Unconventional Monetary Policy in a Nonlinear Quadratic Model

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Abstract

In order to evaluate the response of central banks to the Great Recession in moving from conventional to unconventional monetary policy (UMP) we use an nonlinear quadratic (NLQ) model. The NLQ has a quadratic objective function and three nonlinear state equations. It uses a nonlinear Phillips curve, an IS equation, and captures the recently discovered importance of credit cycles – representing an essential nonlinear financial market-macro link – in a third state equation. Thus, we add to Okun’s output gap and Phillips’ employment gap, a credit gap in a macrodynamic model. Although the model is small scale it is similar in spirit to more complicated large scale macroeconometric models which many central banks currently employ. Yet, in the context of the small scale NLQ model including credit flows, risk premia and credit spreads, essential issues such as the evolution of academic and central banks views on the need, effectiveness and long run side effects of UMP can be assessed.

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1 Introduction

Many observers agree that the financial market meltdown of the years 2007-2009 and its world wide repercussions have been a great challenge to conventional macro models and monetary policies. The question was whether advanced economies will face a meltdown comparable to the 1930s, or if they were capable of implementing macro policies, in particular monetary policy, to rescue their economies from large scale output losses and persistent, high levels of unemployment.

It has been acknowledged that the role of credit cycles, and their international synchronization, make financial markets, banks and the macroeconomy vulnerable to sudden shocks and defaults. Moreover, it has been recognized that credit collapse, rising risk premia and credit spreads for short and long term credit are capable of accelerating and magnifying meltdowns.

On the other hand, moving back to some normal state of the economy through an increase of bank lending and credit flows after the Great Recession was needed. Yet, there was not only a prolonged negative output and employment gap – the former related to the latter by Okun’s law – there was also a prolonged negative credit gap, measured against some trend, that central banks attempted to close by encouraging and stimulating bank lending to households and firms. This was accomplished by a new type of policy, unconventional monetary policy, UMP.\(^1\)

Given the synchronization of credit cycles and vulnerabilities across economies, macroeconomic policies face great challenges in learning to control credit cycles. In this context the following questions may be asked:

- Is there sufficient empirical evidence on internationally synchronized credit cycles?
- Should the dynamics of asset prices be targeted by central banks or are credit cycles the main issue?
- Can conventional monetary policy effectively deal with credit cycles and credit spreads?
- How did academic views on the Great Recession, the role of credit risk, and appropriate policy responses evolve?
- What triggered the urgency of unconventional policy tools, such as UMP?
- What mix of policies is needed to overcome the prolonged recession (monetary and fiscal, as well as other policies)?
- Are the wider issues involved, such as the long run side effects of UMP, understood sufficiently?

\(^1\)Concerning the significant role of credit cycles and credit gaps (as measured against some trend), for the monetary transmission mechanism, see Aikman et al. (2017), Antoshin et al (2017), Ajello et al. (2016), see also Biggs et al. (2009), and Krishnamurthy and Muir (2016).
• How can vulnerabilities spanning across borders be addressed by monetary policy?

These questions can be discussed in the context of a stylized monetary policy model that central banks and international institutions, by and large, followed in conducting their policies.

The remainder of the paper is organized as follows. In section 2 we discuss the academic views of the Great Recession. Section 3 introduces our NLQ model. Section 4 implements the numerical evaluation of the model. Section 5 summarizes the implementation of UMPs in the US and the euro area. Section 6 presents results of some relevant scenarios. Section 7 discusses whether a mix of policies was need and section 8 discusses some perils of UMP in the long run. Section 9 concludes. Annex A shows how borrowing constraints are implemented in our model. Annex B provides some stylized facts on credit cycles. Annex C discusses the link between credit spreads and the shadow rate. Annex D presents the sequence of policy steps towards UMPs in the US and the euro area. Annex E presents results from regime switching VAR estimates.

2 Related Literature

The literature on the “causes of the Great Recession” and the suggested policies is quite diverse, but has generated new macro models. Pertaining to the Great Recession, a variety of causes has been stressed: 1) the role of housing sector and mortgage policies, 2) the role of financial market deregulation and the issuing of complex securities with insufficient regulatory controls, 3) financial market behavior and counterproductive financial incentives, 4) sectoral imbalances (too large a banking sector), 5) the role of asset prices and net worth, see Brunnermeier and Sannikov (2014a) who emphasize asset market reactions, such as the price of real estate assets, and banking assets. The latter may be falling, leading to reduced net worth in the banking system and higher risk premia, 6) there is also the issue of rising vulnerabilities of banks because of overleveraging (see Gross et al. 2017), and bank runs (Gertler et al., 2017). Finally, 7) there is the general view that the Great Recession was mostly caused by the financial sector (Mishkin, 2011) due to insufficient regulation, screening and monitoring, see Hartmann et al. (2018) and Goodhart (2018), and fiscal policies were too tight.

The views on possible causes of the Great Recession is strongly related to the economic paradigm pursued. At the beginning, there was a lack of models that allowed for studying the rise of imbalances and disequilibria in the context of financial-real interactions, a lack of studies on nonlinear propagation mechanisms, on linked vulnerabilities, missing model components and nonlinearities, a lack of regime switching models and studies of state dependent policy effects and endogenously generated risk, see Brunnermeier and Sannikov (2014a). A

\footnote{In the latter work a mechanism is also estimated in a regime change model where banks reduce their loan supply when they are highly leveraged and in a bad regime.}
recent evaluation of the usefulness of DSGE and NK models is discussed in a special issue of the Oxford Review of Economic Policy, Vol 34 Issue 1-2. There some specific flaws of DSGE models are discussed: the infinite horizon modeling, see Korinek (2017) and Korinek and Simsek (2017), the unresolved issue of parameter estimation – as system estimates, see Blanchard (2018), and Herbst and Schorfheide (2015), misleading micro dynamics (Stiglitz 2018), the reliance on the Ramsey path in macro models (Vines and Wills, 2018), and the missing nonlinearities in the financial market-macro interaction, see Brunnermeier and Sannikov (2014a).

The suggested types of monetary policies are, by and large, also paradigm-based. DSGE theorists view UMP as broadly effective. They are influential on current central banks’ research agenda. They usually use NK model features with financial markets, see Woodford (2012), and with persistence of financial market shocks and financial frictions (Christiano et al. 2014). Enormous empirical research has been put forward to show that in regime change and multi-regime VARs policy effectiveness can be evaluated more precisely, if one allows for differences of normal (good) states and bad states. There was also progress in solution techniques for solving nonlinear models; for regime change models, see Farmer et al. (2009), for global solution techniques, see Maliar et al. (2015) and for heterogeneous agent models in the DSGE tradition, see Den Haan et al. (2009).

Some economists maintain that there was an insufficient attention paid to behavioral macroeconomic models, Agent Based Models (ABMs), as well as other new directions, see Ball et al (2016), De Grauwe (2012), Gabaix (2017), Haldane and Turrell (2017), Korinek (2017) and Mitnik et al. (2013, 2018), see also White (2018) who views the economy as an evolving complex system. Some economists and policy makers have already pointed out some perils of conventional models and strongly endorsed UMP (Eggertsson and Krugman, 2012), among others to avoid the perils of deflationary pressures, see Ernst et al. (2017). Behavioral macro theorists tend to support UMP as well, but some of them point to the problem that central banks have not always been active enough in fulfilling their lender of last resort function and in stimulating effective demand in a more direct way, see De Grauwe (2012). Orphanides (2017) argues that the ECB asset purchasing program (APP) came too late and was too hesitant in the euro area, in particular with respect to purchases of government bonds.

A more recently developed theory pertains to the natural rate. It has been argued that the natural rate of interest has decreased and this might constrain the long run interest rate to increase. Many academics predicted such a downward movement in the natural interest rate, particular after 2008-9. Some economist use filtering methods to study trends of the natural rate (see Laubach and Williams 2003, 2016). Theoretical arguments in favor of a lower natural rate are due to excess saving and diminishing investment demands (Summers, 2014). Further explanations for a falling natural rate include demographic factors and

\[3\text{For an extensive survey of ABMs, see Haldane and Turrell (2017).}\]
productivity decline, measured as direct productivity or total factor productivity, see Benigno and Ferraro (2018). Monetary policy models with a downward revised natural rate can be found in regime switching type models such as Förster (2014). In some sense UMP has attempted to reduce long rates of the term structure and keeping it low through forward guidance. However, changes in policy targets may have to be addressed as well.

A contrarian view based on empirical research was released recently. Greenlaw et al (2018) questions if long rates have been affected by Quantitative easing (QE) and large-scale asset purchases (LSAP) policies. They acknowledge that the long end of the term structure was affected by these policies, while Krishnamurthy and Vissing-Jörgensen (2011), Bernanke (2017) and Yellen (2017) have defended the FED position as long rates have come down considerably through a reduction in the term spread through LSAP. They also argue that shorter rates have been reduced. Also, looking at the BBB bond yields in figures 10 and 11, one can observe the strong effect of LSAP on corporate bond yields. In addition, the contrarian view does not seem to fit the Japanese case, where the 10-year bond has been held at zero yield for a long time. Additionally, in the euro area strong effects of the Asset purchase programmes (APP) on long rates were observed, see Andrada et al. (2016). On the other hand Taylor (2016) points to the perils of low interest rates, as they encourage excessive risk taking and may generate financial instability. In fact, strong defense of central banks’ UMPs and LSAP in the US and APP in the Euro area comes mostly from work inside of central banks, see Hartmann et al (2018) and Andrada et al. (2016). Outsiders have rather been evaluating the more controversial side effects. On the latter see section 8.

From the European perspective, the views on the long run effects of UMPs are quite diverse. Economists and policy makers of more stable and growing economies maintain different views compared to policy makers in stagnating economies. In economies such as Germany and other Northern EU countries, economic expert reports view UMPs skeptically, and point out the perils of low borrowing rates for a surge of future inflation rates, financial risks due to asset price inflation and, given the low interest rates, disincentives for household savings. In addition, excessive risk taking by banks to generate reasonable returns has been mentioned. Moreover, some economists argue that low interest rates and asset purchases of the ECB create budget constraints which are too soft for highly indebted states in Southern Europe. This may entail a reemergence of sovereign debt and default risk in some countries, particularly when UMP is tapered. This danger will hold also for Eastern European countries, which exhibited excess credit flows at very low credit cost. Their banks might be endangered by an exit from UMP.

\footnote{For the German perspective on UMP, see DIW (2016). On the German position see also the statements and evaluations by the Ministry of Finance.}
3 A small scale macro-policy model – The NLQ model

The standard model which has conventionally been used as guidance for monetary policy before the Great Recession was the inflation targeting model put forward by Svensson (1997), usually characterized by a Phillips-curve, output gap dynamics and the Taylor rule for setting the policy rate. Financial market components, such as credit dynamics, were not included in those models. Yet after the Great Recession of 2007-9 many academics and policy makers reached the conclusion that the widely used linear monetary policy models are missing some financial market features and are not sufficient for monetary policy guidance. Small, as well as large scale, nonlinear models have emerged. We propose here a small scale model which is similar in spirit to more complicated large scale macroeconomic models which many central banks currently employ. Yet, in the context of the small scale NLQ model essential issues such as the evolution of academic and central banks views on the need and effectiveness, as well as on the long run side effects of the UMP can be assessed.

3.1 Deficiencies of linear models

An early statement on the limitations of linear (for example LQ or linearized DSGE) macro models neglecting nonlinearities, in particular with respect to financial market-macro economy interactions, can be found in Mishkin (2011):

“The role of nonlinearities in the macro economy when there is a financial disruption implies an important flaw in the theory of optimal monetary policy that was in general use prior to the crisis....The financial crisis of 2007-2009 demonstrates that although the linear-quadratic framework may provide a reasonable approximation to how optimal monetary policy operates under fairly normal circumstances, this approach will not be adequate for thinking about monetary policy when financial disruptions hit the economy”.

(Mishkin 2011:23)

The LQ approach was useful since it allowed for studying effectiveness and long run impacts of monetary policy in normal times.

Many central banks had by and large used LQ form models, augmented by the descriptive Taylor rule, See Gali (2008), as their inflation targeting model before the Great Recession. Central banks have also employed DSGE models frequently. Usually these models were constructed in linearized form, allowing for the interaction of product, labor, and financial markets and a large number

5Mishkin also writes:”the bursting of bubbles throughout history has been followed by sharp declines in economic activity, as Kindleberger’s (1978) famous book demonstrated”

6The LQ model has also been interpreted as an approximation, using the first order approximation of the Taylor series, of more general equilibrium models (Woodford, 2003) and the New Keynesian version of the DSGE model, see Gali (2008). The lack of nonlinearities in DSGE models has recently been pointed out by Gertler and Gilchrist (2018).
of shocks.⁷ This allowed for a more extensive assessment of macroeconomic and monetary policy performances. There is also the remarkable work by Herbst and Schorfheide (2015) who build on Bayesian estimation procedures for obtaining accurate parameter estimates for these models.

Yet, given the large meltdown of the financial market in 2007–9, particular after the Lehman Brothers events in September 2008, dominant macroeconomic forecasting models were not capable of predicting large scale output losses and the meltdown of the financial sector.⁸ Economists realized that these models were missing essential components, such as credit flows and financial sectors (asset prices and banking systems) and their interaction with the real estate sector. For instance, Mishkin (2011), but also Eggertsson and Krugman (2012), point out this weakness of conventional macro models. Recent work has therefore focused on adding financial sector components, such as asset prices and financial intermediaries, to macro and monetary policy models.⁹

It is noteworthy that there was some earlier work on historical credit boom-bust cycles (such as Fisher, Minsky and Kindleberger¹⁰). Of significance was also the development of heterogeneous agent and behavioral macro models, see De Grauwe (2012) and De Grauwe and Macchiarelli (2015), as well as Agent Based Models (ABM), see Haldane and Turrell (2017). Encouraging plurality of alternative views on the macroeconomy came from central banks (see Halaj, 2018) that encouraged ABM for policy analysis. Their main contribution is seen in allowing for heterogeneity, bounded rational behavior and interconnectedness of the agent’s decision that may give rise to boom-bust cycles.

In the tradition of small scale macro dynamic models, Svensson (2014, 2016), Ajello et al (2016), and researchers at the IMF, see Antoshin et al (2017), have pursued an analysis based on recent empirical research on credit cycles. In fact, it was important empirical work which triggered this new research agenda, trying to add credit cycles into small scale macro models of the Svensson (1997) type. Such empirical research was put forward by Jorda et al. (2011, 2013), and Schularick and Taylor (2012), to name a few.¹¹ They also show that there has been an international synchronization of credit cycles. The time varying credit gap is now frequently used as measure for the credit cycle¹² and the linkages

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⁷See for example the model by Smets and Wouters (2007).
⁸For a comparison of the diverse models to forecast the Great Recession and their limited success, see Wieland and Wouters (2011).
⁹See Brunnermeier and Sannikov (2014a), Goodhart (2018) and Gertler and Karadi (2013) to name a few.
¹⁰The Minsky view, by referring to credit flows, is also vividly expressed in Kindleberger and Aliber (2005: 10) when they state in their book: “... the cycle of manias and panics results from the pro-cyclical changes in the supply of credit; the credit supply increases relatively rapidly in good times, and then when economic growth slackens, the rate of growth of credit has often declined sharply.”
¹¹See also Borio (2012) who states that credit cycles may evolve over a longer time horizon than the business cycle.
¹²Credit gap may be defined (i) by the level of credit, (ii) by average growth rates of credits, (iii) or by deviations from a trend, see Antoshin et al (2017), and Biggs et al. (2009). Based on the empirics of credit cycles mentioned above, the use of small scale models has flourished, see Aikman et al (2017), Gross and Semmler (2017), Gourio et al (2016), Herman et al (2015).
of credit gaps with asset prices, real estate and banking sector, as well as with other macro variables are studied in more detail. Building on the empirical literature on credit cycles and gaps, small scale models can give relevant insight into the effectiveness of conventional and unconventional monetary policies.

Recent reviews on the need and effectiveness of UMP, and responses to criticism of them, are undertaken by central banks’ officials, current and former staff members, for example in Mishkin (2011), Bernanke (2017), Draghi (2017), Blinder et al (2017), and Blanchard and Summers (2017). There has also been a large number of central bank conferences where the need and effectiveness of UMP has been evaluated, mostly resulting in positive evaluations (see Hartmann et al. (2018) and Andrada et al (2016) for the ECB, and Bernanke (2017) who address the various kinds of criticisms related to QE policy, but mostly focus on the positive effects of UMP). Those recent evaluations take into account both the stabilizing as well as destabilizing effects that UMP might have set in motion. Yet, those represent to a great extent central banks’ views and academic evaluations from outsiders are still to be published. A first attempt in this direction has been undertaken by Ball et al. (2016).

Imbalances UMPs might have generated are arising from new credit bubbles, the impact of UMPs on income and wealth distribution, and the international spillover effects of UMPs, possibly giving rise to international vulnerabilities due to linkages between economies. Furthermore, there are tapering and exit risks. Employing models that include credit flows, asset prices, financial sector dynamics, the housing sector, fiscal policy, wealth and distributional effects, and open economy extensions with endogenous risk build up might be helpful for evaluating the negative long run effects of UMPs.

3.2 Nonlinear model with credit flows—the NLQ model

In order to study these issues in more detail, we introduce an extended inflation targeting model that takes dynamic equations for credit flows and credit conditions into account. This additional dynamic equation helps in defining the crucial role of credit gaps during and after a crisis. We use short cuts and approximations which can be derived from more general macro models by employing (nonlinear) equations for the Phillips curve with an output gap, an intertemporal IS equation representing the dynamics of the output gap,\(^{13}\) and a (nonlinear) flow (credit gap) equation, which resembles the recent work by Svensson (2016, 2017) and Ajello et al. (2016), Antoshin et al. (2017) and others. Including those nonlinearities responds to the Mishkin criticism and allows us to move from a LQ to a NLQ model. Further model variants, including credit flows and their interaction with credit spread and output, are introduced in Gross and Semmler (2017a) and further detailed and estimated in Faulwasser et al. (2018).

Most recent studies use either an infinite horizon model or a two period model with a two period quadratic loss function of the central bank. For rea-

\(^{13}\)For further derivations such approximations we are using above, see Wu and Zhang (2017) and Curdia and Woodford (2009, 2015).
sons of transparency and measurability, we will also work with a quadratic loss function. However, we will use a finite horizon model with a rolling window. We hereby respond to a criticism of Korinek (2017) who states that infinite horizon models are elegant but unrealistic. As stated above, recent work has also re-estimated many parameters of this type of models, using Bayesian methods, which could be taken for some rough model calibration. Our model contains strong nonlinearities and allows for amplifying feedback effects. We want to note that, if desired, the subsequent set up can be derived from a conventional New Keynesian macro policy model, yet with some essential modification by allowing for nonlinearities.\footnote{See Wu and Zhang (2017).}

Following up on the above mentioned criticism of missing nonlinearities in the interaction of financial and real sectors, we consider the following system of nonlinear differential equations. The system contains two regime switches, one for the inflation rate in the Phillips curve, and one for credit flows and credit risk in the financial market. This setup is a sufficient approximation for policy purposes. The stylized NLQ model that allows for credit dynamics dynamics looks as follows:\footnote{For details of such a model which is numerically solved with estimated parameters and with evaluation of policy scenarios, see Faulwasser et al. (2018).}

\begin{equation}
\begin{bmatrix}
\dot{\pi} \\
\dot{y} \\
\dot{l}
\end{bmatrix} = 
\begin{bmatrix}
\pi\alpha_0 + \alpha_1\pi + \alpha(y) \\
\beta_1 y - \beta_2(i + \delta(l) - \pi - r) + \beta_3(e^l - \hat{l})/\hat{l} \\
\gamma_1 l + \gamma_2 y + \gamma_3(i + \delta(l)) - \gamma_4 \pi
\end{bmatrix}
\end{equation}

The nonlinear state equations comprise the dynamics of the inflation rate, $\pi$, output gap, $y$, and credit flows, $l$. What we are proposing is a NLQ model – a nonlinear quadratic model, nonlinear in the state space and quadratic in the objective function.

Note that the deviations from the standard inflation targeting models consist of regime switching behavior of the inflation dynamics, $\alpha(y)$, in the first equation and credit flows and credit spreads, $\delta(l)$, representing state dependent credit spreads, in the second equation. Gross and Semmler (2018) estimate the nonlinearity in the Phillips curve, and Gross and Semmler (2017a) highlight the empirical relevance of nonlinearities in credit flows and credit spreads by estimating the parameters involved. Though we work here with a small scale model, suitable for monetary policy studies, the nonlinear model can be extended to higher dimensions.

As we show in appendix C, Hamilton and Wu (2011) and Wu and Zhang (2017) define the double term $i + \delta(l) = s$, as the shadow rate, an expression for credit cost in the 2nd and 3rd equations. They derive the shadow rate as a summary variable, consisting of the interest rate (the policy rate) and a risk and liquidity premium, driven by the term spread and a bond risk premia.\footnote{See also Curdia and Woodford (2009, 2015) who define the credit spread as interest markup set by banks.}
they show, although there is a lower bound for the interest rate, the zero lower bound (ZLB), the shadow rate can be negative, due to UMP.

Compared to the standard LQ model, the term $\beta_3(l - l_s)$ in the IS (or output gap) equation is added (see Ajello et al. 2016). This term represents the impact of credit conditions on output gap dynamics and then indirectly on the inflation rate. This term can also be derived from a more general NK model with credit constraints for households and firms, see Wu and Zhang (2017).\footnote{A constraint could be given by a loan to value ratio (LTV) which will be relaxed when the LTV is allowed to rise. This ratio is usually impacted by macro prudential policies. With respect to the shadow rate, Wu and Zhang (2017) make also a distinction between normal times and crisis times. This distinction is important when evaluating the effects of UMP, see below.} Note that by taking account of credit constraints we can distinguish between credit constrained and unconstrained agents. This allows us to study the impact of credit volume, macro prudential policy, and selective credit policies on our macro dynamics.

The third dynamic equation represents the flow of new loans affecting the dynamics of the credit gap. Much effort has been spent on estimating and evaluating the parameters of the third equation, see Ajello et al. (2016), Svensson (2016), and Faulwasser et al. (2018). Instead of modeling financial intermediaries explicitly, we may let the change of the flow of new loans be driven by a feedback mechanism related to loans, the output gap - for example a negative output gap co-varying with nonperforming loans and reduced loan flows by banks\footnote{There is much recent research on this issue, see Gross et al. (2017)} - and the shadow rate of the cost of loans $i + \delta(l) = s$, with $\gamma_3 < 0$, and $\delta(l)$ defined by a credit gap. The inflation term appears here too, since we are considering real loan flows.\footnote{For further evaluation of the parameters of the third equation in system (1) using a Bayesian approach, see Ajello et al. (2016), see also Faulwasser et al. (2018) who use the Marquard algorithm for system estimation.}

What a typical credit boom-bust cycle and the corresponding credit spreads would look like in the context of our NLQ model, without monetary policy effect and with monetary policy effects (conventional and UMP), is demonstrated in Annex A and B. The interaction of inflation dynamics, output dynamics and credit dynamics is sketched in system (1). A sketch of how monetary policy (conventional and UMP) can be introduced in our model is shown next.

### 3.3 Monetary policy in a NLQ model

Instead of spelling out the general equilibrium effects in a NK DSGE model which takes the mentioned nonlinearities into account, we use our NLQ model of system (1) that exhibits some shortcuts, but uses an optimal Taylor rule. The quadratic objective function for the domestic central bank - with nonlinear state equations - is equivalent to the minimization of a sum of weighted quadratic losses:

$$\frac{1}{2} \sum (y_t - y)^2 + \frac{\lambda}{2} (\pi_t - \pi^*)^2 + \frac{\gamma_3}{2} (\lambda - \lambda^*)^2 + \frac{\delta}{2} (\pi_t - \pi^*)^2$$

The weights $\lambda$, $\gamma_3$, $\delta$, and $\pi^*$ are chosen so as to reflect the relative importance of inflation, output gap, and credit conditions. In this way, the central bank’s policy can be steered to achieve a desired combination of inflation, output gap, and credit conditions.
\[
\min_{i(t)} \int_0^T \frac{1}{2} e^{-\rho t} \| \Delta(t) \|^2 \Lambda \, dt
\]  
(2)

with
\[
\Delta(t) = [\lambda_\pi (\pi(t) - \pi_s), \lambda_y (y(t) - y_s), \lambda_l (l(t) - l_s), +\lambda_i (i(t) - i_s)]^T
\]  
(3)

subject to (1)

with \( \lambda_\pi, \lambda_y, \lambda_l, \lambda_i \) being the weights for the targets. The target points are \([\pi_s, y_s, l_s]\), possibly the steady state of the system. Note that we use a finite horizon model here.

The nonlinearities refer to inflation dynamics, output gap and credit dynamics in a NLQ model. The essential nonlinearities we want to track in our NLQ model are derived from regime switching models such as Markov switching and threshold models.\(^{20}\) In general these kind of nonlinearities may be stylized by using some kind of step function such as:

\[
\tilde{H}_c(x) = \frac{1}{1 + e^{-c_1 (x-c_2)}}
\]  
(4)

Such a function is often employed as basis function in nonlinear models, for example in logistic smooth transition models (LSTR) via the logistic function (Granger and Terasvirta, 1993). Our nonlinearities are based on the logistic function.

The function \( \delta(l) \), which represents the rise of credit spreads depending on the credit gap (spread rising with negative credit gap), is depicted in figure 1.

Using parameter estimates (Faulwasser et al., 2018) we obtain the approximation as shown in figure 1 for the credit gap dependent credit spread \( \delta(l) \) – or, equivalently, for the shadow rate, \( s \).\(^{21}\)

Moreover, pertaining to the impact of the output gap and its impact on inflation dynamics – the regime switching of the inflation rate in the Phillips-Curve – we can use an approximation similar to figure 2.\(^{22}\)

\(^{20}\)For the former see Hubrich and Tetlow (2017) and for the latter, see Granger and Terasvirta (1993) and Mittnik and Semmler (2018).

\(^{21}\)Note that we could also use a double sided heavy side function instead of using only the contractionary part of the credit spread \( \delta(l) \), or the shadow rate \( s \). The shadow rate and credit spread stay low for some period in the expansion, but the risk premia (and shadow rates) may move up again due to the rising leverage and rising risk perception, accompanied by a rise in the vulnerability of banks and financial market risk. This situation is depicted in figure 10 in Annex B. This case could be econometrically explored by using the ESTAR model suggested by Granger and Terasvirta (1993) instead of an LSTAR model as shown in figure 1.

\(^{22}\)For details on the data base and parameter estimation see Gross and Semmler (2018).
Figure 1: $\delta(l)$ function based on step the function $\tilde{H}_c$; note that credit spread jumps up, as credit flow declines below the threshold of $l - l_s = 0$, or near zero. The vertical axis represents the actual $\delta(l)$ and the horizontal axis the actual $l - l_s$. The credit spread rises with negative credit gap, $l - l_s < 0$, and decreases with the positive credit gap, $l - l_s > 0$. Note that such jumps of course have to be smoothed out in estimation and solution procedures.

Figure 2: $\alpha(y)$ function based on the basis function $\tilde{H}_c$; note that $\alpha(y)$ moves up beyond a certain threshold of the output gap $y$; note also that such a function produces different inflation dynamics in a recession than in an expansion. The jumps are again smoothed out in the estimation and solution methods.

Extensive scenarios using the NLQ model of system (1) can be solved globally, by employing estimated parameters, which are discussed in Faulwasser et al. (2018). A further evaluation of numerical solution methods is also done in Faulwasser et al. (2018) where it is shown that there are tipping points for credit flows to generate expansions or contractions in output with the corresponding inflation rates. The critical or tipping points are created by a sudden fall or rise in credit spreads, as depicted in figure 1. Next we try to answer how
Table 1: Parameters for dynamics (1)-(4).

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<td>$l$</td>
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<td>$\mu_2, \mu_3$</td>
<td>13.35, -13.34</td>
<td>$c_1, c_2$</td>
</tr>
<tr>
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<td>$\lambda_{x,y,l}$</td>
<td>10</td>
<td>$\lambda_i$</td>
</tr>
</tbody>
</table>

conventional and unconventional policies, as they were implemented after the meltdown in 2007-09, fit into this framework.

4 Numerical implementation

Solving the finite-horizon decision problem with regime switches creates some challenges. We solve the following finite-horizon discounted decision problem, with the above mentioned two nonlinearities, as a finite horizon dynamic decision problem, DDP, (1)-(4) with $\lambda_x = \lambda_y = \lambda_l = 10$ and $\lambda_i = 1$ to be used as weights for the targets in equation (3). This means the central banks give little weight to interest rate smoothing. The target point $[\pi_s, y_s, l_s]$ is a steady state of (1) as discussed below when we describe the numerical solution procedure.

We consider the parameter values as listed in Table 1. The data sources, the parameter value estimates and confidence bands are discussed in Faulwasser et al (2018). Some parameters are obtained from Ajello et al. (2017). Note also that the natural rate, $r = 0.03$, does not have to be fixed, but can be endogenous and time varying, see section 6. In our numerical procedure we may have a moving steady state. The computational methods to do so are explained below.

To obtain solutions of system (1), we have to to compute a steady state for the proposed model with two switches first. The steady state may depend on the natural rate, $r$. To this end, we introduce a state vector $x := (\pi, y, l)^\top$ which allows writing (1) at the steady state as

$$f(x, i) = 0,$$

where $f : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$. This is a nonlinear system of three equations and four unknowns. We compute economically meaningful steady states by solving the following simple problem numerically:

$$\min_{x, i} \|x_s - x_{ref}\|^2 \text{ subject to } f(x, i) = 0.$$

23 Other weight parameters were also used but did not make much of a difference in the dynamic paths.

24 Note that the interest rate is here a control variable that assists in reducing macroeconomic imbalances due to (5). We are not applying the descriptive Taylor rule as in Gali (2008, sect. 4.3.1.1) where the issue of determinacy and indeterminacy is discussed within a linearized New Keynesian framework.
Here, $x_{\text{ref}}$ is a chosen economically reasonable reference value. This way we obtain

$$\pi_s = \ldots, y_s = \ldots, l_s = \ldots, i_s = \ldots$$

which are used in the penalty function (2). Solving the dynamic decision problem (1)-(3) entails some challenges, which are discussed in Faulwasser et al. (2018). For example there might exist multiple steady states.

To solve our system (1)-(3), the ODEs are discretized using a fixed-stepsizes Runge-Kutta scheme with 15 integration steps per shooting interval. The input $i$ is discretized as a piece-wise constant function. For the different present results, we consider equi-distant shooting intervals of length $\Delta T = 10$, i.e. the number of shooting intervals equals the horizon length $T$.

The dynamic decision problem, discretized in both control and state variables which allows for solving the NLQ problem reads as follows:

$$\min \sum_{i=0}^{N-1} \rho_k \|x_k - x_s, i_k - i_s\|^2_{\Lambda}$$

subject to, for all $k = 0, \ldots, N - 1$,

$$x_{k+1} = f_d(x_k, i_k),$$

$$x_0 = x(0)$$

$$x_k \in X, i_k \in [-0.015, 3].$$

Here $x_k := (\pi(t_k), y(t_k), l(t_k))^T$ is the discretized state variable, $i_k := i(t_k)$ is the discretized input, $\rho_k := e^{-\rho t_k}$, and $f_d : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$ is the state transition map, arising from the employed fixed-stepsizes Runge-Kutta scheme.

Concerning the considered parameter values, as the horizon length is limited, we do not employ a receding horizon approach as suggest in Gruene et al. (2015). Instead, we solve the NLQ problem directly. However, as the construction of feasible initial guesses could be challenging, one may want to solve simplified instances of the NLQ problem to construct guesses.

Next we present the empirical system estimation procedure as well as the used data set. For the purpose of empirical estimates of our nonlinear model we again approximate the above described continuous time dynamic model by a discrete time version and perform system estimation.

Our empirical system is estimated based on a quarterly data sample covering the 2003Q1-2017Q2 period (58 quarters) for the euro area, see Faulwasser et al. (2018). Some parameters are taken from Ajello et al. (2017). In discrete time, the nonlinear system to be estimated has the following structure:

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$25$This problem is solved using CasADi [Andersson, 2013] where the optimization algorithm IPOPT is used.
\[ \Delta \pi_t = -\alpha_1 \pi_{t-1} + \alpha_2 + \frac{\alpha_{22} - \alpha_{21}}{1 + \exp(-c_1 (y_{t-1} - c_2))} + \epsilon_t^{\Delta \pi} \]

\[ \Delta y_t = -\beta_1 y_{t-1} - \beta_2 \left( i_{t-1}^{pol} + \delta_{t-1} - \pi_{t-1} - r \right) + \beta_3 l_t^{dev} + \epsilon_t^{\Delta y} \]

\[ \Delta \ln (l_t) = -\gamma_1 \ln (l_{t-1}) + \gamma_2 y_{t-1} - \gamma_3 \left( i_{t-1}^{pol} + \delta_{t-1} \right) - \gamma_4 \pi_{t-1} + \epsilon_t^{\Delta l} \]

We have replaced the \( \dot{x} \) notation and denote the incremental change from period to period by \( \Delta \)'s. The variable \( \pi_t \) denotes the annualized quarter-on-quarter (QoQ) GDP deflator inflation rate, i.e. is defined as \( \pi_t = 4 \ln \left( \frac{p_t}{p_{t-1}} \right) \) where \( p \) denotes the GDP deflator index. The variable \( y_t \) denotes the real output gap, defined as \( \ln \left( \frac{y_{a_t}}{y_{trend_t}} \right) \), where \( y_{trend} \) was computed using the Hamilton filter (Hamilton, 2017). The variable \( i_{t-1}^{pol} \) is the ECB main refinancing rate and \( \delta_{t-1} \) denotes the euro area aggregate private sector loan interest rate spread (spread to policy) pertaining to new loans to businesses and households within a quarter.

The sum of \( i_{t-1}^{pol} \) and \( \delta_{t-1} \) therefore yield the loan interest rate for firms and households. Moreover, the term \( r \) denotes the real interest rate which we set to 0.03.\(^{26}\) The variable \( l^{dev} \) is the real loan flow gap. It was computed by first deflating quarterly nominal private sector loan flows with the GDP deflator and obtaining a real trend estimate, \( l_{trend} \), using the same Hamilton filter we used for calculating the output gap. The real loan gap variable is defined as \( l^{dev}_t = \left( l_t - l_{trend_t} \right) / l_{trend} \).

The following equation captures the state-dependent nature of the loan interest spread, denoted by \( \delta_t \). The loan interest rate spread is modeled as a nonlinear function of real credit flow:

\[ \delta_t = \mu_1 + \mu_2 \left( \frac{1}{1 + \exp(-c_3 (\ln (l_t) - c_4))} \right) + \epsilon_t^{\delta} \quad (10) \]

Note that figure 1 visualizes the state-dependent credit spread (loan interest rate spread to policy rate) as a function of the credit flow gap. The credit flow gap is estimated for equation (10) which, in general, is allowed to become negative, similarly to the shadow rate as in Wu and Zhang (2017).

5 Policy steps toward UMP in the US and Euro area

Before we run scenarios using our NLQ model we want to demonstrate that the model outlined in system (1)-(3) is well suited for discussing conventional and unconventional monetary policy steps in the US and Euro area. The details for

\(^{26}\)It has been argued that in recent times that since 2008-9 the natural rate, \( r \), has fallen significantly. Since we work with a longer time series, we keep the natural rate at some text book level first, but reduce it following some more recent research later on, see section 6.2.
It has to be mentioned here that the role of financial markets and nonlinearities has been studied well before the onset of the 2007-9 meltdown, see Bernanke (1983) on the former and Granger and Terasvirta (1993) for the latter. Important contributions after the 2007-9 meltdown are Mishkin (2011), criticizing linear inflation targeting model, and Curdia and Woodford (2009, 2015), pointing out the missing financial market component in inflation targeting models. The latter authors, in particular, are proposing to reduce markups on loans by a banking sector struggling with asset price deflation. Many recent contributions discuss tools and policies to reflate asset prices, support risk taking and encourage bank lending.

The main purpose of UMP when it evolved from the monetary policy debate in the years 2007-9, was to explore how the above nexus works, and what the channels of reducing risk premia and credit spreads are, or, to follow the concept of Wu and Zhang (2017), to bring down the shadow rate, see Annex C. Increasing credit flows and reducing large negative credit gaps, as suggested by Aikman et al. (2017) and Antoshin et al. (2017), was seen as a way for achieving this.

In order to avoid a meltdown of the financial market and the real side comparable to the 1930s – as described in Bernanke’s (1983) work on the Great Depression – the policy rate has been lowered toward the ZLB, or to an Effective Lower Bound (ELB), which is allowed to be slightly below zero. Reaching these lower boundaries, UMPs were implemented as an escape route from policy rate limitations.

1. Conventional monetary policy: in the US the policy rate was lowered. In the beginning this was done in small steps only: 9/2007 from 5.25% to 4.75%, then 12/2012 to the ZLB (0.25%). In the Euro area policy rates were lowered with a delay, first 10/2008 from 4.75 % to 3.75%, then 5/2009 to 1%.

2. Expansion of lending facilities and rescue operations (mainly financial institutions): in the US: 2007-2009 Term Auction Facility (TAF) and Troubled Asset Relief Program (TARP) by the FED and treasury; EU national rescue operations of financial institutions; in the euro area: Covered Bond Purchase Programme (CBPP1) in 2009.

4. Forward guidance decisions: by the FED as interest rate forward guidance in 2012, at least until 2015, in the Euro area the ECB in 7/2013 (for an extended period of time), and APP with forward guidance until Fall 2018.

5. Discussion on the downward revision of the natural rate: discussion in the US started 2015 and in the Euro area 2016-17.

6. Considerations on tapering and exits from UMP, in the US: exit from ZLB started slowly 12/2015, small step increases of the policy rate. In the euro area the policy rate is still kept low, at least until Fall 2018.

Since steps 1. and 2. were not as effective as expected in closing the output and credit gaps initially, UMPs of points 3. and 4. were actively pursued, known as QE policy and forward guidance, with the intention of reducing credit spreads (shadow rate), increase credit flows, and initiating new lending through financial intermediaries. In particular measures 3. and 4. appear to have been effective in reducing credit spreads and shadow interest rates and in encouraging risk taking and lending, see Annex C and D.28

However, as output and employment gaps were not closing despite active policy approaches, many economists were wondering whether a period of secular stagnation has emerged, resulting in declining output (Blanchard and Summers, 2017). Policies with the aim of changing inflation targeting models were discussed as well. Discussions evolved around a lower natural rate, $r^*$, and an increased inflation target $\pi_s$. Negative policy rates and increasing inflation targets were discussed as well. These changes were supposed to increase the monetary policy space, see Bernanke (2017).

Concerning the monetary-fiscal policy mix, it has to be stressed that policy rate reductions have only been partly accommodated by fiscal expansions.30 This was especially the case in Europe. In addition it has also been recognized, that policies focusing on credit volumes were needed to reduce credit constraints, see Zdzenicka et al (2015). As stated in the literature this was not sufficiently done in the EU, see Brunnermeier and Sannikov (2014b).

The effects of conventional and unconventional monetary policy have been studied in great detail for the US, the Euro area and other countries as well. Some studies have also focused on scenarios where the output gap is negative and disinflationary or deflationary pressures may arise, in particular when the interest rate is at the ZLB, and credit risks drive up risk premia and credit flows.

The NLQ model as sketched in (1)-(3) is useful in explaining the effects of

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28Empirical studies also show that the equity risk premia came down, see Croce et al. (2017).
29For the US see Foerster et al (2014).
30Many IMF studies were already exploring the regime (business cycle) dependent effects of fiscal policy, but in practical policy advise there was still much consideration of fiscal policy as a tool for budget consolidation, see below.
policy steps 1.-6.\textsuperscript{31} and for understanding deflationary perils that may arise in a protracted period of negative output and credit gaps. In such a model one may trace price channel effects of UMP, as well as quantity channel effects, see Faulwasser et al (2018).\textsuperscript{32} Yet, we want to note, that the nonlinear term $\delta(l)$ is sometimes, and for some time periods, difficult to identify and to handle, as time series data for $\delta(l)$ is often very short.

6 Results from some scenarios

Next we are exploring policy effects in the context of some relevant scenarios. The most important scenarios we want to consider here using our NLQ model will be a scenario with deflationary pressures. We will consider credit supply and credit demand policies. In the first three scenarios we assume a given natural rate. Subsequently we will study scenarios involving uncertainty about the natural rate and a changing rate.

6.1 Deflationary regime with negative output gap

Scenario 1: Credit supply policy– Deflationary regime with negative output gap; defeating the ELB through QE and high credit flows

We consider large negative shocks, moving the initial values of the state variables further away from the steady state, but allowing a large positive shock to credit flows. We consider initial conditions $\pi(0) = -0.01; y(0) = -0.03; l(0) = 13.2$, with $\beta_3 = 0.03$, and $\gamma_1 = 0.0$. The results are shown for our four variables in figure 3.

\textsuperscript{31}For a detailed study of the policy effects of the steps 1.-6., in the context of a model type of equations (1)-(3), see Wu and Zhang (2017) and Annex B.

\textsuperscript{32}Other scenarios can be considered as well, and a model version can be studied where one could use $\delta(y)$ instead of $\delta(l)$ for the 2nd and 3rd equations, see Gross and Semmler (2018).
Figure 3: Results for $\pi(0) = -0.01; y(0) = -0.03; l(0) = 13.2; \beta_3 = 0.03; \gamma_1 = 0$; deflationary regime with negative output gap, defeating the ELB through QE

The main monetary policy change in figure 3 is a positive credit shock, by setting $l(0) = 13.2$, which is equivalent to an increase of the central bank’s balance sheets by purchasing private assets and governments bonds.

As in the previous case the spread moves down with a positive credit shock due to QE while credit flow remains high initially but declines eventually. In addition inflation rate and output gap are mean reverting and move toward their steady states.\footnote{Here we by-pass modeling a slowly recovering banking system and improving asset markets which are likely to aid in reducing risk premia, supporting the recovery of the output gap and the success of inflation targeting.} This has also, as demonstrated in figure 1, an effect on risk premia and the credit spread, see figure 4 where the credit spread is first pushed down through QE. Yet, as the credit flow gap becomes negative, the credit spread starts rising again, see figure 4, after 35-40 periods. Thus, QE policies are effective by influencing credit flow and credit spreads. Still, credit spreads may rise again at a later stage of the business cycle and may produce another downturn. After credit flow falls below its threshold, roughly at period $t = 35 − 40$, all macro variables are affected adversely.

Note, however, that our setup builds on relaxed credit demand by assuming $\beta_3 = 0.03$. The lowering of the constraints worked as an additional stabilizing force.

In sum, the case of a deflationary economy, resulting from strong negative shocks, and an endogenous jump of risk premia and credit spreads, can be managed by increasing credit flows by unconventional monetary policy (increasing credit flows to $l(0) = 13.2$). The subsequent nonlinear reduction of credit spreads, and reduced credit constraints, with our positive feedback effects of
credit flows through $\beta_3 = 0.03$, is likely to reverse the negative output gap and negative inflation rate. Also note the significant role of asset markets and financial intermediaries conditions here. Yet new perils can arise as credit flow is passing its threshold from above, becoming negative and resulting in a quickly rising credit spread.

Note, however, that this recovery effect is not arising from changes in the policy rate, but rather from the strong reduction in credit spread and increasing credit flows. On the other hand, the inflation rate, see upper left figure in figure 3, increases due to the nonlinear Phillips curve. Inflation rates are only slowly rising when the output gap is recovering. On the other hand beyond period $t = 35 - 40$, new perils may arise when the credit gap becomes negative.

**Scenario 2: Credit supply policy—Insufficient credit flows and insufficient QE result in contraction**

Next we consider the case of a negative shock to credit flows. This scenario simulates an insufficient QE response by policy makers. We start with a mild recession and a small negative output gap, but a positive inflation rate. We assume the following initial conditions and parameters, for $\pi(0) = 0.03; y(0) = -0.03; l(0) = 10.8; \beta_3 = 0.03, \gamma_1 = 0$. A negative shock to credit flow is represented by setting $l(0) = 10.8$. Figure 5 shows the results of a negative credit shock. Note that credit flow dynamics start below their steady state value.

As mentioned before, we do not model asset markets and balance sheets of the banking system explicitly. Negative credit shocks in credit market have been studied in detail recently. It has been pointed out that negative credit shocks may arise from a decline in asset prices, affecting banks balance sheets

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$^{34}$The decreasing behavior of the variables at the end of the trajectories is due to our finite horizon decision problem.

$^{35}$Again, this situation may arise due to a negative net worth shock of financial intermediaries as in Brunnermeier and Sannikov (2014b).
by reducing their net worth and triggering fire sales of assets. This will reduce asset prices further leading to second round effects. For such a mechanism, see Brunnermeier and Sannikov (2014a), De Grauwe and Machiarelli (2015), Schleer and Semmler (2015) and Gross et al. (2017). In the latter work a mechanism is also estimated in a regime change model where banks reduce their loan supply when they are highly leveraged and stuck in a bad regime.

As figure 5 shows, there is some instability arising due to insufficient QE and insufficient credit flows. As expected risk premia are rising when the output gap is negative and asset prices are falling. In addition non-performing loans and a regime switch to high credit spread are observed. This is due to declining credit flow which are moving below some threshold, as shown in figure 1 and figure 4. Insufficient credit flow, creating a further fall of credit flows, and a jump into a regime of high credit spreads, entails a reduction in output. This is likely to generate negative inflation rates eventually with the nonlinear Philips Curve introducing a floor in this process. With regard to our credit spread variable and risk premia, we observe a rapid rise for both of them, accompanied by a rapid decline of credit flows, not depicted here.

Finally we observe that the ELB is only partially binding. The effect of small negative changes in credit flows, generating large risk premia, rising credit spreads and credit contractions could also occur in case of a slow and premature tapering of QE. This is in particular a peril if the economy and financial markets have been locked in a low credit cost economy for long period of time.
Scenario 3: Credit demand policy – Credit expansion reduces credit spread and allows output stabilization

Next, neglecting the feedback effect of credit flows on itself, we explore a stronger effect of credit flows on the output gap, setting \( \beta_3 = 0.15 \). We thus explore the following initial conditions and parameters: \( \pi(0) = 0.05; y(0) = 0.04; l(0) = 12.8; \beta_3 = 0.15, \gamma_1 = 0.0 \). In this case credit demand constrains are further relaxed, for example by reducing collateral requirements, allowing for a higher loan-to-value ratio in real estate mortgages, or by relaxing borrowing standards through central banks’ regulations and guidelines. As mentioned before, many of those measures fall into the area of macroprudential regulation.

![Figure 6: Results for \( \pi(0) = 0.05; y(0) = 0.04; l(0) = 12.8; \beta_3 = 0.15, \gamma_1 = 0.0 \); disinflation dynamics, credit expansion, policy rate moves to ELB, QE reduces credit spread, and achieves output stabilization](image)

The results are presented in figure 6. We observe small disinflation dynamics, credit remains high, while the policy rate moves to the ELB. QE keeps credit flows high, reduces credit spread, and achieves output stabilization. Note that in figure 6 we observe small disinflation dynamics, but credit flow moves to its steady state, and the policy rate moves to the ELB. Output, inflation, credit flows, risk premia premia and credit spreads are stabilized. In this scenario macro prudential policies seem to be working, which of course assumes that the temporary contraction of the output gap, see the chart for the output gap in figure 6, is presumably not accompanied by an asset price fall and banking

\[^{36}\text{Note that Ajello et al (2016), appendix, explore a parameter uncertainty of } \beta_3 = +(-)0.28 \text{ and its effect on the stabilization of the variables. Also note that this term of the credit flow equation can be derived from a NK DSGE model, when linearized, see Wu and Zhang (2017).}\]
instability.\footnote{The latter two scenarios point to the limits of macroprudential policies as fine tuning policies. Others have also expressed doubts whether macroprudential policies are available and sufficient. See Gourio, Kashyap, and Sim (2017: 1) where it is stated: “Many countries such as the United States have a limited set of macroprudential tools, and suffer from dispersion of regulatory authorities. The tools are difficult and slow to adjust, and their effects remain fairly uncertain.”} This at least holds as long as the credit flow gap is positive. As it becomes negative the credit spread starts rising again, as in figures 3 and 4. After credit flow falls below its threshold, roughly at period $t = 38 - 40$, all macro variables are affected adversely.

### 6.2 The Uncertainty of the Natural Rate

One other important issue in recent studies on monetary policy was the uncertainty of the natural rate. We explore here two scenarios, one pertaining to a relatively low natural rate and a second scenario with a high natural rate. We want to note that although in standard models the natural rate is driven by forces other than monetary policy, monetary policy and financial conditions such as risk premia and credit flows might also drive the natural rate, as for example Borio et al. (2018) claim.

#### Scenario 4: Higher Natural Rate Requires Adjustments in Targets

Indeed, recently much academic research predicted a change in the natural rate, in particular a downward movement after 2008-9. Economists used either filtering methods to show these trends to a lower natural rate (see Laubach and Williams (2003) and Laubach and Williams (2016) ), or elaborated on some long term forces to explain such presumed facts: such as excess savings and diminishing investment demands, Summers (2014)), demographic factors, productivity decline, measured as direct productivity or total factor productivity. Some recent work expresses the view that demand constraints for a protracted period of time can turn into supply constraints, namely when demand is weak, the constraints in the product, labor and financial markets are extensive, for example credit constraints are severe (since collaterals are down, and risk premia up) and households and firms can borrow less and thus expenditures for R&D, innovation, skill building and productive activities are declining, affecting the supply side, i.e. generating declining productivity growth, see Benigno and Fernaro (2017). In addition, Borio et al. (2018) state monetary policy and financial conditions could be an important factor as well.

Thus, in our next scenario we explore the effect of such downward trend in the natural rate. In monetary policy models this has been done in terms of regime change models, see Forster (2016).

We are first using a natural rate $r = 0.03$, as many authors have suggested for the long run. This is a rough approximation of the estimates of Borio et al. (2018:6) for the average of the period before and after 2007. The results for this case are shown in figure 7.
Figure 7: Results for $\pi(0) = -0.01; y(0) = -0.04; l(0) = 13.2$ (note that $\hat{l}$ is still at 12.8); $\beta_3 = 0.03$, $\gamma_1 = 0.0$; deflation dynamics, but credit expansion, stabilizes inflation rate at 0.0133, output gap at 0.0331, and the policy rate which moves to ELB, QE thus reduces credit spread, and achieves output and inflation stabilization stabilization, with the natural rate, $r = 0.03$, and the policy rate is about zero.
Figure 8: Results for $\pi(0) = -0.01; y(0) = -0.04; l(0) = 13.2$ (note that $\hat{l}$ is now at 13.1 instead of 12.8) ; $\beta_1 = 0.03$, $\gamma_1 = 0.0$; the natural rate is $r = 0.005$; first with deflation, but credit expansion, inflation rate stabilize at 0.012, output gap at 0.025, the policy rate moves to the ELB; QE thus reduces credit spread, and achieves output and inflation stabilization; but the policy rate is moving further into a negative region.

Scenario 5: lower natural rates – requires adjustment in targets

Next we let the natural rate decrease to $r = 0.005$, as some literature has suggested recently for the time period after 2007, see Borio et al. (2018), Eggertsson et al. (2017) and Benigno and Fernaro (2017).

As figure 8 shows, moving the natural rate to $r = 0.005$, gives rise to a higher steady state value of credit flows. Note that $l^*$ is now 13.1 instead of 12.8. But the decline in the natural rate also lowers the inflation target, and lowers the output target. Moreover the policy rate has to come down slightly, as compared to figure 7, to stabilize the economy. Thus, the ELB has moved down slightly as compared to the case of the higher natural rate of figure 7.38 Thus if the natural rate moves down, this requires adjustments in the central banks’ targets, in particular for credit flows.

In both natural rate variants we have not depicted the credit spread movements which behaved very similarly to figure 4. As the credit gap becomes negative the credit spread starts rising again, as in figure 4 and after the credit flow falls below its threshold, here roughly at period $t = 32 - 38$, all macro variables are affected adversely.

Overall, the above scenarios illustrate the perils of strong negative shocks.

38Note that, as discussed in section 3, the regime change to a different natural rate will also make the control variable, the policy rate, change.
to inflation and output gap when including credit flows and credit conditions.\footnote{On the dangers of a possibly arising debt deflation through this situation, see Ernst et al. (2017).}

The dangers come from the decline of credit flows and the rapid rise of risk premia, credit spreads and credit costs, that may not be reversed be reversed by interest rate or QE policies. Although there are scenarios of successful mean reverting policies, central banks are not always able to control private sector dynamics through monetary policy. Effective policy intervention depends on the state or regime of the business cycles, the expansion or contraction of credit flows, model specifications (the role of $\beta_3$), and the magnitude of credit flow shocks. Additional uncertainty is added when the natural rate changes — which may itself be impacted by monetary and financial conditions — and cannot be estimated exactly. As shown in figures 7 and 8, central banks need to adjust targets if the natural rate changes, although uncertainty about changes in the natural rate remains an obstacle in designing effective policy responses.\footnote{For a detailed discussion of the role of uncertainty in parameters and monetary policy, see Borio et al. (2018) who claim that the natural rate itself is impacted by monetary policy.}

7 A need for a mix of policies

Not only monetary policy options, but also the relevance of other policies has been discussed in the context of the practically implemented UMPs. Issues are:

- Credit price and credit volume policies: The (credit) price set by conventional and unconventional monetary policy may miss the specific needs of credit flows and liquidity to sectors, regions, households and businesses (for example to SMEs), see Brunnermeier and Sannikov (2014a), and the work on China by Chen et al. (2017). There is a need to study the relevance for selective credit policies in more detail, though these may involve the treasuries of different countries or currency regions. This holds both ways, namely when credit flows are incentives to encourage expenditure as well as for macroprudential policies to avoid sectoral credit booms.

- Fiscal policies complementary to UMP: recent studies have investigated the use of fiscal policies complementary to UMP; after some hastily implemented budgetary consolidation policies after the 2007-9 recession some revisions were made, also in the IMF policy advise, see Blanchard et al. (2013). It is now recognized that fiscal policy (either consolidation or expansionary fiscal policy) should be country and business cycle regime specific. Expansionary fiscal policy should be considered with more care if there is unsustainable debt and perils of a slow moving debt crisis, see Bobola and Lorenzini (2017). In addition, it is important to keep the business cycle dependency of budget consolidations in mind, i.e. budget consolidations are less advisable in recessions than in expansions, see also Mittnik and Semmler (2013) on the multi-regime multiplier.
• Growth oriented policies: Many countries support other policies, such as growth, innovation, industrial and climate policies to be enacted complementary to UMP. Here the EU’s Lisbon agreement has to be mentioned. The European commission has also initiated expert reports and policy advise concerning infrastructure, climate and growth oriented policies.

• Macro prudential policies: These are usually considered by central banks as “fine tuning policies”. These policies may not operate efficiently in currency unions like the Euro area due to a lack of cooperation between regulatory institutions within currency union. However, Gourio et al. (2016) observe inefficiencies for the United States as well. Thus, for policies of “Leaning Against the Wind” (LAW), i.e. policies tackling financial risk build up, other policies may have to substitute macro prudential policies when the latter is not so effective.

Such complementary policies to UMP — or even other first best policies — have been discussed in many countries and regions. However, central banks are usually bound to operate within their mandate, making complementary policies more difficult to implement. Yet additional policies have been implemented, often initiated through parliamentary oversight committees and through political and public pressures. Moreover, in a currency union such as the Euro area, the interaction between monetary and fiscal policy, as initially discussed by Tabellini (1986) in a game theoretic setting, needs to be discussed in greater detail.

Since the IMF is not bounded by mandates like central banks, it might initiate a discussion of a wider range of policies. As some observers mention, there are overburdened central banks, and if they retreat to the policy of rules and limited tasks as some observer suggest, see Taylor (2016) and Issing (2018), there will be a lack of institutions pursuing financial stabilization policy.

8 Long run side effects of the UMP

Central banks tend to defend the need, as well as the effectiveness, of UMP. This can be found in statements of the FED (see Bernanke, 2017 and Yellen, 2017) and the ECB (see Draghi, 2017). In addition, central banks’ research departments have been very active in demonstrating the effectiveness of UMP to their own constituencies, their home countries. Yet some critical views arose from some outside economists and institutions. Mainly three side effects have been addressed.

Financial crisis risks

UMP may lower - through increased credit flows - risk perception, shadow rates, and credit spreads. Asset price booms (stock market and real estate bubbles) and the risk of financial crisis may also be generated by excessive UMP. In terms of model variants this can be studied as a positive credit gap. Analytically these dangers can be integrated as an important trade-off in macroeconomic models,
The studies focus on the rise of financial bubbles, driven by excess credit gaps, and the probability of a bursting bubble. This is usually done by looking at cumulative growth rates of credit expansions for a number of years, typically over 4 to 5 years. More specifically, these studies focus on the benefits and costs of Leaning Against the Wind (LAW) when UMP is activated. The literature shows that the benefits and costs of LAW are presumably regime dependent and the benefits of avoiding future financial risks appear to be lower than the cost in terms of unemployment increases (see Svensson 2016, 2017), though this position is not uncontroversial.

Impact on income and wealth distribution

Another important side effect concerns the income and wealth effects of UMP. This has been noticed even by central banks and was discussed by high-level representatives of the FED (Janet Yellen) and the ECB (Peter Praet). For recent empirical studies on the distributional effects of UMP, see Otrok (2017) and Furceri et al. (2017). Yet, an empirical evaluation would necessitate more recent data from the Survey of Consumer Finance. Central Banks are now slowly providing these. Preliminary work in simpler models have been undertaken by the research departments of the FED and ECB. In this context it is probably interesting to study the effects of UMP on net wealth (or net worth), as they are affected by both asset prices and leveraging. On the other hand, analyzing central bank behavior suggests that accelerated credit expansion to close credit and employment gaps - which allows for reaping benefits from re-employment and wage income - remains a priority. Furthermore, a possible new housing boom, and wealth effect from housing wealth for lower income groups, should also be taken into account when judging the distributional effects of UMP. Ultimately the question becomes if central banks should be thinking about compensating losers? So far only few quantitative studies have been undertaken on these issues, though partially addressed in Otrok (2017). There is however substantial research on capital account liberalization which has shown to have little effects on growth rates, but significant effects on inequality, see Furceri et al. (2017), a research framework that might be applied to the issue at hand.

International spillovers and linked vulnerabilities

Many studies have recently focused on the issue of international spillovers. Some of the proponents of UMP point to the positive international spillover effects (capital flows and lower borrowing cost in other countries), see Cesa-Bianchi et al. (2017). But a great majority of studies point to negative spillover effects, see Rajan (2013). Studies on the globalization of capital markets and international spillover effects of UMP have shown that 1) there is an international synchronization of booms and busts in international capital flows, and 2) quick transmission of international liquidity is dominantly occurring through internationally operating banks. When there was a provision of liquidity through UMP
and a decline of credit spreads and shadow rates in advanced countries, large capital inflows in emerging market economies were the result. This led to the appreciation of currencies, and current account problems. This created not only volatility in financial markets for these countries, but exposed them to the “sudden shocks”, in particular if they are hold large amounts of debt denominated in foreign currencies, for example in US dollars. Large scale models of global VAR (GVAR) type can be used to study how these international spillovers take place, see Gross and Binder (2015). One should also note, as stated in the literature, that there is a more general problem involved, namely the extent to which financial markets in emerging markets have stabilizing mechanisms in place which guarantee screening, regulation and oversight on the financial sector in case of strong credit cycles (see Mishkin, 2011, for more details).

9 Conclusions

1. It appears appropriate to use a simplified macro-policy framework that includes credit flows, risk premia and credit spreads as suggested above for studying the dynamics of the financial market meltdown of 2007-9. The evolution of views on the crisis and prolonged recession, policy steps toward UMP, the effects of the “new normal”, tapering, as well as international spillovers, policy actions and multilateral responses, can be studied with the help of such a model. It can also be used (partly through extensions) for studying the side effects discussed above, for example for considering spillover effects of UMP.

2. How are UMPs more specifically justified ex post? For the US an extensive analysis is given by Bernanke (2017). He addresses ex-post various kinds of criticism related to QE policy, such as: QE 1) produces precarious asset price bubbles, 2) leads to over-borrowing, sowing the seeds for the next cycle and bust, 3) creates wealth and income inequality due capital gains affecting mostly the top of the wealth pyramid, 4) does not leave any suitable instrument to manage the next crisis, and thus 5) points to the loss of potent monetary instruments for any subsequent recession. Bernanke (2017) responds to all those criticisms by defending QE and UMP. Yet, some further research should be undertaken along those lines, in particularly addressing the issue of what monetary tools are available for the next recession.

3. The official view of the ECB is similar, as stated by Draghi (2017): “... We introduced a number of unconventional measures including negative deposit facility rates, targeted longer-term refinancing operations (TLTROs) and an expanded asset purchase program. These measures – known as our “credit easing” package – were aimed at combating the impairment of the transmission mechanism...”, Draghi ex-post evaluates UMP as a convincing policy to avoid a credit meltdown and to turn credit growth in the
EU around, see Draghi (2017). But concerning financial crisis perils he also states: "Of particular concern is the development of so-called credit-fueled bubbles, which previous experience has shown to be particularly detrimental to financial stability."

4. Yet, in the Euro area critical views have also been raised by Northern countries pointing to long term low returns for savings of low income groups, see German Bundesbank (2015). In Northern countries less benefits and more deficiencies with respect to the long-run effects of UMP are seen. The quite big disparity between the diverse fiscal policies (without risk sharing) and the single EU monetary policy may have generated those effects.

5. Concerning the containment of international spillovers and fallouts, one should mention the need for better stabilization of financial and banking sector in emerging market economies. This should be done by financial market regulation, screening and monitoring, see Mishkin (2011) and Rodrik (2012).

6. Are there other policies that should be implemented? What would be a wider range of policies suitable, to be supported by the IMF? Credit volume policies? Inclusive growth? Climate policies and support of sectoral change and adjustments? Policies to support impact investment and climate finance? These policies often come up in parliamentary hearings and policy circles, but may not need be directly related to UMP.

7. An additional conclusion could be that the link between central banks, aiming at both monetary and financial stability, and the IMF should be strengthened. This should be done on the basis of macro policy models with a strong financial side. The design and use of macro-models without well-characterized financial systems, prone to instability (including financial intermediaries, markets, frictions and regimes of major distress and dislocation) should be promoted less. It is important to acknowledge that institutions and banks (or other financial intermediaries) are creating inside money and finance. The financial stability models with only exogenous shocks and no endogenous risk creation, no feedback loops and endogenous propagation mechanisms should not be viewed as reliable models for policy decisions.

41 Draghi (2017) confirms the crucial mechanism of credit flows and credit spread as main drivers for recent expansions and contractions: “The unconventional monetary policy...[allowed] to transmit the ECB’s credit impulse to firms and households across the monetary union and to ensure sufficient financing to sustain the recovery – which is exactly what we have seen. In the first quarter of 2017, annual loan growth to Euro area households stood at 2.6% and non-financial corporations at 1.6%, up from respective troughs of -0.6% in the second quarter of 2014 and -3.6% in the third quarter of 2013. Bank lending rates for both firms and households have dropped by around 110 basis points over the past three years and are now at historical lows.”

42 For further details see the book by Hartmann et al. (2018) in honor of Charles Goodhart.
8. Another important discovery is that there are large challenges spanning across borders. National central banks cannot work in isolation. Cross-border linkages in the international monetary and financial system create important interdependencies. This issue also requires more attention by the IMF. If central banks limit themselves to their traditional role of inflation targeting again, focusing on their domestic constituencies, then the IMF is more than ever needed to move forward by studying and pursuing policies of medium and long term externalities and spillover effects. It also needs to pay attention to the interconnectedness of cross-border vulnerabilities, and focus on wider policy issues and policy tools.

9. A special IMF international committee might be suggested with the aim of studying, screening, and responding to international spillovers, challenges and interconnected vulnerabilities across borders. This might be similar to the Systemic Risk Committee as suggested by the Dodd-Frank act for the US.
References


Blanchard, O., G. Dell’Ariccia, and P. Mauro (2013). Rethinking Macro Policy II: Getting Granular, IMF Staff Discussion Note 13/03.


[37] DIW Vierteljahreshefte zur Wirtschaftsforschung, 01, 2016.


A  The dynamics with borrowing constraints –
   The role of credit volume policies

The second equation of system (1), and in difference form in equation (9), need some further derivation. There are numerous studies on endogenously generated borrowing constraints for agents who want to borrow. How agents are endogenously constrained by the swings in asset value is presented in Kiyotaki and Moore (1997), for agricultural farming, and for a model with industrial production in Miller and Stiglitz (1999), among others.

Formally borrowing constraints for the agents in an economy could be formulated as follows:

\[ l_t = \phi(q_t k_t) \]  

with \( q_t \), asset price, \( l_t \), the loans taken on as liability and \( \phi \) the fraction of collateralized assets, defining the borrowing constraints. Thus, the borrowing is constrained by the value of the collateral and \( \phi \), the collateralized fraction of the asset. In Kiyotaki and Moore (1997) being concerned with agricultural production, the constraint is driven by the harvest in the last period, and in Miller
and Stiglitz (1999) by the last period’s sales of the firm.\footnote{Stiglitz (2018) in a recent review article on macro theory claims that the analysis of the Great Recession, with its dynamic financial-real interaction, cannot be properly analyzed without studying the role of credit constraints.}

In Wu and Zhang (2017) the borrowing capacity is driven by a fraction of the value of capital as in equation (11), determined by both the volume of capital and Tobin’s q, the value of a unit of capital. Spending for consumption is derived from the first order condition of the Euler equation through an (infinite time) optimization problem, but the borrowing capacity, depending on the value of assets as well the fraction of collateralized assets, enters the consumers’ budget constraints. Thus relaxing the budget constraints, for example through an extension of lending facilities, via an increase of the loan to value ratio, affecting $\phi$, allows for an ease of borrowing and decline of the shadow interest rate in their model. They also show how the credit constraint gets relaxed through QE, entailing a decline of the shadow rate to an even negative region so that the credit cost can move even below the ZLB.

Another interesting version is suggested in Eggertsson and Krugman (2012). They study an endowment economy and focus on consumers only—on savers with a high discount rate, and borrowers with low discount rate. The size of borrowing could be high, for the borrowers when the discounted income is high. Yet when the asset value is the collateral for borrowing, and the asset value falls, this entails lower borrowing, and a switch from the high to a low borrowing case, with consumption spending reduced. This collateral value shock and the switch from high to the low borrowing of households is what they call the Minsky deleveraging effect. This could give in their view rise to a cumulative downward spiral, with asset value and income falling, and real debt rising— the Fisher debt deflation process.

In our model of system (1)-(3), in continuous time, and (5)-(8) and (9), in discrete time form, we have a term $\beta_3(e^l - \hat{l})/\hat{l}$ included that represents the above considerations of constrained borrowing. We propose that the excess (positive) credit gap strongly co-varies with the asset prices (stock market and real estate, as in Jorda et al. 2011, 2012), and a fraction of assets are collateralized as in Wu and Zhang (2017). We assume, however, that in the economy there is only a fraction of economic activities that are eligible to obtain collateralized loans. For example $\beta_3 = 0.03$ can mean that $\phi = 0.3$ and only 10 percent of agents can obtain collateralized loans, see scenarios 1 to 2. In scenario 3 the latter ratio is then increased to 50 percent through central banks’ lending facilities. Note that whenever excess credit is rising (increase of the positive credit gap), for example through CBs’ increase of borrowing facilities, the output gap is likely to close faster. In other words, the credit gap $\beta_3(e^l - \hat{l})/\hat{l}$, besides its effect on the credit spread, and direct effect on the output gap, aids to overcome loan rationing. Surely, a number of macro prudential policy tools will have their impact on this term, see scenario 3.

Note that our inclusion of the excess credit gap term is closely related to Eggertsson and Krugman (2012), where credit volume policies also become im-
important, and resembles Wu and Zhang (2017), where a term such as $\beta_3(e^l - \hat{l})/\hat{l}$ is derived from a (consumer) budget constraint, based on an infinite horizon decision model. See their derivation of the Euler equation with bond issuing and their local linearization study in their appendix A.1. In all of these cases with collateral constrained borrowing, heterogeneity and fragmented capital markets, a volume oriented credit policy and macroprudential policy tools might also be advisable, see Brunnermeier and Sannikov (2014b).

B Credit cycles

The credit cycle without UMP

A typical credit boom-bust cycle, accompanied by credit spread and output variation, is presented in Krishnamurthy and Muir (2016). In figure 9 one can observe the relationship between credit flow, credit spread and GDP growth. The data are assembled from many countries.

Figure 9: Credit spreads, GDP growth and credit flows around a financial crisis starting with at date 0. GDP and credit expressed as cumulative annual percentage growth, spreads are normalized by dividing by the unconditional mean, all variables are demeaned at the country level; Krishnamurthy and Muir (2017).

A typical stylized interaction of credit cycles, credit spread and output is depicted in figure 9. While credit flows go up, credit spread is low, and the GDP growth path moves up. When the credit path is declining, following GDP growth decline, accelerated by the credit spread strongly rising, credit flows and GDP growth are further reduced. Credit spread is cause and effect of credit and output growth and decline. Yet data for figure 9 comes from a large set of countries from earlier times, with mostly UMP not yet implemented. Such a dynamic interaction is sketched in system (1)-(4). A sketch of how UMP is interacting with the variables displayed in figure 9, is discussed next.
The credit cycle and UMP

Now we can add, in a stylized way, the kind of effects interest rate policy and UMP (QE, or APP) would have. We focus mainly on the impact of UMP on credit flows and credit spreads.

There are stylized facts related to credit flows, credit spread measures and UMP, for both the US as well as the Euro-area. Corresponding to our model (1)-(4) the following is shown to hold: 1) a strong negative co-movement of credit spread (shadow interest rate) and credit flows, 2) a strong negative co-movement of the QE (in the US called LSAP, and APP in the EU) with credit spread, and 3) a strong positive co-movement of credit flows and QE. Those three claims can be seen to hold in our figures 10-14.

![Figure 10: US credit flows and credit spread represented by BBB bond yields; upper graph: credit flows (time varying trend), lower graph: BBB bond yields, data from FRED](image)

In Figure 10 we use smoothed data for BBB bond yields, to measure credit spreads, and the flow of new loans to household and non-financial business sector. A clear negative co-variation is observed, though with some phase shift, i.e. loan peaks occur later.

Figure 11 shows BBB bond yields, our measure of credit spread, and how the yields were driven down by the effects of QE since 2009. Note that for the effect of QE policy we use the inverted log of the FED’s balance sheets.

Figure 12 establishes smoothed credit (annual) credit flows, and the inverted log of FED’s balance sheets. A quite strong correlation between those two is observable.

For the Euro area similar results are obtained. Figure 13 shows credit spreads and credit flows for the Euro area.
For the Euro area credit spreads were reduced through QE, though this has development set in later than in the US, see figure 14. Due to the lack of corporate bond markets we measure credit spreads by the banks’ lending spread to firms in the Euro area the, see figure 14.

In figure 14 we can observe a strong co-movement of credit spreads for firms (lower chart) and the QE policy (upper chart).

Figure 14 shows the ECB balance sheets (upper panel) and credit flows (lower panel). Again regarding QE policy and credit flows in the Euro area, we see the same co-movement as in the US, with increased asset volume in the balance sheets of the ECB credit flows are moving up. But there is a particular period observable between 2012-13, where QE and the asset purchasing program is retarding, entailing a decrease of credit flows and a temporary increase in credit spreads. This is presumably related to the sovereign debt crisis in the Euro area.

C UMP and the shadow interest rate

Many recent studies use a computed shadow rate for measuring the movement of risk premia and credit spreads, in particular to provide some measure for negative interest rates. For the US economy, as well as for the euro area, there is extensive empirical literature that points to a strong co-movements of UMP, credit flows and the shadow interest rate, see Wu and Zhang (2017) for the US and for the Euro area Lemke and Vladu (2017) and Mouhabbi and Sahuc (2017). The focus is on QE and the central banks’ balance sheets and their impact on the shadow rate instead on the credit spread, though there is a strong correlation between the latter two.

Theoretical, as well as empirical studies for the US show a strong negative co-movement of the shadow rate and QE, see Wu and Zhang (2017). Their shadow
rate is computed for two regimes, for normal and bad times. For normal times the shadow rate is obtained from an estimation of the policy rate using the Taylor rule. Thereby it is assumed that in good times asset purchases of the central bank are just normal and the central bank does not increase its balance sheets. Yet in a bad regime when the policy rate reaches the ZLB a shadow rate is computed by taking into account estimates of the term structure risk premium, liquidity premium, and corporate bond premia. In the model by Wu and Zhang (2017) the monetary policy of QE type is affecting the shadow rate as follows:

\[ r^B_t = r_t + rp - \zeta (b^{CB}_t - b^{CB}) \]  

(12)

Hence in normal times \( b^{CB}_t = b^{CB} \) one has \( r_t + rp \) and monetary policy operates through the Taylor rule, and \( \zeta \) is a positive coefficient. When the policy rate reaches the ZLB, \( r_t = 0 \), the bond rate becomes \( r^B_t = s_t + rp \) with \( s_t < 0 \), due to the CB’s asset purchasing program. Hereby, in the first step \( rp \) taken as a constant risk premium that can be made stochastic.

In Wu and Zhang equation (12) allows for a negative shadow rate when the ZLB is reached, and the central bank pursues UMP, see Wu and Zhang (2017, figure 1). In our case, when we have QE and \( (e^l - \hat{l}) > 0 \), the shadow rate, can be negative given the optimal policy rate, \( i_t \), which could be negative – so could be the credit spread in figure 1 under the impact of QE. This corresponds to what has been observed for the Euro area where at some time period after 2014, the Deposit Facility and also the EONIA became negative. Consequently the yields for bonds of shorter maturity became negative, see Lemke and Vladu (2017).

In our model (1)-(4) we allow the credit spread, due to a rising risk premia \( \delta(l) \), to jump up for \( (e^l - \hat{l}) < 0 \), with \( \hat{l} \) as threshold, defining the credit gap. This would increase the markup on the policy rate, \( i_t \), as in figure 1. This case is neglected in Wu and Zhang (2017) since they start their study only with the
downward trend of the shadow rate, in 2009, and define the time before 2009 as normal times, with $b_{t}^{CB} = b^{CB}$. By using data before 2009, we can show a strong (negative) co-movement of credit flows and credit spreads, as in figure 13.

Note that in our case of system (1)-(4) the optimal policy rate itself could also be negative. Whenever we have $(e^{i} - \hat{l}) > 0$, through QE, the credit spread would move down toward zero, see figure 1. The optimal policy rate, $i_{t}$, could become negative, even if the policy rate is bounded at zero. The optimal negative interest rate and the QE impact on the spread allow for reducing the output gap and lifting the inflation rate. The actual real interest is of course affected by the inflation rate as well.

For the Euro area a shadow rate, $s_{t}$, has also been computed by Mouhabbi and Sahuc (2017). The rate turns negative in 2014. In the Euro area the shadow rate was driven down by the Asset Purchasing Program (APP) and forward guidance. The shadow rate for the Euro area, computed in Mouhabbi and Sahuc (2017), roughly replicates the figures for the US shadow rate by Wu and Zhang (2017). The former use a principle component and dynamic factor model to extract information on the shadow rate.

Figure 13: Credit flows and credit spread in the Euro area, data source, ECB
D Sequence of monetary policy actions in the US and Euro area

1. The US FED decisions

<table>
<thead>
<tr>
<th>Date</th>
<th>Action</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 09.2007</td>
<td>Interest Rate Policy</td>
<td>Federal Funds target rate is lowered by 50 basis points to 4.75% due to tightening of credit conditions.\textsuperscript{44}</td>
</tr>
</tbody>
</table>

\textsuperscript{44}https://www.federalreserve.gov/newsevents/pressreleases/monetary20070918a.htm
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007 - 2009</td>
<td>TAF*, TARP* and other</td>
<td>Providing loans, capital and credit, primarily to financial institutions; the Fed participates in stabilization of &quot;too big to fail&quot; institutions such as AIG.</td>
</tr>
<tr>
<td></td>
<td>programs</td>
<td></td>
</tr>
<tr>
<td>10.2008</td>
<td>Interest Rate Policy</td>
<td>Interest on bank reserves introduced.</td>
</tr>
</tbody>
</table>
| 11.2008    | QE1                        | Large scale asset purchases (LSAPs): $100 billion in government sponsored enterprise (GSE) debt from Fannie Mae, Freddie Mac, and the Federal Home Loan Banks and $500 billion in mortgage-backed securities (MBS) are purchased.  

45

12.2008    | Interest Rate Policy       | Zero lower bound (ZLB): Target range for the Federal funds rate 0% to 0.25%.                                                                                                                   |
| 03.2009    | QE1                        | LSAPs are expanded: $300 billion in long-term treasuries and an additional $750 billion and $100 billion in MBS and GSE debt, respectively.                                                                 |
| 08.2011    | QE2                        | Purchase of $600 billion in treasuries.                                                                                                                                                           |
| 03.2010    | Switch to calendar-based   | "The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period."  

46

8.2011     | Switch to calendar-based   | Low rates at "least through mid-2013."                                                                                                                                                           |
|            | guidance                   |                                                                                                                                        |

45https://www.federalreserve.gov/newsevents/pressreleases/monetary20081125b.htm  
46https://www.federalreserve.gov/newsevents/pressreleases/monetary20090318a.htm
<table>
<thead>
<tr>
<th>Date</th>
<th>Event Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>09.2011</td>
<td>MEP</td>
<td>Maturity Expansion Program: purchase of $400 billion of treasuries with remaining maturities of 6 to 30 years; sales of an equal amount of treasuries with remaining maturities of 3 years or less.</td>
</tr>
<tr>
<td>01.2012</td>
<td>Calendar-based guidance extended to 2014</td>
<td>Low rates &quot;at least through late 2014.&quot;</td>
</tr>
<tr>
<td>09.2012</td>
<td>QE3</td>
<td>Purchase of $40 billion of MBS per month as long as &quot;the outlook for the labor market does not improve substantially ...in the context of price stability&quot;</td>
</tr>
<tr>
<td></td>
<td>Calendar-based guidance extended to mid-2015</td>
<td>Low rates at &quot;least through mid-2015.&quot;</td>
</tr>
<tr>
<td>12.2012</td>
<td>Switch to state-contingent guidance</td>
<td>Low rates will be appropriate while unemployment is above 6.5% and inflation is forecast below 2.5%.</td>
</tr>
<tr>
<td>09.2013</td>
<td>Reverse repos introduced</td>
<td>Support the target range for Federal funds rate, upon exit from the zero lower bound.</td>
</tr>
<tr>
<td>12.2015</td>
<td>Interest Rate Policy</td>
<td>Exit from ZLB: Target range for the Federal funds rate raised to 0.25% - 0.50%.</td>
</tr>
</tbody>
</table>

Source: The table is based on Borio and Zabai (2016), Greenlaw et al. (2018), and own research.
Figure 16: EU shadow interest rate and other interest rates, from Mouhabbi and Sahuc (2017)

2. The Euro area ECB decisions

<table>
<thead>
<tr>
<th>Date</th>
<th>Action</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 10.2008</td>
<td>Interest Rate Policy</td>
<td>Main refinancing rate lowered from 4.25% to 3.75%; main refinancing operations via fixed rate tenders.</td>
</tr>
<tr>
<td>05.2009</td>
<td>Interest Rate Policy</td>
<td>Main refinancing rate lowered to record low of 1%.</td>
</tr>
<tr>
<td></td>
<td>Non-standard measures</td>
<td>Covered Bond Purchase Programme (CBPP1) introduced.</td>
</tr>
<tr>
<td>10.2010</td>
<td>SMP</td>
<td>Securities Market Program: intervention in the euro-area public and private debt securities announced; purchases will be sterilised.</td>
</tr>
<tr>
<td>10.2011</td>
<td>Non-standard measures</td>
<td>Covered Bond Purchase Programme (CBPP2) introduced.</td>
</tr>
<tr>
<td>Date</td>
<td>Event</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>----------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>06.2012</td>
<td>OMT</td>
<td>Outright money transaction: countries applying to the European Stability Mechanism (ESM) for aid and abide will be eligible to have their debt purchased in unlimited amounts on the secondary market by the ECB.</td>
</tr>
<tr>
<td>07.2013</td>
<td>Open-ended guidance</td>
<td>Key interest rates are expected &quot;to remain at present or lower levels for an extended period of time.&quot;</td>
</tr>
<tr>
<td>06.2014</td>
<td>Interest Rate Policy</td>
<td>Deposit facility enters negative territory: -0.10%; flattening of short- to medium-end of yield curve.</td>
</tr>
<tr>
<td></td>
<td>Non-standard measures</td>
<td>Introduction of targeted longer-term refinancing operations (TLTROs) and longer-term refinancing operations (LTROs)</td>
</tr>
<tr>
<td>09.2014</td>
<td>APP/ABSP and CBPP3</td>
<td>Asset Purchasing Program/Asset-Backed Securities Purchase Program and Covered Bond Purchase Programme 3; further depression of term structure of interest rates.</td>
</tr>
<tr>
<td>01.2015</td>
<td>APP/PSPP</td>
<td>Public Sector Purchase Program: purchase of bonds of euro-area central government, agencies and European institutions.</td>
</tr>
<tr>
<td>03.2016</td>
<td>Interest Rate Policy</td>
<td>Zero interest rate policy: Main refinancing rate lowered to 0%; deposit facility at -0.40%.</td>
</tr>
<tr>
<td></td>
<td>APP</td>
<td>APP is expanded to $80 billion monthly starting in April 2016; investment grade euro-denominated bonds issued by non-bank corporations established in the euro-area included in the list of eligible assets for regular purchases; introduction of TLTRO II starting in June 2016.</td>
</tr>
<tr>
<td>12.2016</td>
<td>APP</td>
<td>APP is reduced to $60 billion monthly starting in April 2017.</td>
</tr>
</tbody>
</table>
APP is reduced to $30 billion monthly: January 2018 - September 2018.

Source: The table is based on Borio and Zabai (2016) and own research.

**E Empirics using a regime-switching VAR**

Since the proper empirics on a change of the long-run natural rate is still missing, particular due to an insufficient quality of long time series data, we restrict ourselves to an RS-VAR estimation without attempting to specify a time varying natural rate, using shorter time series data at monthly frequency. We use monthly data (January 2003 to April 2016) of the Euro area. For further details see Faulwasser et al. (2018).

Empirically, we look at four types of shocks and exploring their effects: 1. conventional monetary policy shock (policy rate changes), 2. UMP as change of the CB’s balance sheet (pure QE, see scenario 1 of sect. 6), 3. loan supply shocks affecting directly the credit spread and loan rates of banks (represented by scenario 2\textsuperscript{47}), and 4. loan demand shock (reducing the credit constraints, increase of $\beta_3$, see scenario 3 in sect. 6). Thus, here in the RS-VAR, we have decoupled UMP, as change of the CB’s balance sheet, from the asset market and credit spread effects captured in shocks 3.- 4. All shocks are summarized for our RS-VAR in table 2, rows 1-4, each of the shocks affecting six macro variables.\textsuperscript{48}

The RS-VAR model structure, with 2 lags, can be written as follows.

$$y_t = c_r + \sum_{i=1}^{p} A_r y_{t-i} + B_r z_t + u_{rt}$$  \hspace{1cm} (13)

where $y_t = (y_{1t}, ..., y_{Kt})'$ is a vector of dimension $K \times 1$ comprising $K$ endogenous variables, $c_r$ are the intercept coefficients under the two regimes ($r = 1, 2$), $A_r$ are $K \times K$ matrices of coefficients, and $B_r$ are $K \times G$ coefficient matrices loading an exogenous variable vector $z_t = (z_{1t}, ..., z_{Gt})'$ of length $G$. The $u_{rt}$ is a $K$-dimensional error term whose covariance matrix $E(u_{rt}u_{rt}') = \Sigma_r$ is allowed to be regime-specific, too. For the linear variant of eq. 13 without regime dependence we let the $r$ subscripts drop.

The macro variables that the vector $y$ contains include real GDP in natural log (ln) differences month-on-month (MoM), HICP inflation lnMoM, new business bank loan volume flows to the nonfinancial private sector lnMoM, banks’

\textsuperscript{47}Note that though figure 4 represents a rise in spread, one would obtain a declining spread corresponding to credit flow increase.

\textsuperscript{48}For details of the following RS-VAR estimation, impulse-response functions, and data sources, see Faulwasser et al. (2018).
new business loan rates, ECB total assets in lnMoM, and a nominal short-term interest rate (3-month Euribor). Our choice of employing flow-based measures (credit flows) and interest rates instead of stock-based measures corresponds to the theoretical model since the loan volume variable is supposed to reflect credit to new business (and households) instead of stocks or changes of stocks.\footnote{See Biggs et al. (2009) who develop a theoretical model that highlights this point and shows that consumption and investment flows are related primarily to new lending rather than to the stock or changes in the stock of loans.}

Slightly modifying our theoretical model, we take in the empirics not the credit gap as variable driving the credit spread but the output gap, which is easier to measure and much work has been done here to obtain robust measures of gaps.\footnote{In the Annex B we show that they behave quite similar over the business cycle.} To measure the output gap, we use the procedure by Jarocinski and Lenza (2016) who operate with a Bayesian dynamic factor model for the euro area to imply the estimates of the unobservable output gaps which are consistent with observed inflation dynamics.

Based on the Jarocinski and Lenza (2016) we derive a 0-1 indicator, which is 1 if the output gap is positive and zero otherwise. The results that we present in the following are robust to the various different output gap and regime inference schemes as presented in Gross and Semmler (2018). The RS-VAR is done with sign restrictions, displayed as in table 2. Note that rows 2 and 3 of table 2 corresponds to scenarios 1 and 2 with loan supply shocks of sect. 6, and the row 4 corresponds to the scenario 3 with loan demand shock of sect. 6. The shocks of row 1 are not evaluated in sect. 6 because there the policy rate is endogenous.

To reveal the linear and nonlinear model’s dynamics we simulate sign-

### Table 2: Sign restriction settings for linear and regime switching model-based impulse response analysis

<table>
<thead>
<tr>
<th>#</th>
<th>Description</th>
<th>RGDP</th>
<th>INF</th>
<th>NGDP</th>
<th>NBV</th>
<th>NBI</th>
<th>ECBTA</th>
<th>STN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conventional expansionary monetary policy shock (price-based)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-25bps</td>
</tr>
<tr>
<td>2</td>
<td>Unconventional expansionary monetary policy shock (volume-based)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-25bps</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Positive loan supply shock (price-based)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-25bps</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Positive loan demand shock (volume-based)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-25bps</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
restricted impulse responses (SR-IRs).\footnote{As an entry point to the literature related to sign-restricted SVARs see Faust (1998), Canova et al. (2002), and Uhlig (2005), for details of our results see Faulwasser et al. (2018).} To derive the impulse responses from the regime-switching models, we take the coefficient sets and covariance matrix estimates that are specific to the regimes of the regime-switching version of the VAR and simulate the impulse responses assuming that the regimes keep prevailing.\footnote{See e.g. Ehrmann et al. (2003) who use the same regime-dependent impulse response simulation scheme. Other model settings are conceivable, whereby the regime process would be endogenous, for shocks to possibly imply, depending on their size, a transition between regimes.}

We have simulate the above mentioned four shock scenarios, whose results are summarized in table 3. All four scenarios are positive shock scenarios and they correspond to a conventional interest rate-based monetary policy shock (CMP), an unconventional volume-based monetary policy shock (UMP as change of CB’s balance sheets), a loan supply shock (LS), and a loan demand shock (LD), the LS works through the loan rate - interest rate spread, and the LD through relaxing the credit constraints, a volume effect. There are no constraints imposed on real GDP and inflation in any of the scenarios. A positive constraint is under all shock scenarios imposed on nominal GDP which is an off-model variable whose paths are proxied by the sum of real GDP growth and inflation during the simulation. Under the LS and LD scenarios the shocks are meant to originate more directly in relation to the banking system instead of the central bank, in which case the central bank total assets and the short-term interest rates are in fact not constrained.

Overall, the first three shock scenarios reflect supply side-type shock scenarios which can be seen by the fact that bank interest rates are assumed to fall or are shocked negatively. In addition to the responses of the core model variables, two off-model variables’ reactions are shown, that is, the aforementioned nominal GDP proxy as well as a credit spread which is defined as the difference between the loan interest rate and the short-term money market rate. That variable corresponds to the credit spread $\delta$ in the theoretical model, its called NBISTNspread in table 3.

The table 3 summarizes all our results of the RS-VAR and impulse-responses.\footnote{The corresponding figures of the IRs can be found in Faulwasser et al. (2018).} We report in table 3 the differential effects of contractions and expansions, $R_2-R_1$. Across all four shock scenarios, the response of inflation is positive and it is more positive under an assumed expansion regime, both on impact in the first month and in cumulative terms after 18 months. This outcome was not pre-informed by any sign constraints, and is as robust a finding as in Gross and Semmler (2018); also when considering various different measures of economic slack to inform the regime process.

The nominal GDP response was constrained to be positive; yet we see the same feature that was not pre-informed insofar as the nominal GDP response is more positive under the assumed expansion regime; under all four shock scenarios. Real GDP responses are generally positive, and by the end of the...
18-month period are again more positive under the assumed expansion regime. Concerning nominal loan flow growth, we see a similar pattern as for nominal GDP, that is, positive responses, which are more positive under the expansion regime.

With respect to the credit spread responses (NBISTNspread), we can observe that they are negative and quite persistent under all scenarios, meaning that lending conditions are eased not only because level interest rates fall but also the spreads on top fall, reflecting in turn that the borrowers’ default risk would fall under the scenarios. It is noteworthy to observe for (NBISTNspread) in table 3 that under all shock scenarios, the differential between the contractions and expansion regime-conditional response of credit spreads is positive, meaning that the fall in spreads is more negative under the expansion regime, except under the loan supply shock, where the effects are roughly similar in contractions and expansions.

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Table 3: Differences of cumulative effects – responses for recession and expansion regimes after 18 months (differential effects of contractions and expansions, R2-R1)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Conventional expansionary monetary policy shock (price-based)</th>
<th>Unconventional expansionary monetary policy shock (volume-based)</th>
<th>Positive loan supply shock (price-based)</th>
<th>Positive loan demand shock (volume-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGDP</td>
<td>-0.06</td>
<td>-0.22</td>
<td>-0.97</td>
<td>-1.48</td>
</tr>
<tr>
<td>INF</td>
<td>-1.14</td>
<td>-2.08</td>
<td>-0.78</td>
<td>-0.91</td>
</tr>
<tr>
<td>NGDP</td>
<td>-1.16</td>
<td>-2.27</td>
<td>-1.75</td>
<td>-2.47</td>
</tr>
<tr>
<td>NBV</td>
<td>-9.51</td>
<td>-17.73</td>
<td>-4.61</td>
<td>-4.31</td>
</tr>
<tr>
<td>NBI</td>
<td>-1.95</td>
<td>-3.04</td>
<td>-3.22</td>
<td>-9.28</td>
</tr>
<tr>
<td>ECBTA</td>
<td>13.47</td>
<td>20.76</td>
<td>11.13</td>
<td>-11.93</td>
</tr>
<tr>
<td>STN</td>
<td>-3.74</td>
<td>-6.14</td>
<td>-3.96</td>
<td>-15.62</td>
</tr>
<tr>
<td>NBISTNspread</td>
<td>1.73</td>
<td>2.99</td>
<td>0.67</td>
<td>6.03</td>
</tr>
</tbody>
</table>

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54 This is reported in the impulse-response figures in Faulwasser et al. (2018), Annex 4.