$
(1 + \sigma) N > A

- Because the typical producer does not produce her share of the first product, there are only 
  \( \frac{A}{K} \frac{n-1}{n} \) units of the first product per producer, as opposed to \( \frac{A}{K} \) units of all the other products. The typical agent supplements this with her diversified production, of one unit of her product. Her payoff is determined by her consumption of the good she produces, \( \frac{A}{K} + 1 - \frac{A}{nK} \), which is greater than her payoff following the spontaneous socialist action if and only if 
  (1 + \sigma) N > A, remembering that \( nK = N \).

- While we have assumed \( A > (1 + \sigma) K \), if \( n > 1 \), it
is possible to have \((1 + \sigma) K < A < (1 + \sigma) N\). In this case the typical producer's best response to the spontaneous socialist action of other producers is to drop out of the socialist specialized division of labor and diversify.

- Spontaneous socialism is a best response to itself, and therefore a sustainable Nash equilibrium if \(A > (1 + \sigma) N\).

- When spontaneous socialism is a Nash equilibrium, produced goods become freely available.

- The assumption of strict complementarity among products in the typical producer’s payoff function is favorable to the spontaneous socialist
equilibrium. If producers could substitute other products for their own, they would be more likely to defect to diversification, or to specialize but refuse to share their product.

With strict complementarity the use-value of the producer’s own product to her is limited by the amount of other products she receives through sharing.
Implications of the spontaneous socialist analysis

- We can read the condition for the viability of spontaneous socialist organization, \( A > (1 + \sigma) N \), in two ways.

- For a given productivity of specialization, \( A \), the condition tells us that spontaneous socialism is a self-reproducing equilibrium if \( N \) is not too big, that is, if the community which is organizing itself is relatively small. We would expect to see spontaneous socialist arrangements in activities like baby-sitting collectives or barn-raisings in local neighborhoods, where the productivity incentive to cooperation is high and the number
of participants is limited.

- For a given community size, $N$, the condition tells us that spontaneous socialism is possible only when specialized production has an advantage over diversification on the order of the size of the population itself.

- One strand of the socialist tradition, which we find in some of Marx’s writings, is the idea that socialism can be achieved through very large increases in productivity. In this view, “scarcity” is a social, not a physical fact, and when productivity is high enough, products will no longer be scarce, but freely available, as they are in the spontaneous socialist scenario when it is a
Nash equilibrium.
Methodology of the social interaction model

- This example illustrates the methodology of the social interaction model.

- In order to test whether some feasible symmetrical social behavior is sustainable as a social action equilibrium, we have to determine whether the behavior is a best response to itself.

- If the behavior is a best response to itself, it is an equilibrium, and we can go on to consider whether it is stable.
Exchange in the hub-and-spoke model

- Adam Smith envisions a system in which the division of labor is sustained by private property and quid pro quo exchange.

- Each producer is free to choose her own line of specialized production or to diversify and remain self-sufficient.

- In contrast to the sharing model, however, each producer bargains with other producers to exchange a part of her product with them.

- In this system, Smith observes, there is a
distinction between the *use-value* of a product and its *exchange value*.

- The products have the same use-value whether they are produced with the diversified technology or through specialization.

- In an economy based on a specialized division of labor, the product is not a use-value to its producer (she has too much of it) but an exchange value. The producer meets her needs by exchanging her specialized product for the goods she needs produced by other producers, for whom her product is a use-value.
Marx and the theory of the commodity

- Marx called Smith’s system *commodity production*, and analyzed the commodity as a unity of use-value and exchange value.

- Marx views commodity production as *irrational*.

- The socially coordinated specialized division of labor seems rational enough. It is the outcome a typical individual producer would choose if she could choose for the system as a whole.

- Presumably Marx regards the exchange of products as commodities as irrational because it fails to achieve the social coordination solution in
a specialized division of labor.
Smith’s “early and rude state”

- Exchange of products as commodities separates the typical individual producer further into a seller of her specialized product and a buyer of all the other products she needs.

- In these roles the typical producer has no interaction with other specialized producers of the same good, that is, the others located at the same spoke.

- All the producers located at other spokes are essentially identical.
Barter

- The simplest behavior to consider to achieve the advantages of the specialized division of labor through the exchange of products as commodities is *barter*.

- In the barter system the typical producer starts with her own output of her specialized commodity and no other products.

- She somehow interacts with other producers through *transactions*.

- When she meets another producer whose product she does not have and who does not
have her product, a *double coincidence of wants*, they exchange $\frac{1}{K}$ of their products with each other.

- Two producers cannot transact under the rules of the barter system if either one already holds the product of the other. As a result as transactions take place it becomes less and less likely that two producers will be able to transact.

- The result of barter is that each producer will hold an equal share of all the produced commodities, as in the social coordination outcome of the hub-and-spoke model.

- If, however, transactions are costly, say in effort, to the typical producer, the average payoff from
commodity exchange through barter will be smaller than at the social coordination outcome.
Transaction costs and barter

- It can take many transactions to complete a round of barter when there are several distinct commodities being traded.

- On the first round of interactions it is almost certain (assuming that agents can avoid interacting uselessly with producers at the same spoke) that the typical agent will find a successful transaction, because no producer has yet transacted and everyone wants every product except their own.

- The exact computation of the number of rounds
of transactions required to achieve complete distribution in a barter system is a technically challenging and tedious task.
Money

- The analytical details of barter in the hub-and-spoke model are, however, largely irrelevant to political economy because, as both Adam Smith and Karl Marx explain, barter exchange is unstable.

- Suppose that one producer decides to accept either commodities she needs or some particular commodity, say the first, whether she needs it or not in exchange for her specialized product.

- This slight change in the barter system will probably make little difference to an isolated
individual producer. She will find it slightly easier to sell her product because she can always exchange with the producers of the first commodity. In the process of exchange she may accumulate holdings of the first commodity that she does not need, but given the structure of the exchange situation, sooner or later her holdings would converge to the balanced social coordination outcome bundle.

But the more individual producers decide to accept the first commodity in excess of their needs in exchange for their own product, the higher the payoff to any other producer of making the same decision. The reason is that if many producers accept the first commodity in
exchange for their product, then a producer who does the same is much more likely to find a successful transaction involving the first commodity.

In the terminology of the social interaction model there is a strong strategic complementarity associated with the strategy of accepting one particular commodity in exchange for a producer’s specialized commodity even if she already has more of that commodity than she needs.
The instability of barter and the stability of monetary equilibria

- If specialized producers in the hub-and-spoke model are choosing between specialization and exchange through barter and specialization and exchange through barter and the acceptance of one particular commodity in excess of their needs, strategic complementarity will make the barter equilibrium unstable.

- Without taking into account any further factors (which would mean descending to a lower level of abstraction and a higher degree of
concreteness) we can confidently conclude that the hub-and-spoke type of society will find an equilibrium in which all producers accept one commodity in excess of their needs in exchange for their specialized commodity. This commodity, whichever it turns out to be, is the *money commodity* in the economy.

- Marx describes the money commodity as the *socially accepted general equivalent commodity*.

- At this level of abstraction there is no reason to suppose that any one commodity will be more likely to emerge as the money commodity than any other. This is an example of the path
dependency characteristic of social interactions with strategic complementarity.
Separation of sale and purchase in monetary economies

- The emergence of money does not necessarily suppress barter altogether, though it tends to relegate barter transactions to a minor part of economic production.

- Once money is widely used, the typical producer undergoes another split, now into seller and buyer, corresponding to the separation imposed by money between sale of the typical producer’s own product and purchase of other commodities from other producers.

- Each of these splits opens up the possibility of
social coordination problems, due to the fact, for example, that the typical producer as seller cannot directly coordinate with other producers as buyers.

Marx expresses this by saying that the separation of sale and purchase introduces the “possibility of crisis”.

From this point of view the principle Keynes made famous as “Say’s Law” amounts to the assumption that some mechanism exists to overcome the social coordination problem inherent in the separation of sale and purchase in monetary economies.
Concrete factors in the emergence of a money commodity

- In the real world a variety of concrete factors influence which commodity emerges as money in a specialized division of labor.

- One set of factors have to do with the properties of particular products, such as durability, verifiability, and the like, which textbooks on money argue make precious metals such as copper, silver, and gold particularly well-suited to be money commodities.

- Just which commodity emerges as money in a
particular economy can also depend on socially coordinated decisions such as the policy of the state. If the king (or legislature) designates a particular commodity as money, the producers who make up the public are likely to go along.

- There are a few curious historical cases where sectors of economies stubbornly use a different money, at least for local transactions, from the politically designated money. This is somewhat parallel to the survival of local language groups.

- Along with the emergence of money we almost always see the development of various kinds of credit based on deferred payment. In some
historical situations commodity money hardly ever actually appears in transactions, which are almost universally conducted in terms of tally-sticks or credit contracts. Typically, however, it is understood that credit balances that do not clear against each other (which may be quite small in closed systems) are in principle payable in the money commodity.
Non-commodity money

- There is no theoretical reason why money necessarily has to be a produced commodity. The main requirements for an asset or liability to function as money are that it be exchangeable against produced commodities, that it be hard to counterfeit (so that individual producers cannot cheaply create their own money), and that it be generally accepted.

- In the twentieth century in almost all advanced capitalist countries the debt of the state came to be used as money. The acceptability of liquid state debts such as reserve accounts of central
banks is strongly supported by the fact that they can be used to settle tax liabilities.

- It is something of a misnomer to call state-debt money “fiat” money, since “fiat” (which means “let it be”, as in the phrase “fiat lux”, “let there be light”) suggests that the money asset is widely acceptable simply because the state says so. There are many counterexamples, going back to the “assignats” of the French revolution, that illustrate the limits of state intervention in imposing monetary standards on economies.

- In modern state-debt money systems, the money asset that is socially accepted is generally the reserve accounts of the central
bank, or close substitutes such as central bank notes or bank deposit accounts. The value of the reserve account (the “dollar”, for example) is indirectly regulated through the macroeconomic impact of monetary policy on the price level.

- An interesting account of the origins of this system in the breakdown of the gold standard after the First World War is in Liaqat Ahamed’s book *Lords of Finance*, which explains the process by which Benjamin Strong, then president of the Federal Reserve Bank of New York discovered the rudiments of inflation targeted monetary policy.
Smith on the equalization of returns

- The whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighbourhood, be either perfectly equal or continually tending to equality. If in the same neighbourhood, there was any employment evidently either more or less advantageous than the rest, so many people would crowd into it in the one case, and so many would desert it in the other, that its advantages would soon return to the level of other employments. This at least would be the case in
a society where things were left to follow their natural course, where there was perfect liberty, and where every man was perfectly free both to choose what occupation he thought proper, and to change it as often as he thought proper. Every man’s interest would prompt him to seek the advantageous, and to shun the disadvantageous employment. (Smith, 1937, Book I, ch 10)
The labor theory of value

- In the real world when a producer chooses to specialize in some particular line of production (spoke of the production wheel) she incurs effort costs, perhaps over her life-cycle, in acquiring the skills, tools, knowledge, access to inputs, and in production itself. These are the “disadvantages” of specialization in a line of production, which are frequently lumped together as labor effort or labor.

- If producers are free to choose which product to specialize in, Smith argues that they will
gravitate, given the assumption that all the spokes involve equal effort, to the product that has the highest money price. This movement will tend to equalize the ratio of money price of different commodities to this broad concept of labor effort.

■ In order to achieve an equilibrium in which all of the products are produced in the proportions necessary to meet social needs, the money prices of the commodities will have to be proportional to labor effort.

■ This is the substantive core of the labor theory of value, which is a theory both of the money prices of commodities and of the allocation of
producers in the specialized division of labor.
The separation of sale and purchase with money

- In order to apply the social interaction model to the hub-and-spoke model with money-mediated exchange, suppose that each producer of a non-money commodity can set a money price, $p$, at which she will sell her specialized produced commodity for money.

- Each producer also must choose a frequency distribution, $\{f_1, \ldots, f_K\}$, describing the frequency with which she chooses each line of production.

- According to the principle of the labor theory of value, the typical producer will be indifferent
among the lines of production so long as their payoffs are equal.

- The typical non-money producer and the money commodity producers then decide how much of each of the other commodities they will purchase.

- In terms of the social interaction model, the average action of the other producers is \( \overline{p} \), the price at which they are selling their specialized commodities, which is also the price the typical producer will pay in money for the commodities she does not produce.

- Before we tackle the question of the determination of price, it is helpful to understand how this economy would operate given a
uniform price, $\bar{p}$. 
Commodity exchange at a given money price--money producers

If all non-money producers offered their commodity at the money price $\overline{p}$, each money producer would maximize her payoff by buying the quantity $x_m[\overline{p}]$ that equalizes the amount of the money commodity she retains with the amounts of the other $K - 1$ commodities she buys:

$$A - (K - 1) \overline{p} x_m[\overline{p}] = x_m[\overline{p}] \text{ or}$$

$$x_m[\overline{p}] = \frac{A}{1 + (K - 1) \overline{p}} \text{ or}$$

(3)
\[
\frac{A}{\chi_m[\bar{\rho}]} = 1 + (K - 1) \bar{\rho}
\]

The payoff for a money producer would also be \(\chi_m[\bar{\rho}]\), since the producer would wind up holding that much of each of the commodities, including the money commodity.
Commodity exchange at a given money price---non-money producers

- If the number of money producers is the same as the number of producers of every non-money commodity, the amount of the money commodity the average non-money producer would receive would be $\bar{p} x_m[\bar{p}]$. They would then buy the same amount, $x[\bar{p}]$, of each other commodity:

$$x[\bar{p}] = \bar{p} x_m[\bar{p}] = \bar{p} \frac{A}{1 + (K - 1) \bar{p}}$$  \hspace{1cm} (4)

- This would work in the following way. Once the
money producers had bought their commodities they would drop out of the process. On average each non-money producer would then have 
\( A - x_m[p] \) of her own commodity and \( \bar{p} x_m[\bar{p}] \) of the money commodity, which we will arbitrarily assign to the \( K \)th position. On average these producers would spend the money on some other commodity, buying 
\[
\frac{\bar{p} x_m[\bar{p}]}{\bar{p}} = x_m[\bar{p}]
\]
of the commodity in order to equalize her holding with her holding of the money commodity.

At the same time some other non-money producer would buy her commodity, renewing her holding of the money commodity, until on average her holding of each of the commodities
she does not produce would equal her holding of the money commodity. At the end of the process the typical non-money producer (of the $K = 1$st commodity) would have the bundle:

$$\{A - (K - 1) x_m[\bar{p}], x_m[\bar{p}], \ldots, \bar{p} x_m[\bar{p}]\} =$$

$$x_m[\bar{p}] \left\{ \frac{A}{x_m[\bar{p}]} - (K - 1), 1, \ldots, \bar{p} \right\}$$  \hspace{1cm} (5)

- The typical non-money producer’s payoff when everyone charges $\bar{p}$ is $\text{Min}[1, \bar{p}] x_m[\bar{p}]$.

- Since the money producer payoff is $x_m[\bar{p}]$, in order for expected payoffs in all lines of production to be equal, $\bar{p} \geq 1$.

- The typical non-money producer cannot control
the amount she sells, so her final holding of her own commodity may exceed her purchases of others, but she will not operate in a region where her payoff is limited by her residual holding of her own commodity.

\[ \frac{A}{x_m[\bar{p}]} - (K - 1) \geq \text{Min}[1, \bar{p}] \]  \hspace{1cm} (6)

\[ \bar{p} \geq 1 \]  \hspace{1cm} (7)

- These conditions imply

- The money commodity circulates \( K - 1 \) times in this process as an exchange value, but winds up as a residual use-value.
Features of exchange at a given uniform money price of commodities

- If $\bar{\rho} \geq 1$, the average non-money producer payoff will be $x_m[\bar{\rho}]$, equal to the average money producer payoff.
- The average non-money producer winds up with a surplus of her own commodity when the money price is high, $\bar{\rho} > 1$. This represents a kind of inefficiency or unemployment of resources in this specialized economy.
- We have seen that the residual constraint (6) will prevent producers from charging a price $\bar{\rho} < 1$.
- When $\bar{\rho} = 1$, the residual constraint of
equation (6) will be binding. This is the efficient “full employment” regime of this type of specialized economy.

The regime where $\bar{\rho} = 1$ has some other interesting features.

- From a political economic point of view, when $\bar{\rho} = 1$ the money commodity circulates as money at the same value as its ratio to other use-values for the typical producer.

- When $\bar{\rho} = 1$, the typical final holding of all producers will be:

$$\left\{ \frac{A}{K}, \ldots, \frac{A}{K} \right\} \text{ with payoff}$$

(8)
net of specialization effort $\frac{A}{K} - \sigma$

- When $\overline{\rho} = 1$, the monetary exchange outcome is the same as the social coordination outcome. Money is a veil.
Distribution with money-mediated exchange

- When all producers choose the same price for any commodity, buyers will be indifferent among them.

- In the absence of any kind of rationing scheme, the demand will be allocated across the producers to maximize entropy subject to the residual constraint (6), since once a producer has sold enough so that her residual holding of her own commodity as a use-value is equal to her holding of all the other commodities, she will drop out of the exchange process.

- The distribution of holdings of the money
commodity as a use-value and other non-produced commodities will be systematically unequal in this monetary exchange economy unless it is in the full-employment regime.
Determination of the money price

- What determines the money price of commodities in the monetary exchange specialized division of labor economy? According to the social interaction model method, it will depend on the typical non-money producer’s best response choice of her own price, \( p \), when all the other producers are selling at price \( \bar{p} \).

- This problem has an interesting and important history, because it is essentially the problem of Bertrand competition in which a typical competing seller chooses a price assuming its
competitors will not change the price they charge. (This is the “dual” of the Cournot model where the typical competitor chooses a quantity to supply to a market assuming that its competitors will not change the quantity they supply and that the market will somehow equilibrate at a single price.)

What is the typical non-money producer's best response \( p[\bar{p}] \) when the price other producers are charging is \( \bar{p} \)? This depends on the quantity the typical producer expects to sell of her own commodity, with these prices, \( x[p, \bar{p}] \), which determines the money revenue she will have to buy other commodities, \( p x[p, \bar{p}] \).
The payoff of the typical producer who divides her money income equally over the $K-1$ commodities she does not produce will be:

$$u[p, \bar{\rho}] = \text{Min} \left[ \{ A - x[p, \bar{\rho}], \frac{p x[p, \bar{\rho}]}{(K-1) \bar{\rho}} \} \right]$$

The typical non-money producer setting a money price $p$ when everyone else (including especially the other competing producers of her specialized commodity) is charging a money price $\bar{\rho}$ will sell $x[p, \bar{\rho}]$ of her own commodity and will have $A - x[p, \bar{\rho}]$ left for her own use. Her money income will be $p x[p, \bar{\rho}]$, which she will divide equally among all the other commodities, winding up with a quantity $\frac{p x[p,\bar{\rho}]}{(K-1) \bar{\rho}}$ of each of the
other commodities.
The typical producer’s best-response price

- As the typical producer raises her price, she sells less of her own output (and has more left for her own use) and after some point will lower her money income and as a result will purchase less of all the other commodities.

- The typical producer’s best-response payoff will always be limited by her purchase of other commodities, and therefore will:

\[
\text{Max}_{p} \frac{p \times[p, \overline{p}]}{\overline{p}(K - 1)} \quad \text{with the first order condition}
\]
\[ x[p, \bar{p}] + p \frac{\partial x[p, \bar{p}]}{\partial p} = 0 \]

\[ \frac{p}{x[p, \bar{p}]} \frac{\partial x[p, \bar{p}]}{\partial p} \equiv \eta_p x[p, \bar{p}] = -1 \]

- Here \( \eta \) is the elasticity of price with respect to quantity.
- This is the Bertrand model of competition among the sellers of whatever commodity the typical agent produces.
- In the usual analysis of Bertrand competition, the individual seller faces a step function demand curve:
\[
x[p, \bar{p}] = \begin{cases} 
0 & p > \bar{p} \\
\text{Indeterminate} & p = \bar{p} \\
A & p < \bar{p}
\end{cases}
\]

- Clearly the best response of the typical producer with this step function individual demand schedule is

\[
p[\bar{p}] = \bar{p}
\]  \hspace{1cm} (12)

- This is the bifurcation point in the social interaction model, at which any price is an equilibrium, and scale-free fluctuations of price are likely to occur.
The Bertrand model of competition

- The condition of equation (12) is often taken in discussions of Bertrand (“cut-throat”) competition, such as the “contested markets theory”, to imply that the equilibrium $p$ will be forced down to marginal cost, which in the hub-and-spoke model is $\bar{p} = 1$, corresponding to an efficient monetary exchange.

- But in the context of the social interaction model, the typical producer best response $p[\bar{p}] = \bar{p}$ has different implications.

- In this case the equilibrium price is indeterminate
because the typical producer best response coincides with the equilibrium 45° line.

- In reality under these conditions we would expect price to fluctuate wildly (“scale-free”) rather than to be forced to its minimum level.

- In the real world, in fact, prices seem to be quite “sticky”, rather than fluctuating freely.
Typical producer’s demand with entropy-constrained behavior

- If the agents are entropy-constrained, however, the typical producer will not lose all of her sales when she raises the price above the average.
Typical producer’s best-response price

The typical buyer’s best response

- If only one producer (seller) of a commodity charges a price $p \neq \bar{p}$, while the $n – 1$ other sellers continue to charge $\bar{p}$, the typical buyer (who in this model produces some other commodity) has the choice of buying from the one seller at the price $p$ or from any one of the other $n – 1$ sellers at the price $\bar{p}$.

- Consider the response of a money-commodity producer. If she buys from the seller who is
charging price $p$, in order to equalize her holdings of all the commodities, she will buy a quantity $x_m[p, \bar{\rho}]$ that satisfies the condition:

$$A - ((K - 2) \bar{\rho} + p) x_m[p, \bar{\rho}] = x_m[p, \bar{\rho}]$$

or

$$x_m[p, \bar{\rho}] = \frac{A}{1 + p + (K - 2) \bar{\rho}}$$

(13)

- Note that $x_m[\bar{\rho}, \bar{\rho}] = x_m[\bar{\rho}]$ in terms of our earlier discussion.
The payoff to a buyer

- The money-commodity producer who buys from the seller charging $p$ will have a payoff representing $x_m[p, \bar{p}]$, since she will wind up with the bundle:

$$\{x_m[p, \bar{p}], \ldots, x_m[p, \bar{p}]\}$$  \hspace{1cm} (14)

- The buyer’s logit quantal response behavior will depend on the von Neumann-Morgenstern function, $v[.]$, that represents her degree of risk-aversion, $v[x_m[p, \bar{p}]]$.

- In these notes we will take $v[.]$ to be a constant relative risk aversion function with the elasticity
of substitution equal to 1/2.
The seller’s expected revenue charging $p \neq \bar{p}$

- The typical producer thinking of charging a different price from her competitors will have a payoff limited by her money revenue, and will predict a money revenue equal to the frequency of purchases from buyers times the revenue on the sale:

$$R[p, \bar{p}] = f[p, \bar{p}] \rho x_m[p, \bar{p}]$$  \hspace{1cm} (15)

- The typical seller will gain or lose some, though not all, of the market by changing price, and will also change the quantity of the commodity she sells. Her best-response price exploits the
degree of monopoly power she has to maximize her expected money revenue.
The buyer’s logit quantal response

- When all but one seller offer commodities at the price $\bar{p}$, and one seller offers one commodity at the price $p$, the typical buyer has to choose from which producer to buy the particular commodity.

- The frequency with which the typical buyer will choose the producer offering the commodity at a price $p \neq \bar{p}$ will be the logit quantal response given the payoffs of these options at some behavior temperature $T$.

- Using the von Neumann-Morgenstern function to evaluate the minimum consumption levels:
\[ f[p, \bar{p}] = \frac{\Theta^T v[x_m[p, \bar{p}]]}{(n - 1) \Theta^T v[x_m[p]] + \Theta^T v[x_m[p, \bar{p}]]} \]

- Note that when \( p = \bar{p} \), \( x_m[p, \bar{p}] = x_m[\bar{p}] \), so that the typical producer gets a fraction \( f[\bar{p}, \bar{p}] = \frac{1}{n} \) of the market sales.

- Assume that the buyers make these decisions independently, but that the numbers are large enough that producers hardly ever “stock out” by having too many buyers.

- There are \((K - 1)n\) buyers of the commodity. The expected money revenue of the seller with price \( p \neq \bar{p} \) will be
\[ R[p, \bar{p}] = f[p, \bar{p}] n(K - 1) p x_m[p, \bar{p}] \] (17)
*Mathematica* functions
The seller’s best-response price

- The assumption that the typical producer’s payoffs are strict complements gives the seller considerable bargaining power: the only loss of sales the seller expects is through the entropy effect.

- For any $\bar{\rho}$, the typical producer as seller can find an expected revenue maximizing price:
The best-response function of the typical producer, $p[\overline{p}]$, is difficult to solve explicitly.
Computing the equilibrium price

- The equilibrium price occurs when the derivative of expected revenue with respect to the typical producer's price, \( p \), is zero (which guarantees \( p \) is a best response) and \( p = \bar{p} \) (so that it is a best response to itself).

- This turns out to be a quadratic expression in this model:
Clear[eqcond];
eqcond[A_, K_, n_, T_,
  opts : OptionsPattern[{freq}]] :=

eqcond[A, K, n, T] // Simplify

\[
\left\{ \left( (K - K n) \ p^2 + A n T + p \ (1 + n (-1 + A (-1 + K) T)) \right) / \n\left( n p \ (1 + K p)^2 T \right) == 0 \right\}
\]

- It is possible to compute the equilibria numerically:
Clear[eqp];
eqp[A_, K_, n_, T_,
  opts : OptionsPattern[{{freq}}]] :=
Solve[
  Simplify@eqcond[A, K, n, T, opts], p]
Visualizing the dependence of the equilibrium price on the behavior temperature

- In this case the equilibrium price is very close to a linear function of the behavior temperature of the buyers:
Money price equilibrium

$A = 1000$, $K = 20$, $n = 10$
The linear coefficient can be found from the limit:

\[
\text{Limit}\left[\left(\frac{p}{\text{eqp}[A, K, n, T]}\right) / T, T \to \infty, \text{Assumptions} \to \{A > 0., K > 2, n > 1\}\right]
\]

\[
\left\{ \frac{A(-1+K)n}{K(-1+n)}, 0 \right\}
\]

The linear relation between \(T\) and \(p^{eq}\) extends over several orders of magnitude:
Visualizing the seller’s best-response and the social interaction equilibrium price
From a social interaction point of view, this is a case of weak strategic complementarity. The social coordination solution, $p = 1$, is unstable.

The equilibrium price is higher than $p = 1$, leading to unsold excess inventories of non-money commodity producers. Monetary exchange when the behavior temperature is positive does not achieve the social coordination outcome in the hub-and-spoke economy.
Methodological implications

- This example illustrates that money exchange creates a social coordination problem in separating sale and purchase for a typical specialized producer.

- This problem is inherent in Bertrand competition. But the knife-edge assumption that buyers will uniformly move to the lowest-price supplier (a version of the “law of one price”) has prevented economists from realizing that Bertrand competition does not necessarily lead to an equilibrium with price equal to marginal cost.

- With entropy-constrained behavior, Bertrand
competition, like Cournot competition, results in a determinate equilibrium with price above marginal cost.

- The equilibrium of the hub-and-spoke model with these parameters will leave all the non-money producers with an unsold excess supply of their product.

- The advantage of the Bertrand model of competition is that it explains how prices actually get set, as opposed to the Walrasian treatment of equilibrium prices as a kind of public good conferred on the economy by an external agent such as the Walrasian auctioneer.

- This is a striking example of the non-neutrality of
money arising from the non-convex production technology and the monetary character of exchange.

The monetary character of exchange leads to a different outcome from that predicted by market-clearing equilibrium in the economy with an individual production technology that is the convex hull of the non-convex hub-and-spoke technology.
Money, social coordination problems, and economic heterodoxy

- We have seen that the specialization inherent in the hub-and-spoke model fragments the typical producer into $K$ types, each occupying one specialized sector of production.

- The separation of sale and purchase creates a further splitting of the typical producer with further potential for social coordination failures.

- A central question is the degree to which the organization of a specialized division of labor
through the exchange of products as commodities can approximate the social coordination outcome of the hub-and-spoke economy.

On the whole the presumption of economic orthodoxy is that the commodity exchange system and money do a good job of securing the potential social benefits of the social division of labor, because money is a “veil”. This is the point of view, for example, of the Fundamental Welfare Theorems of general equilibrium theory.

On the whole the presumption of economic heterodoxy is that the commodity exchange system and money realize only a part of the
potential social benefits of the social division of labor, and involve inherent socially negative side-effects, such as endogenous inequality in wealth and income distributions, instability of aggregate demand, and involuntary unemployment.

These side-effects of commodity production seem closely connected to the social coordination problems that arise because of the fragmentation of the typical producer in the specialized division of labor and commodity exchange system with money modern societies rest on.
General equilibrium theory and convexity

- The theory of general equilibrium developed from marginalist principles through neoclassical economics rests on the assumption that production technologies give rise to convex individual and social production sets. In general convexity leads to social and individual behavior that can be characterized as an interior optimization of payoff functions that satisfy Lagrangian first-order conditions.

- In the hub-and-spoke model the production set of the typical producer is non-convex. This non-convexity implies that payoff-maximizing
individual and social behavior is generally at the extreme limits of feasible sets where Lagrangian first-order conditions are not satisfied with equality. For example, the typical producer in the hub-and-spoke model chooses to specialize to produce one product rather than to diversify.

The problem of non-convexity is a delicate issue for general equilibrium theory, often reserved for advanced analysis involving arcane and unfamiliar mathematical ideas.
The convex hull approach

- We have seen that when there are enough producers to populate the spokes of the hub-and-spoke model densely, they can (in the social coordination outcome) approximate diversified production with the economies of scale inherent in specialization.

- By allocating a large number of producers to the spokes, it is possible to reach or approximate any plan of production that is in the convex hull of the non-convex production set.

- One important line of thinking in general
equilibrium theory that includes the work of Farrell, Rothenberg, Bator, Aumann, Starr, Khan, and their followers investigates various mathematical conditions under which the convex hull of a non-convex production set can be approximated by allocations in the non-convex production set.

- This is one motive for studying economies in which there is an atomless continuum of producers and consumers.

- The example of monetary exchange, however, raises important questions as to the adequacy of this research program, because phenomena like monetary exchange mark a *qualitative* difference
between production at the spokes and at the hub.

Diversified, largely self-sufficient producers at the hub will see much smaller strategic complementarity effects leading to the emergence of money than specialized producers distributed on the spokes of the production wheel. Even though a socially coordinated production plan can approximate any point in the convex hull of the hub-and-spoke production model, it does not follow that an economy with a convex production set coinciding with the convex hull of the hub-and-spoke model will have the same qualitative properties as non-convex hub-and-spoke production.
Market and natural prices

■ We have been analyzing the hub-and-spoke model on the assumption that the prices of all commodities are identical, so that expected payoffs to choosing any line of specialization are equal.

■ Under these assumptions (which correspond to the establishment of natural prices in the terminology of the classical political economists) producers will see the same payoff ex ante to any line of specialization and the quantal response will be a uniform distribution.

■ But because choice of line of production is not
centrally coordinated, the mean outcome, equal numbers of producers at each spoke, will not always occur.

If there are fewer producers at some spokes than others in some realizations of the quantal responses, the corresponding commodities will be in short supply.
Market equilibrium with unbalanced supplies in the hub-and-spoke model

- Under the assumption of strict complementarity of commodities in the typical producer’s payoff function, if producers as buyers maximized expected payoff, the scarcest commodity would command an infinite price and all the others would have zero prices.

- But if buyers behave according to the entropy-constrained quantal response pattern, they will not shift all of their demand to the scarcest commodity.

- The scarce commodities will have higher payoffs
and will attract some of the market demand and higher prices in these unbalanced realizations, but not infinite prices.

This line of thinking provides a framework for a quantitative modeling of the fluctuation of market prices around natural prices in successive realizations of the hub-and-spoke model.

In this analysis the fluctuations of market prices will be related to the behavior temperature of the buyers.