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Crisis and theoretical methods: equilibrium and disequilibrium once again

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Crisis and theoretical methods: equilibrium and disequilibrium once again

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Abstract

The financial crisis of 2007-8 damaged the credibility of macroeconomic analysis based on price-taking Walrasian intertemporal general equilibrium models. This talk explores methodological alternatives, particularly stable Nash-Cournot equilibria of social interaction models that center on agents' response to other agents' actions rather than on agents' forecasts of future paths of prices and production. Social interaction equilibria in conjunction with constraints from information theory highlight the social coordination problems at the root of macroeconomic policy questions. Equilibrium concepts enhance the explanatory power of economic theories in contrast with the limitations of disequilibrium dynamical systems analysis and agent-based modeling. Constrained maximum entropy methods offer a general approach to macroeconomic modeling. Various conceptions of equilibrium in economics arise from distinct conceptions of expectations.

Keywords: economic equilibrium, statistical equilibrium, Nash-Cournot equilibrium, expectations, maximum entropy. JEL codes: B22, B40, C62, C72, D50, E12.

1 The wrong sort of bees

The wrong sort of bees

- A memorable episode in A. A. Milne's *Winnie-the-Pooh* recounts Pooh's attempt to steal the honey from a bees' nest by floating himself to the top of the tree where the nest is located. This attempt ends in disaster as the bees attack Pooh who is helplessly suspended as their target. Pooh philosophically concludes that these were "the wrong sort of bees".

The wrong sort of equilibrium

- Economics has had a similar experience with equilibrium theories in the aftermath of the financial crisis of 2007-8 with equilibrium theory.

- As a result many people, including myself in some incarnations, have concluded that it would be as well to avoid equilibrium concepts altogether in modeling economic outcomes.

- There are, of course, innumerable ways not to do something, so this resolution can lead in various directions, including the exploration of non-linear dynamical models, agent-based modeling, and other explicitly disequilibrium modeling methods (almost all Markov models in one form or another).

Sorts of equilibrium

- There are, however, other conceptions of equilibrium than the price-taking Walrasian general equilibrium model that has provided the theoretical framework for most economists' education.

- The prestige of the fundamental existence and welfare theorems that we continue to teach as "economic thinking" has led us to identify "equilibrium" with "Walrasian general equilibrium", and, as a result, to conclude that the only alternative to price-taking Walrasian equilibrium modeling is one or another framework of disequilibrium dynamics.

- I'd like to argue that this is a case of the "wrong sort of equilibrium", and that we can make more progress applying non-Walrasian equilibrium methods, which have a stellar track record in other fields of intellectual inquiry, including statistical physics, to economic problems.

Information theory

- At this level I am far from without company, and an important part of my thinking has been shaped by the "post-Walrasian" school of economics.

- I will argue that post-Walrasian theory can realize itself completely only by incorporating key elements of information theory, in particular the principles of entropy maximization that in E. T. Jaynes' formulation (Jaynes, 2003) provide the framework for theories of statistical equilibrium and equilibrium fluctuations.

Constraints and behavior

- Statistical equilibrium models can be formulated in terms of maximizing the entropy (unexplained fluctuations) of complex systems subject to constraints imposed by physical laws, or, in the case of social phenomenon, social institutions and purposive human behavior.

- Except in the unattainable limit of zero entropy (corresponding in physical systems to a temperature of absolute zero) the resulting theories provide both a model of central tendencies (equilibrium in the traditional economic sense) and endogenous fluctuations.

- When this method works, it provides parsimonious and tractable theories of complex system behavior, in which the forces of equilibrium are explicitly modeled.

Equilibrium in political economy

- The power of equilibrium methods in social theory has a long history.

- In political economy Adam Smith's analyses of the equalization of profit rates is a leading example.

- It is striking that John Maynard Keynes' macroeconomic thinking had much greater influence when it was couched in equilibrium terms in the *General Theory* than in the disequilibrium framework of the *Treatise on Money* (Keynes, 1936, 1930).

- The simplifications of equilibrium thinking seem to be particularly appropriate as a framework for abstraction in the context of human social interactions, which feature multiple causal pathways at multiple levels that give rise to intricate feedback loops, self-fulfilling prophecies, and other unavoidable and familiar social phenomena that pose difficulties for dynamical systems.

2 What I learned from Sam Bowles

The Bowles program

- In the first chapter of his *Microeconomics*, Bowles lays out a strong methodological program for explanation of human social phenomena.

- His idea is to represent the outcome of human interaction as the Nash-Cournot equilibrium of a game, in which the actions of the players are constrained by particular institutions and motivated by the expected payoffs they associate with various outcomes.

Bargaining as an example of Bowles' program

- For example, in the classic Edgeworth-box problem of bilateral bargaining to distribute some bundle of resources between two individuals with given private preferences over the bundles they will wind up with, the outcomes are the feasible allocations of the total resources between the individuals, and their expected payoffs can be represented by ordinal utility functions representing their preferences.

- Various social institutions, such as first-mover ability to make take-it-or-leave-it or price-setting offers might structure this type of interaction in different contexts.

The problem of multiplicity of equilibria

- One of the limits of Bowles' program is that in many cases, even when the social institutions governing a social interaction are well-defined, there are multiple Nash-Cournot equilibria in the resulting game, and the explanatory power of the program is reduced.

- The explanatory power of Bowles' program is enhanced if we also require that the equilibria proposed as explaining phenomena are stable in some robust and demonstrable sense.

Social coordination problems

- Bowles emphasizes the important observation that Nash-Cournot equilibria, because each player responds by choosing her action assuming the other player's (or players') actions are given, in general can deviate from the set of socially coordinated (Pareto-efficient) outcomes some player might choose if given control of the other player's actions.

- The Prisoners' Dilemma represent one class of such social coordination problems, in which there is a unique, stable, but socially-uncoordinated Nash-Cournot equilibrium.

- Another important class are institutions that give rise to Assurance Games in which there are multiple stable Nash-Cournot equilibria, only one of which is socially coordinated.

Other suspects

- Bowles shares the blame or credit for this line of thinking with a distinguished company, including his co-author Herb Gintis and Peyton Young, Rajiv Sethi, Peter Diamond and his co-authors, Martin Weitzman, and Cooper and John who have explored the impact of strategic complementarity on aggregate outcomes, Martin Shubik, Ross Starr, Ali Khan, and others who have insisted on the necessity of modeling equilibrium in monetary economies as Nash-Cournot equilibria of well-defined games, Joseph Stiglitz, George Akerlof and Michael Spence, Brian Arthur, and others. The thoroughgoing application of these fundamental ideas to macroeconomic problems remains a piece of unfinished business for economic theorists.

The social interaction model

- There is a subset of symmetrical models for which Bowles' program is particularly well-suited, the *social interaction model*.

- In this setting the two or more participants each choose some action variable, x, which may be subject to constraints, $x_{\text{lo}} \leq x \leq x_{\text{hi}}$ with payoffs, identical for all the participants, that depend on the (typical) individual's action and the mean (or some other statistic) of the other participants' actions, $\bar{x}$, $u[x,\bar{x}]$.

- The best response of the typical individual is $x[\bar{x}] = \arg \max_{x_{\text{lo}} \leq x \leq x_{\text{hi}}} u[x,\bar{x}]$. The symmetrical Nash-Cournot equilibria, x^e satisfy the condition $x[x^e] = x^e$. The socially coordinated outcome is $x^{sc} = \arg \max_{x_{\text{lo}} \leq x \leq x_{\text{hi}}} u[x,x]$.

Keynes' cross as a social interaction model

- As Figure 1 illustrates, in the case of weak strategic complementarity the (once) well-known equilibrium analysis of aggregate demand Keynes and Richard Kahn developed and presented in Keynes' *General Theory* is an example of the social interaction model.

- The identical agents are firms that generate household wage and profit incomes by producing and selling commodities.

- The typical firm chooses its level of production (and hence income generation, x) depending on the average level of incomes generated by all the other firms in the economy, $\bar{x}$.

Keynes' cross

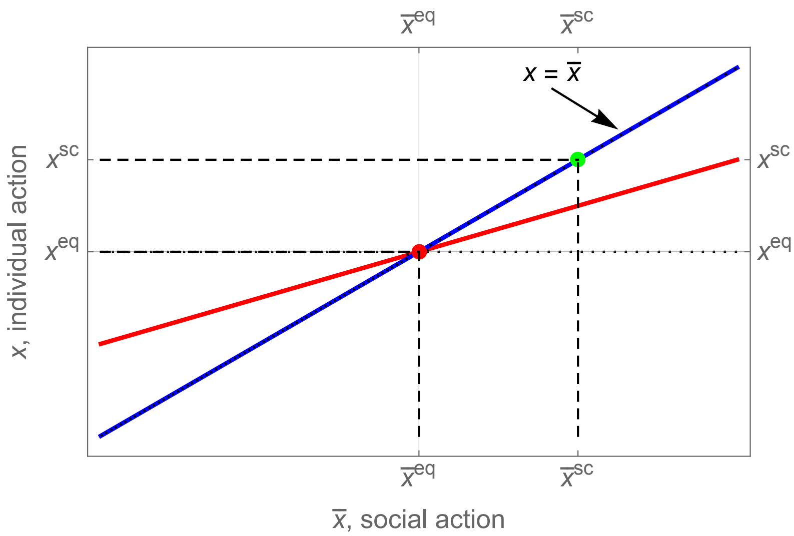

Figure 1: The average level of firm production is on the horizontal axis, and the best-response typical firm reaction on the vertical axis. The equilibrium level of output falls short of the socially coordinated level, or, in Keynes language, full employment.

Keynes' cross and weak strategic complementarity

- Households are assumed to have a positive marginal propensity to spend out of income less than unity, leading to an upward-sloped best-response schedule, $x[\bar{x}]$, with $0 < x' < 1$ indicating weak strategic complementarity of firm spending.

- A positive equilibrium level of aggregate demand and income is sustained by the typical firm's commitment to an autonomous level of investment expenditure that does not vary (or vary as much) with demand as household spending.

- The social coordination problem is due to the dependence of the typical household/firm's spending on the spending of all the others.

Social interaction as micro-foundations

- This interpretation of "Keynes' cross" shows how the social interaction model can derive "macroeconomic" outcomes from a genuinely "microeconomic" model.

- The microeconomics of the social interaction model are parsimoniously summarized in the best response of the typical participant, which determines the macroeconomic equilibrium outcome.

- The relation of the macroeconomic equilibrium to changes in behavior represented by shifts of the best-response schedule can be counter-intuitive and highly non-linear, representing the effects of "fallacies of composition" and the underlying social coordination problem being modeled.

Typical agents and representative agents

- The concept of the *typical agent* in the social interaction model from the concept of a *representative agent*.

- The typical agent acts taking the actions of the other agents as given, leading to a socially uncoordinated second-best outcome.

- The representative agent internalizes all of the interactions in the system, leading to a socially coordinated first-best outcome,

In Keynes' words

Keynes remarks in chapter 3 of *GT:*

> The classical doctrine ... which used to be expressed categorically in the statement that 'Supply creates its own Demand' and continues to underlie all orthodox economic theory, involves a special assumption as to the relationship between these two functions ... that $f(N)$ and $\phi(N)$ are equal ... for all levels of output and employment .... That is to say, effective demand, instead of having a unique equilibrium value, is an infinite range of values all equally admissible; and the amount of employment is indeterminate except in so far as the marginal disutility of labour sets an upper limit. If this were true, competition between entrepreneurs would always lead to an expansion of employment up to the point ... where a further increase in the value of the effective demand will no longer be accompanied by any increase in output. Evidently this amounts to the same thing as full employment.

Keynes and the price-taking Walrasian model

- As Keynes describes it, the "classical" or "Say's Law" view he is critiquing is not the simple "invisible hand" claim that in price-taking Walrasian competitive equilibrium all available resources will be fully employed (or have a zero price).

- In the social interaction model the invisible hand corresponds to the knife-edge case where the best response of the typical participant does not depend on the actions of the other players at all.

- Keynes, however, argues that the classical Say's Law view is based on the quite different claim that whatever the level of production the other firms choose, the typical firm's best response will be the *same* level of production, so that there is a continuum of equilibrium levels of production.

Indeterminacy and scale-free fluctuations

- Keynes correctly characterizes this (also knife-edge) situation as one in which the equilibrium level of production is *indeterminate*.

- From the analogy with similar phase-changes in physical systems, we would expect in fact to see fluctuations of production at all scales in this indeterminate case.

- Keynes, on the other hand, seems to believe that weak forces of competition will lead the equilibrium level of production to drift toward full employment.

Keynes and the Bowles program

- From the point of view of the Bowles program, Keynes' theory involves two social coordination problems.

- First, in Keynes' thinking output and labor markets the institution of seller price-setting leads to a price-setting equilibrium with excess capacity, or involuntary unemployment.

- Second, the institutions of monetary exchange imply that when the typical buyer spends money, she relieves the liquidity constraint of other buyers, creating a social coordination problem in the level of spending, effective demand.

3 Stability and dynamics in the social interaction model

Dynamics and the social interaction model

- The "Bowles program" calls for explanation of social phenomena through embedding the individual behavior that supports the phenomenon in an institutionally appropriate game and exhibiting the observed outcome as a *stable* Nash-Cournot equilibrium.

- The Achilles' heel of this program is the difficulty of demonstrating the highly intuitive requirement of stability.

- It is one of the bedrock principles of reasoning in other branches of science that we are extremely unlikely to see systems anywhere but in or near stable equilibrium states.

- Game theoretic reasoning as an explanatory theory in social sciences constantly runs into the limitation that Nash's version of Nash-Cournot equilibrium is not based on explicit dynamics, and, as a result, the set of Nash equilibria of typical social interactions can be large and include many irrelevant unstable outcomes.

Stability with one dimension of social interaction

- In the one-dimensional social interaction model, the stability issue is much more tractable at an intuitive level, but for reasons that raise troubling questions for the more general program of basing social explanations on explicit system dynamics.

- In the case illustrated in Figure 1, for example, it is very difficult to avoid the conclusion that the stability of an equilibrium depends on whether the best-response schedule cuts the equilibrium 45° line from above or below.

- If we imagine the average level of the action on the horizontal axis to be perturbed upward or downward from the equilibrium, when the typical agent best response cuts the equilibrium condition from above, the model implies universal pressures to restore equilibrium through the adjustment of individual actions.

When explicit dynamics are no help

- In the one-dimensional social interaction model, then, there seems to be almost no role for explicit dynamics in explanation.
- *Any* dynamics that lead the typical agent to track her best response to fluctuations in aggregate outcomes will stabilize the equilibrium.
- From a methodological point of view the researcher may be much better off precisely because there is no need to specify some particular dynamical response to support the conclusions of the model.
- To put the issue another way, the stable interior social interaction equilibrium has explanatory power because it represents a very large class of dynamical models, all of which will have the same qualitative implications, namely, the strong tendency for the system to return to the neighborhood of the equilibrium.

Empirics

- Furthermore, from an empirical point of view it may be very difficult to identify social interaction dynamics near stable interior equilibria, because the strong equilibrating forces represented by the best response of the typical participant mean that there will be very little variation in the aggregate outcome variable on which to base statistical inferences about dynamics.
- The main thing we can learn about a one-dimensional social interaction model is that it will stay close to a stable interior equilibrium.
- In the social interaction context Samuelson's Correspondence Principle is much more convincing than in supply-demand models.

Unstable interior equilibrium

- It is instructive to consider what happens in the social interaction model when the interior equilibrium is unstable because strategic complementarity is strong, and the typical individual best-response schedule cuts the equilibrium 45° line from below.
- In these cases limits on the typical individuals best response play a critical role.
- Figure 2 illustrates the unstable case where there are three equilibria, two stable equilibria at the limits of the typical individual action, x_{lo} and x_{hi}, and one interior unstable equilibrium that divides the basins of attraction of the two stable equilibria.
- This is an Assurance Game: one of the stable equilibria is the socially coordinated outcome (the "good equilibrium") and the other ("bad equilibrium") is payoff-inferior (Pareto-inferior) for the typical agent.

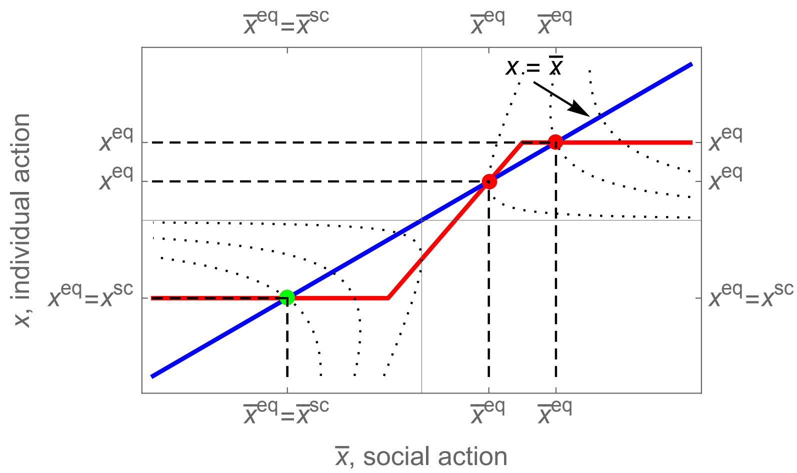

Figure 2: When strategic complementarity is strong the slope of the best-response schedule is greater than 1, and the social interaction model with limits on the individual action generically has three equilibria, two stable equilibria at the limiting actions and one unstable interior equilibrium that is the boundary of their basins of attraction. The socially coordinated outcome coincides with one of the stable equilibria.

Strong strategic complementarity

Dynamics of assurance games

- The dynamics of the social interaction model with strong strategic complementarity are well-known from the analyses of network externalities.
- We are almost certain to observe the system at or close to one of the stable equilibria, where everyone is choosing the action or no one is.
- *Which* stable equilibrium we will see, however, is *path-dependent*, because it depends on the initial conditions or fluctuations that set the system on a path toward one or another of the stable equilibria.

Social conventions

- Once the system is close to one of the stable equilibria, the stabilizing forces are very strong, typically much stronger than at a stable interior equilibrium of the system.
- The typical participant faces enormous payoff penalties for deviating from equilibrium behavior.
- This is the territory of *social convention* (such as the use of money in exchange) in social theory, patterns of behavior that are strongly self-reinforcing and can be challenged only through some form of (usually political) coordinated action.

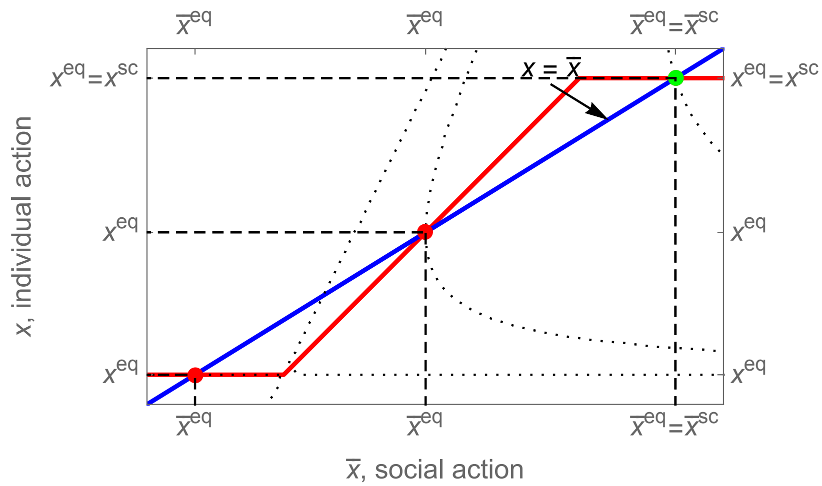

Figure 3: The cusp catastrophe. The basin of attraction of a stable equilibrium shrinks due to drift of the typical agent best response.

- Just as in the case of the stable interior equilibrium it may be very difficult to recover the detailed dynamics of the best-response function of the typical agent precisely because there is so little variation in the aggregate outcome.

Empirics of strong strategic complementarity

- The social interaction model with strong strategic complementarity also poses serious problems for econometric investigations.

- Consider, for example, the possibility that drift in the parameters of the typical agent's best-response function can drag the best-response schedule down far enough to eliminate the high-level equilibrium altogether in a "cusp catastrophe" where the unstable interior equilibrium collides with and annihilates the stable extreme equilibrium. It is clear enough that the system will "relax" (or "collapse") to the remaining low-level stable equilibrium.

- But this process (analogous to the "irreversible" transformations of physical thermodynamic systems) is likely to be chaotic, short-lived, and very difficult to reconstruct econometrically.

The cusp catastrophe

Rational expectations

- The recurrence of events of this type makes the idea of "rational expectations" highly suspect for macroeconomic behavior, as Keynes seems to have concluded at an early stage from his investigation of the fundamental principles of statistical inference.

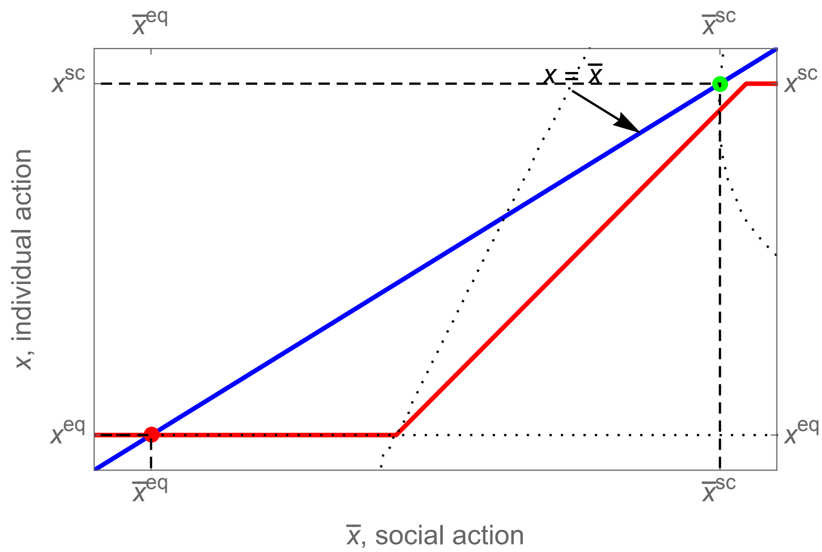

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Figure 4: The cusp catastrophe. After the shift in the best response the system moves irreversibly and chaotically to the remaining stationary equilibrium.

- Because macroeconomic outcomes consist of substantial periods in which the system remains close to a stable equilibrium and as a result does not reveal much dynamic structure, punctuated by crises that are hard to track in data and not replicable enough to provide much leverage for statistical inference, the project of reconstructing a "rational expectations" model of macroeconomies statistically faces enormous obstacle.

4 Statistical fluctuations

Fluctuations

- Figure 2 raises another fundamental modeling question.

- If the typical individual is an expected payoff maximizer, and the action under consideration is a quantal decision, $x \in \{0,1\}$, the typical individual best response will be a mixed strategy described by a frequency $p[\bar{p}]$, $0 \leq p \leq 1$, with which the individual chooses the action $x = 1$, which will in general depend on the frequency, $\bar{p}$, according to which the other participants choose $x = 1$.

- The typical individual's expected payoff can be written $pu[1,\bar{p}]+(1-p)u[0,\bar{p}]$, and if $u[1,\bar{p}] \neq u[0,\bar{p}]$, the expected payoff maximizing mixed strategy will be either $p[\bar{p}] = 1$ or $p[\bar{p}] = 0$.

- With strategic complementarity, $p'[\bar{p}] > 0$, and the typical individual's best-response will be an upward step function, which will determine the location of the unstable equilibrium, with two stable equilibria at $p^e = 0$, $p^e = 1$.

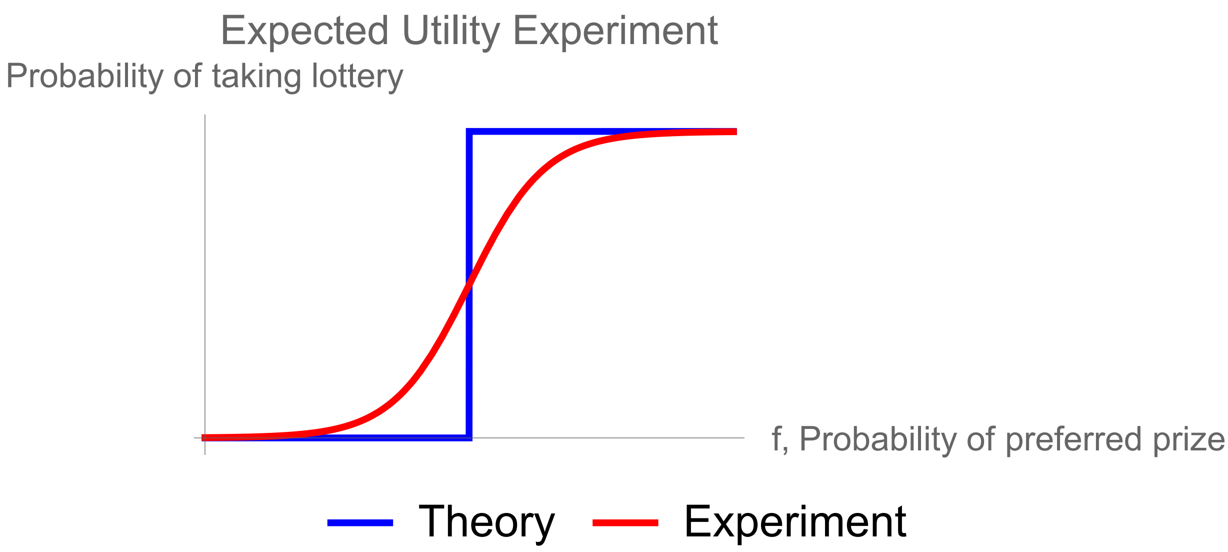

Figure 5: Quantal response approximates a step function

Quantal responses

- As the study of quantal response behavior going back to Duncan Luce and even the Weber-Fechner Law has established empirically, individual behavior can only approximate a step function.

- A very typical empirical outcome is a *logistic quantal response* of the form

$$p[\bar{p}] = \frac{\exp[\frac{u[1,\bar{p}]}{T}]}{\exp[\frac{u[1,\bar{p}]}{T}] + \exp[\frac{u[0,\bar{p}]}{T}]} \tag{1}$$

where the parameter T determines the steepness of the quantal response, or, equivalently, its closeness to a step function (which is the limit of the logistic quantal response as $T \to 0$).

Quantal response approximates a step function

Information constraints and quantal responses

- There are various models from which the logistic quantal response can be derived.

- One parsimonious derivation, which Christopher Sims (Sims, 2005) has explored, arises from maximizing expected payoff subject to a constraint on the informational entropy in Shannon's sense of the frequency distribution describing the mixed strategy.

- The parameter T in the logistic quantal response arises directly in this derivation as the Lagrange multiplier on the entropy constraint.

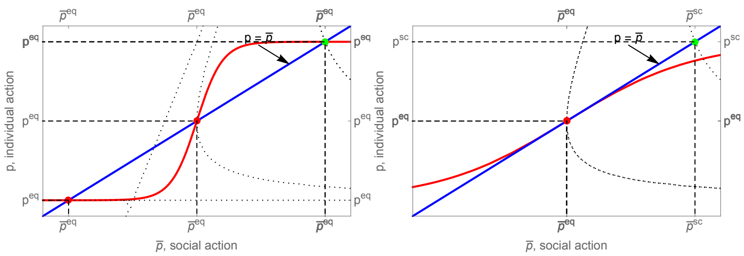

Figure 6: When the typical agent in a social interaction model has a positive behavior temperature, her best response is a logistic quantal response curve. When the behavior temperature is low relative to the difference in payoffs, as in the first panel, there be two stable extreme equilibria and one unstable interior equilibrium at the boundary of their basins of attraction. In contrast to the case of zero temperature, the extreme equilibria have a large proportion, but not all, of the agents choosing the same action. When the decision temperature is high, as in the second panel, there can be a unique interior stable equilibrium, but the actions of individual agents in equilibrium will fluctuate endogenously.

- This derivation is an example of the general approach of maximizing entropy subject to a constraint (in this case on expected payoff) originated by Gibbs and developed by E. T. Jaynes.

Constraints and maximization

- Note that maximizing entropy subject to a constraint on expected payoff is mathematically equivalent to maximizing expected payoff subject to an entropy constraint, because both problems have the same Lagrangian.

- From this point of view the logistic quantal response is one example of the broad class of Gibbs functions familiar to statistical physicists, and it is appropriate to regard the parameter T as expressing the "behavior temperature" of the quantal responder.

Social interaction with quantal response

- If the typical individual in a social interaction model has a positive behavior temperature, her best-response schedule will be a logistic quantal response function.

- With strategic complementarity at low behavior temperatures there will again be two stable extreme equilibria and one unstable interior equilibrium (or at higher temperatures for which the quantal response is flatter, a single stable equilibrium), as Figure 6 illustrates.

The effect of behavior temperature on social interaction equilibrium

Equilibrium fluctuations in social interactions

- Two important modifications of the social interaction model follow from the introduction of a logistic quantal best response.

- First, in equilibrium the behavior of the typical agent is described by the equilibrium frequency, and as a result fluctuates "endogenously".

- We expect in empirical data to observe typical agents' behavior distributed around some central tendency (not necessarily symmetrically, depending on the constraints imposed by the institutions of the interaction).

- Second, with strong strategic complementarity (or, equivalently, a low behavior temperature) at the extreme stable equilibria some fraction of the population deviates from the equilibrium norm. The pressures toward conformity to the social norm are still very strong, but are limited by the entropy inherent in individual behavior.

Heterogeneity and quantal response

- The introduction of logistic quantal response in the social interaction model addresses to some extent the limitations of the model imposed by the assumption that all the participants in the interaction are identical.

- Even if all the participants are identical in the sense of having the same payoff function and behavior temperature, if their behavior temperature is not the unattainable extreme of absolute zero, their observed behavior will not be uniform even when the system is in equilibrium.

5 Walras one more time

Expectations

- The example of the social interaction model interpretation of Keynes' macroeconomics underlines the ambiguous role of the concept of "expectations" in economic theory.

- Basically the question is whether we are talking about the expectations the typical individual has of *other individuals' collective action*, or about the expectations the typical individual ("representative agent") has of the unfolding of the paths of concrete variables such as prices of assets and commodities, incomes, and output in real historical time.

Varieties of expectational equilibrium

- This ambiguity has troubled economic theorizing throughout the twentieth century.

- Nash's concept of equilibrium (following Cournot) is firmly located in terms of fulfilled expectations as to the behavior of other participants in the system.

- Walrasian intertemporal equilibrium and its rational expectations variant, on the other hand, attempt to define equilibrium in terms of the correctness of the typical individual's expectations about the unfolding of prices, incomes, and output.

- One way to understand the claims of general equilibrium theory and the fundamental welfare theorems is that they conjecture that complete information, provided by complete markets, can serve as a coordinating device to overcome the social coordination dilemmas posed by decentralized economic decision making.

Supply and demand and the Bowles program

- The difference in point of view, however, goes deeper than this dichotomy suggests.

- The problem is that the Walrasian supply-demand conception of equilibrium does not successfully meet the requirements of Bowles' program for the explanation of social phenomena.

- Basically this is because, as many people have pointed out over the years, the Walrasian conception cannot explain the determination of prices.

Are supply and demand curves genuine best responses?

- This is slippery territory, because we tend to teach supply-demand equilibrium as if the supply and demand curves were typical agent (or aggregated) best responses.

- But, as Walras himself recognized in invoking the figure of the auctioneer, supply and demand curves do not represent best responses in the Bowles sense of responses to other participants in a well-defined institutional context.

- "Competitive" supply and demand schedules represent the best response of a typical seller or buyer to the budget constraint determined by a system of prices called out by an auctioneer, without explaining just who it is in the real economy who sets these prices.

Price-taking and price-making

- From the social interaction point of view it is not unreasonable to think that some market participants, say, buyers, view prices as beyond their control and take them as given because they are determined by the actions of other participants.

- But Walrasian supply-demand analysis seems to make this assumption about *all* the participants in decentralized markets, leaving the question of a theory of the determination of prices hanging.

- It is tempting to think that the intersection of the supply and demand curves in a market can be interpreted as a Nash-Cournot equilibrium intersection of best-response schedules, but the game giving rise to these best responses remains elusive and in the end impossible to pin down. In particular, Walrasian theory cannot explain how actual decision makers set prices.

The fundamental welfare theorems

- Price-taking equilibrium prices, as Pareto, Barone, and a long line of successors have realized, contain an enormous amount of information in an economy with a convex production technology.

- If this information were available as a public good, it would greatly help the participants in a commodity-producing economy where production is organized through decentralized exchange to realize their potential economic surpluses.

- The first welfare theorem says just this: if price-taking equilibrium prices were known to households and firms they could achieve a Pareto-efficient outcome. The problem is that everyone cannot be a price-taker in a genuinely decentralized economy.

Cournot competition

- Auguste Cournot's analysis of competition established the idea that equilibrium requires self-fulfilling expectations about the behavior of other participants (in Cournot's case, competing firms).

- Unfortunately Cournot's approach to this problem supposes that the demand schedule for the industry is given.

- Cournot can explain why competition among a finite number of firms will realize only a fraction of the potential surplus available to a monopolist, but does not address the question of who sets prices in the industry.

- Like Walras, Cournot seems to assume some black-box process by which the industry will find itself on the industry demand schedule at the equilibrium level of output.

Bertrand competition and entropy constraints

- Joseph Bertrand argued that a Cournot-type equilibrium among even a small number of price-setting firms implies equilibrium price equal to marginal cost and thereby reproduces the conditions of price-taking equilibrium.

- Bertrand's conclusion depends crucially on the implicit assumption that the buyers in the market are operating at a behavior temperature of absolute zero.

- These buyers shift (according to a step function) all of their purchases to the seller with the lowest price.

- As I have argued above, this type of reasoning is inherently flawed on informational entropic grounds. If the buyers in a market are operating at a positive behavior temperature, no matter how low, their response to price differentials among sellers will follow a logistic quantal response law.

Entropy-constrained Bertrand equilibrium

- A seller deviating from competitors' prices will not lose or gain the whole market, and as a result the Nash-Cournot equilibrium price with Bertrand-like price competition will generally exceed marginal cost.

- From the point of view of Keynes' analysis, this type of equilibrium also implies that the typical firm operates with excess capacity in equilibrium, or as Keynes phrased it, at an output level below full employment.

Models of competitive equilibrium and macroeconomics

- The financial crisis of 2007-8 convinced many people that there was something fundamentally wrong with the real business cycle research program that explains macroeconomic fluctuations as the result of exogenous shocks to a price-taking Walrasian economy.

- Keynes would have agreed, reasoning as he did on the basis of second-best models in which equilibrium is based on anticipations of other participants actions, not on unbiased forecasts of paths of prices and output.

- The reunification of macroeconomics and microeconomics depends on shaking off the flawed conceptions of the first welfare theorem and analyzing economic interactions as stable Nash-Cournot equilibria of socially uncoordinated interactions.

6 Constrained maximum entropy

Constraints in complex system modeling

- Scientific investigation has confronted the problem of large state and hypothesis spaces in many contexts, most famously in Maxwell, Boltzmann and Gibbs' investigations of the statistical regularities of thermodynamic systems.

- In retrospect three aspects of these enormously successful statistical theories seem critical for economics and social science in general: the framing of the scientific problem in terms of an *ensemble* specifying a frequency distribution over possible states of a system; the formulation of the relevant substantive theoretical understanding of the system in terms of *constraints* on the relevant moments of the ensemble; and the realization that the residual fluctuations of the system possible within constraints maximized the *entropy* of the ensemble subject to the constraints.

Statistical mechanics of gases

- For example, in the analysis of a "perfect gas" consisting of a large number of weakly interacting particles, the ensemble assigns frequencies to each of the possible states of the gas describing the positions and momenta of its constituent particles; the key theoretical insight is the conservation of energy expressed as a constraint on the mean energy of the ensemble; and the statistical equilibrium exponential (Gibbs) distribution over possible states maximizes informational entropy subject to the mean energy constraint and the conservation of the number of particles.

- What we mean by explanation of observed features of the gas such as its pressure and temperature is the derivation of these properties from the underlying particle ensemble.

The Jaynes program

- E. T. Jaynes was particularly inventive in generalizing this paradigmatic example to a general method of scientific investigation and discovery.

- Jaynes argues that the high road of scientific discovery lies in maximizing residual entropy (the unexplained fluctuations of a system) subject to relevant constraints imposed by our knowledge of its structure.

The Wolf dice parable

- Jaynes illustrates this procedure (Jaynes, 1978) in his entertaining analysis of data generated by the nineteenth-century researcher Wolf, who, believing that he could "test" the laws of probability empirically, rolled a pair of dice thousands of times and counted the resulting outcomes in the number of visible spots.

- Jaynes' first model maximizes entropy with no constraints other than the total number of spots on the dice faces, which intuitively leads to a mean of 3.5 spots, far lower than Wolf's measured 3.5983, an outcome astronomically improbable on the basis of purely entropic fluctuations.

- For Jaynes this discrepancy does not call into question the maximum entropy method, but indicates that further constraints are necessary to explain the data.

Jaynes' inferences about Wolf's dice

- Jaynes investigates three further constraints in order: the first representing the likely asymmetry resulting from cutting wooden dice with crude tools; the second taking account of the weight imbalance created in the dice by the removal of mass in gouging the spots; and the third (barely necessary to maximize entropy) attributable to a chip on the corner of one of the dice.

- Once these constraints are introduced, the maximum entropy frequency distribution explains the data to a very high degree of accuracy.

- The introduction of each constraint, of course, reduces the maximum entropy and brings the model closer to the data.

Expected payoff maximization in social models

- One fundamental difference between physical systems and social systems is the forward-looking purposefulness in human behavior.

- One way to introduce this concept mathematically into a model is the assumption that the relevant agent responds to incentives that can be quantified as payoffs, and chooses a mixed strategy frequency of responses that maximizes expected payoff.

- As I have pointed out, the introduction of a constraint on the entropy of the mixed strategy introduces a parameter, the Lagrange multiplier on the entropy constraint, that represents a behavior temperature.

Expected payoff maximization as a constraint

- The Lagrangian can be written $E_p[u] + TH[p]$ where u is the payoff to each action, p is the frequency distribution describing the mixed strategy over available actions, $E_p[u]$ is the expected payoff with frequency p, T is the behavior temperature (Lagrange multiplier on entropy) and $H[p]$ is the informational entropy of the frequency distribution. (By analogy with the concept of "free energy" in statistical physics the Lagrangian is sometimes called the "free payoff".)

- A straightforward, but crucial, insight is that this is the *same* Lagrangian as would arise from the program of maximizing entropy subject to a constraint on the expected payoff.

Expected payoff maximization as an explanatory concept

- If we consider this from Jaynes' perspective, we see that the assumption that agents in a system maximize expected payoff (for example, through their entropy-constrained best response to the institutions through which they are interacting) is mathematically equivalent to the addition of a constraint.

- In social science contexts the purposeful behavior of agents (the pursuit of a higher profit rate in Adam Smith's discussion of capitalist competition, for example) is often the substance of a theory.

- In terms of Bowles' program, the best responses conditioned by institutions governing interactions are indeed the content of theoretical explanations of outcomes.

The Jaynes program for social science

- Requiring agents to be best responders in particular institutional situations (in macroeconomics, for example, as spenders and recipients of income in a monetary economy) formulates theoretical insights in the form of constraints.

- Maximizing entropy subject to feasibility and best-response constraints according to Jaynes' reasoning expresses in a parsimonious form all of the information we have about a system.

- If the resulting model fits the data badly, Jaynes recommends investigating further constraints.

Ensemble reasoning in economics

- The impulse toward modeling complex economic systems as ensembles surfaces in much economic research.

- The idea in general is to express the residual variation of the system as an expression of "random" shocks.

- The critical contribution of statistical equilibrium thinking is the discovery that "random" means maximum entropy, which does not always correspond to some intuitive ideas of randomness such as "independent identically distributed Gaussian disturbances".

- For example, the maximum entropy distribution over the whole real line from negative to positive infinity subject to a constraint on the mean and standard deviation is indeed the Gaussian normal distribution, but the maximum entropy distribution with a mean constraint over the nonnegative reals is an asymmetric exponential distribution.

Statistical theories of fluctuations

- One analytical advantage of statistical equilibrium methods based on maximizing entropy subject to constraints is that the resulting models systematically predict both the central tendencies of systems and the statistical patterns of fluctuations around those central tendencies.

- In this sense statistical equilibrium is an alternative to dynamic modeling of systems, in that it integrates the information we have about fluctuations into an explanatory framework without resorting to explicit assumptions on the underlying dynamics.

- For example, once we introduce entropic considerations into the social interaction models, the model-predicted outcomes will be heterogeneous across agents (though obedient to definite statistical laws) even when we assume homogeneity of the agents *ex ante* in terms of their payoffs.

7 Agent-based modeling

Agent-based models

- The intractability of models that attempt to be faithful to the economy as a complex system together with the enormous increases in computing power available to researchers has led to a strong interest in micro-simulation approaches to economic modeling.

- The most popular of these approaches is "agent-based modeling", which represents economic interactions through object-oriented computer programs that can be interpreted as recording the outcomes of the interaction of individual agents responding to a complex social environment.

- Doyne Farmer and I published a kind of manifesto calling for agent-based modeling approaches to macroeconomic and financial economic problems (Farmer and Foley, 2009). In the years since the appearance of this piece, I have had some major second thoughts on this question.

What are agent-based models?

- Before getting into the second thoughts, let me re-iterate some first thoughts that remain salient.

- Agent-based models (ABMs) mathematically are Markov models that implicitly define transition probabilities among system states.

- From a mathematical point of view ABM simulation is no more and no less than an alternative method to solve a dynamic model of complex system interactions.

Methodological problems of the ABM research program

- As a general research program for modeling macroeconomic fluctuations, however, ABM poses some knotty and unresolved methodological problems due to its freedom from constraints.

- Even in modeling the behavior of relatively small sub-systems of economies, such as default contagion among financial institutions, the number of plausible ABM representations of the interactions is bewilderingly large and hard to discipline.

- Because the implications of ABM models for policy and explanation can depend sensitively on specific modeling assumptions, this range of possible specifications weakens the credibility of any particular model.

Disciplining ABM research

- The main lesson from this observation is the desirability in ABM research of combining other forms of analysis with simulation.

- For example, it is helpful to construct an ABM so that it contains some simple tractable model as a special case (perhaps when all of the agents are parametrically identical) so that it is possible to understand qualitative properties of model paths through closed-form solutions of the special case.

- ABMs have the additional advantage, which Farmer and I highlighted in our remarks, of allowing researchers a much wider choice of behavioral assumptions than closed-form modeling because of the flexibility of the simulation method.

ABM simulation of conventional models

- One remedy for this methodological vulnerability of ABM methods is, ironically, to use ABMs to reproduce the outcomes of simpler, analytically tractable models, preferably some version of the social interaction model rather than of Walrasian intertemporal general equilibrium.

- ABM methods have had success in modeling problems in which social interaction effects are central, such as a theatre audience fleeing a fire or terrorist attack, and the control of contagious diseases by quarantine and vaccination policies.

- The problem of macroeconomic fluctuations shares many characteristics with this class of problems, so that ABM methods can make a significant contribution to understanding them.

8 Disequilibrium modeling

Equilibrium and explicit dynamics

- Mathematically sophisticated economists often adopt the methodological position that explicit dynamic analysis is in principle the correct path of theoretical investigation.

- If we correctly specified the dynamics governing the state of the economy (including its constituent participant firms, households, and other entities) in principle we could explain any observed phenomenon and make whatever predictions that are possible within limits of our ability to measure the state of the system.

- This methodologically dynamic point of view views equilibria as stationary points of the dynamics (and a fortiori addresses the question of the stability of equilibrium points), but de-emphasizes the importance of equilibrium states as being in the end just one subset of states of the system.

Complex systems and dynamics

- These considerations make good sense.

- The limitations of a rigorous dynamical systems approach to modeling economic interactions lie in considerations of complexity.

- Take the social interaction model with a binary action $x \in \{0,1\}$ and N participants, for example. The number of states of the system, $s \in \{0,1\}^N$ already is 2^N, and this is a very simplified case.

- In principle we can write the dynamics of the system (in discrete time to simplify matters) as an N-dimensional map $f : \{0,1\}^N \to \{0,1\}^N$.

- There are $2^{N^{2^N}}$ maps of this type, which is an awful lot of maps once N gets to orders of magnitude relevant to a large economy.

Strategies for simplifying dynamical systems

- With the shade of William of Occam hovering over our computers we might resort to various considerations of symmetry or topology (graph theory, for example, leading to cellular automata or related models) to get a handle on choosing a class of models to study in detail.

- A little experience with this kind of work predisposes the researcher to favor simplifying, dimension-reducing assumptions.

Representative agent models

- In many cases simplifying assumptions are successful in rendering model analysis tractable because they have lower explanatory power.

- For example, the "representative agent" version of the Walrasian intertemporal general equilibrium theory greatly simplifies the inherently difficult problem of computing general equilibrium prices.

- The representative agent differs sharply from the typical agent in the social interaction model because the representative agent internalizes the social coordination problems inherent in decentralized social interaction.

The explanatory poverty of representative agents

- The set of equilibria of the Walrasian system reduce to the set of outcomes (paths for output and consumption and their associated dual prices) that are optimizing for the representative agent, and thereby solve the social coordination problem in the simplified case of identical agents.

- The issue here goes beyond the "realism" of assuming a system of identical interacting agents.

- Even if agents are effectively identical in a decentralized economy they have to cope with social coordination problems.

- The explanatory weakness of representative agent models arises from their assumption that the representative agent internalizes the externalities that lead to social coordination problems.

The explanatory power of stable Nash-Cournot equilibria

- Constraining outcomes to be stable Nash-Cournot equilibria of interactions based on well-defined institutional structures as in the Bowles program, by contrast, can greatly increase the explanatory power of the theory.

- The statement of the conditions for Nash-Cournot equilibrium are simple enough, amounting to the requirement that outcomes reflect reciprocal best responses among all the participants (in general defining stability is more challenging), but the resulting set of constrained outcomes can reflect a mathematically subtle and explanatorily powerful interplay of non-linear feedbacks.

Methodological problems of the dynamic approach

- As I have argued above, in many contexts the empirical identification of disequilibrium dynamics is difficult because key variables in systems near stable equilibrium do not vary enough to provide statistical information, or because transitions due to disappearance of stable equilibria are catastrophic, irreversible, chaotic, and unrepeatable.

- In cases of this kind the discovery of stable Nash-Cournot equilibrium outcomes that will occur in a large class of specific dynamic models is an invaluable explanatory resource.

- The underlying problem is that there are many more ways for a system to be in disequilibrium than in a stable equilibrium.

9 Expectations: What is economic equilibrium?

Varieties of equilibrium

- Keynes' *General Theory* appeared at what in retrospect we see was a period of extreme flux in economic methodology, and in particular in a period where the concept of equilibrium was highly contested and undergoing rapid evolution.

- Within a few years of the appearance of the *General Theory* major advances in understanding different concepts of equilibrium appeared to clarify concepts that remain undifferentiated in Keynes' thought and language.

- For the purposes of this discussion we can distinguish, in hindsight, three conceptual strands of thinkng about equilibrium.

Best-response equilibrium

- The first concept of equilibrium is the Cournot concept of equilibrium as a state of the system where agents correctly anticipate the behavior of other agents.

- This is the concept formalized by Nash (but without successfully addressing the crucial issue of stability) that has gradually taken hold in the movement to reformulate economic reasoning in terms of non-cooperative game theory (what I have called the "Bowles program").

- There is good reason to regard Keynes as a precursor of this line of thinking, and to read the concept of "equilibrium" in the *General Theory* as referring to this concept.

Market-clearing equilibrium

- The second concept of equilibrium is the Walrasian notion of supply-demand or price-taking market-clearing equilibrium.

- This is the concept elaborated by Arrow and Debreu in their formal investigations of Walrasian general equilibrium theory.

Convexity assumptions and market-clearing equilibrium

- One important by-product of the Arrow-Debreu analysis is to highlight the critical role of convexity of preferences and technologies in assuring the continuity of excess demand functions and the existence of equilibrium price systems.

- Given the abundant evidence that real-world economies are based on a highly specialized division of labor supported by pervasive economies of scale, this analysis of continuity in retrospect is one of the weak points in general equilibrium theory.

- (This is not to deny the vigorous attempts of many researchers to fill these gaps with a rigorous account of possible large-number statistical mechanisms for smoothing of aggregate excess demand functions.)

Keynes and market-clearing equilibrium

- I read the chapters in which Keynes frames the *General Theory* as rejecting the relevance of supply-demand equilibrium to understanding real-world economic outcomes.

- Intertemporal market-clearing models attribute current economic decisions to agent expectations of the paths of prices, output, and consumption in concrete historical context.

- This is quite a different sense of expectations from the expectations agents have about other agents' behavior.

Statistical equilibrium

- The third concept of equilibrium is statistical equilibrium based on ensemble reasoning.

- There are a few hints of this line of thinking in the work of Edgeworth and Harold Hotelling, and some notion that economic laws depend on aggregation over large numbers pervades the theoretical economics literature of this transitional period, but no systematic introduction of information theory concepts into economic theory.

The right sort of equilibrium

- Much of the history of economics of my lifetime arises from the tangled intertwining of these concepts of equilibrium.

- The sad history of the attempt of "Keynesian economics" to reconcile Keynes' analysis with Walrasian general equilibrium is one main line of this story.

- Hicks' (1946) attempt in *Value and Capital* to map general equilibrium theory onto real economic experience suggested that any explanatory content of supply-demand theory must depend on a theory of expectations of the future path of prices.

- Unfortunately this line of thinking tangles economic analysis with the existential problem of the finiteness of human knowledge and its limitations precisely in foreseeing the future, leading to the dead end of rational expectations macroeconomics.

Equilibrium, after all

- The theme of these remarks, then, is first of all to affirm the immense scientific value of equilibrium insights for economics and other social sciences.

- The discovery of stable equilibria of systems greatly increases the explanatory leverage of theory.

- The problem as it appears to me is not with equilibrium reasoning, but the sort of equilibrium reasoning.

- In particular, an economics tied to the flaws and fallacies of price-taking Walrasian general equilibrium concepts is barking up the wrong tree and bound to encounter the wrong sort of bees.

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